How Much Is Too Much Debt for South Africa? A Threshold Nonlinear Autoregressive Distributive Lag (T-NARDL) Perspective

Andrew Phiri
Nelson Mandela University, South Africa
phiricandrew@gmail.com

Asanda Fotoyi
Nelson Mandela University, South Africa
Asanda.Fotoyi@mandela.ac.za

Since the global financial crisis, South African fiscal authorities have acquired debt at a faster rate compared to other economies, which, according to recent growth theory, implies that the economy has a lower debt tolerance or threshold level than previously thought. Our study presents country-specific debt threshold estimates for the South African economy based on reduced form regressions derived from an endogenous growth model of public debt that incorporates public investment as a channel through which debt can influence economic growth. We estimate the reduced form regressions using the threshold nonlinear autoregressive distributive lag (T-NARDL) cointegration model which we apply to quarterly time series spanning from 1960:q1 to 2020:q4. We identify debt thresholds of 47 percent which are much lower than those predicted by previous panel-based studies. Similarly, the corresponding public investment threshold estimate of 2.8 percent is lower than that prescribed by previous literature. However, our study shows that both public debt and public investment are too high to be growth enhancing and we provide policy recommendations based on these findings.

Key Words: optimal debt, optimal public investment, economic growth, endogenous growth model, threshold nonlinear autoregressive distributive lag (T-NARDL) model, South Africa
JEL Classification: C22, C51, E63, O47

https://doi.org/10.26493/1854-6935.21.93-120

Received: 22 June 2022 · Accepted: 7 August 2022
Published online: 30 June 2023

Introduction

The economic recession which emerged from the ongoing coronavirus pandemic has resurrected fears of a looming global debt crisis, which could assimilate in a similar fashion to how the 2009 global recession led to the 2010 European sovereign debt crisis. In response to the experienced sharp plunges in gross domestic product (GDP), primarily caused by the abrupt 'shutting down' of economies worldwide, many governments rolled out fiscal stimulus packages, partially financed by loans obtained from international governing bodies such as the International Monetary Fund (IMF) and World Bank, and acquired huge debt in the process. By the second quarter of 2021, a handful of developing and emerging economies defaulted on their sovereign debt (e.g. Argentina, Ecuador, Ethiopia, Lebanon, Zambia) and it is expected that more emerging economies are at sovereign risk of default (Arellano, Bai, and Mihalache 2020).

In the case of South Africa, which is the focus of our study, the levels of debt are not as high as those in other emerging or industrialized economies and yet the sharp increase in debt-to-GDP ratios from 38.2% (2010) to 68.7% (2020) indicates that, over the last decade, the country has accumulated debt faster in comparison to more industrialized counterparts/nations (see table 1). To mitigate the effects of COVID-19, the South African government rolled out a R500-billion 'extraordinary coronavirus budget' (10% of GDP), of which a significant portion has been financed through government debt. The BRICS New Development Bank (NDB) loaned the country $1 billion in June 2020 and in July 2020 the IMF approved a further $4.3 billion loan. Notably, government expenditure programmes supporting the economy through the pandemic have attained some level of success, as economic growth rebounded from –55.7% in the second quarter of 2020 to 66.1% in the fourth quarter. However, given the countercyclical fiscal approach adopted by the Ministry of Finance in response to the pandemic, both the budget deficit and the public debt have exceeded their 'targets' and are expected to sharply rise in the future (Bhorat et al. 2020). To cushion the adverse effects of rising debt, the South African government has committed itself to stabilizing debt so that it peaks at 87 percent by 2023–2024 and starts declining thereafter (Burger and Calitz 2021).

The question our paper poses is 'Are South African fiscal authorities trying to stabilize debt at levels which would compromise long-term economic growth?' This question begets a more specific empirical question of 'how much debt is too much debt for the South African economy'? To answer these questions, we use the T-NARDL framework to capture the nonlinear dynamics of the reduced form regression extracted from the endogenous debt-growth model which encompasses both public debt

TABLE 1 Debt-to-GDP Ratios of 2010 and 2020 for Select Countries

Country	2010	2020	Country	2010	2020
South Africa	34.7	83.0	UK	62.6	84.4
Brazil	51.8	75.8	Argentina	55.4	102.0
Russia	9.0	14.6	Canada	81.3	117.8
India	65.6	69.6	France	85.3	115.7
China	33.9	66.8	Germany	82.5	69.8
Japan	207.7	266.2	Italy	119.2	155.8
Zambia	77.9	91.9	Spain	60.5	120.0
US	91.2	107.6			

NOTES Based on data from World Bank (https://data.worldbank.org/indicator).

and public investment thresholds. One appealing feature of the model framework is that the optimal levels of debt and public capital are not universal for all economies and are dependent on certain economic factors such as the growth rate of public debt and the share of public investment in government expenditure. We further consider this framework suitable in the context of the South African economy since recent empirical evidence by Ncanywa and Masoga (2018) and Burger and Calitz (2021) confirm that public investment is an important channel through which public debt can influence economic growth in South Africa. However, these studies fall short of identifying optimal levels of public investment and debt which maximize economic growth, which is a shortcoming our study overcomes.

The rest of the study is presented as follows. The next section of the paper provides a brief review of the associated literature. The third section outlines the theoretical framework of the study whilst the fourth section outlines the T-NARDL model used to estimate the public debt and public investment thresholds. The empirical analysis is presented in the fifth section of the paper and then the study is concluded in the sixth section in the form of policy implications.

Brief Review of Associated Literature

Traditional economic theory speculates that debt can either be growth-enhancing (i.e. Keynesian hypothesis), growth-retarding (i.e. debt overhang hypothesis) or exert neutral effects on growth (i.e. Ricardian-equivalence hypothesis) (Akanbi 2016; Dombi and Dedák 2019; Mhlaba and Phiri 2019; Rahman, Ismail, and Ridzuan 2019; Yared 2019). Numerous empirical studies have reconciled these opposing views on the effects of debt on growth by assuming that debt only retards growth after it has crossed some optimal threshold level of debt (Smyth and Hsing 1995; Pattillo, Poirson, and Ricci 2002; Chen and Lee 2005; Reinhart and Rogoff 2010; Cecchetti, Mohanty, and Zampolli 2011; Chang and Chiang 2012; Checherita-Westphal and Rother 2012; Baum, Checherita-Westphal, and Rother 2013; Checherita-Westphal, Hallett, and Rother 2014; Herndon, Ash, and Pollin 2014; Pescatori, Sandri, and Simon 2014; Casares 2015; Égert 2015; Chen et al. 2017; Gómez-Puig and Sosvilla-Rivero 2017; Kamiguchi and Tamai 2019; Bouchrara, Rachdi, and Guesmi 2020; Bentour 2021).

Some of this literature has included South Africa in their empirical analysis and notably, the optimal debt threshold estimates obtained in previous studies are above the current debt-to-GDP ratio of 70 percent, which implies that South African fiscal authorities have not acquired too much debt and have some 'fiscal space' to acquire more debt to enhance economic growth (Eberhardt and Presbitero 2015; Chudik et al. 2017; Bitar, Chakrabarti, and Zeaiter 2018; Ndoricimpa 2017). Only a few more recent studies which include South Africa as part of the analysis argue otherwise (Chen et al. 2017; Ndoricimpa 2020; Law et al. 2021).

After surveying and scrutinizing the available empirical literature which has previously estimated optimal debt thresholds for the South African economy (see table 2 for a summary of this literature), we observe that the suggested optimal debt levels based on these previous studies may be biased towards the country and further deliberation on the subject matter is necessary. We present three main reasons supporting our views.

Firstly, we observe that most previous South African studies are panel based, which include South African data amongst a host of high debt outlier economies and generalize the estimated threshold as being applicable to all observed economies (Caner, Grennes, and Koehler-Geib 2010; Eberhardt and Presbitero 2015; Chudik et al. 2017; Arčabić et al. 2018; Bitar, Chakrabarti, and Zeaiter 2018; Mensah et al. 2019; Ndoricimpa 2017; 2020; Liu and Lyu 2021; Law et al. 2021). We argue that the panel data approach masks the country-specific dynamics underlying the true debt-growth relationship for the South African economy. By conducting a country-specific analysis for South Africa we more effectively 'separate the wheat from the chaff' in estimating an appropriate debt threshold for the economy.

TABLE 2 Summary of Associated Literature

Author	Countries	Period	Method	Results
Cordella, Ricci, and Ruiz-Arranz (2005)	79 developing countries (including SA)	1970–2002	Panel Threshold Autoregressive Model	Debt overhang threshold at 15–30% and debt irrelevance threshold at 70–80%.
Caner, Grennes, and Koehler-Geib (2010)	75 developing and 26 industrialized economies	1980–2008	Panel Threshold Autoregressive Model	Optimal debt of 77% for entire sample and 64% for developing countries.
Eberhardt and Presbitero (2015)	118 (including SA)	1960–2012	Kink regression and Nonlinear Autoregressive Distributive Lag model	Optimal debt in range of 60–90%.
Égert (2015)	29 developed and 21 emerging economies (including SA)	1960–2009	Multiple-regime Panel Threshold Autoregressive model	Debt threshold of 60–90% for emerging economies.
Chen et al. (2017)	65 countries (including South Africa)	1960–2014	Panel Smooth Transition Regression model	Debt threshold of 59.72%.
Chudik et al. (2017)	40 countries (including South Africa)	1965–2010	Panel Threshold Autoregressive Distributive Lag model (PTARDL)	30–60% for developing economies. No universal debt threshold once common observed factors are accounted for. Debt trajectory is more important for relationship.

Continued on the next page

Secondly, most previous studies including South Africa in their analysis do not rely on sound theoretical foundations which dictate the channels through which the nonlinear debt-growth relationship emerges. We note that previous studies have either estimated bi-variate regressions with no control variables (Égert 2015; Chudik et al. 2017; Mensah et al. 2019) or have estimated multivariate regressions with inconsistent control

TABLE 2 *Continued from the previous page*

Author	Countries	Period	Method	Results
Ndoricimpa (2017)	38 African countries	1980–2010	Panel Threshold Autoregressive Model	Debt threshold of 92.78%.
Arčabić et al. (2018)	185 OECD and non-OCED countries	1960–2009	Panel Threshold Autoregressive Model	Debt threshold of between 26.68–80.71% for OECD countries and between 43.24–106.33% for non-OECD countries.
Mensah et al. (2019)	38 African countries	1970–2015	Panel Threshold Autoregressive Distributive Lag model	Optimal debt in range 50–80%.
Ndoricimpa (2020)	39 African countries	1980–2012	Panel Smooth Transition Regression model	Debt threshold of 74.3%.
Bouchrara, Rachdi, and Guesmi (2020)	36 countries (SA included)	1990–2013	Panel Smooth Transition Regression model	The effect of debt on growth is dependent on institutional quality.
Law et al. (2021)	71 developing countries (including SA)	1984–2015	Panel Threshold Autoregressive Model	Debt threshold of 51.65%.

variables, or include covariates which are selected based on 'generalizations' of growth theory (Bitar, Chakrabarti, and Zeaiter 2018; Eberhardt and Presbitero 2015; Arčabić et al. 2018; Ndoricimpa 2017; 2020; Liu and Lyu 2021; Law et al. 2021). In our study, we follow the strategy of Chen et al. (2017) who apply optimization techniques to derive reduced form econometric regressions from an endogenous growth model which theoretically sets the foundation for the nonlinear debt-growth dynamics. We use the reduced form regressions to dictate the growth covariates included in our econometric specification when estimating optimal debt thresholds for the South African economy.

Thirdly, many previous studies rely on rather inflexible econometric models such as 'quadratic' or 'kink' regression models (Liu and Lyu 2021), the panel threshold autoregressive (PTAR) framework (Cordella, Ricci, and Ruiz-Arranz 2005; Caner, Grennes, and Koehler-Geib 2010; Ndoricimpa 2017; Arčabić et al. 2018; Law et al. 2021) and the panel smooth transition regression (PSTR) model (Ndoricimpa 2020). For instance, Law et al. (2021) criticize the quadratic term modelling strategy which, as argued by the authors, overestimates the optimal debt threshold level. Moreover, the PTAR and PSTR nonlinear econometric models ignore important long-run and short-run cointegration relations depicted by dynamic growth theory and these econometric models are considered inflexible since they are exclusively compatible with stationary time series.

To the best of our knowledge, only the studies of Eberhardt and Presbitero (2015) and Mensah et al. (2019) have used the more flexible nonlinear autoregressive distributive lag (NARDL) model to investigate long-run and short-run asymmetric cointegration effects between debt and growth for panels inclusive of South African data. Eberhardt and Presbitero (2015) and Mensah et al. (2019) use predetermined thresholds of 60–90 percent and 50–80 percent, respectively, to model threshold debt-growth effects within the nonlinear cointegration framework. Consequentially, these studies are only able to confirm a range over which the optimal debt threshold can lie but fail to 'pinpoint' the exact optimal debt threshold level, which is a shortcoming our study empirically addresses.

In our study, we make use of the threshold nonlinear autoregressive distributive lag (T-NARDL) model of Greenwood-Nimmo, Shin, and van Treeck (2011) to estimate the optimal debt threshold for the South African economy within a nonlinear cointegration framework. Note that whilst the NARDL framework of Shin, Yu, and Greenwood-Nimmo (2014), by default, assumes a 'zero' threshold value, the T-NARDL model of Greenwood-Nimmo, Shin, and van Treeck (2011) extends this framework by using the grid search methods and threshold testing procedures described by Hansen (2000) to obtain and validate optimal 'non-zero' threshold points. Despite the empirical appeal of the T-NARDL in endogenously determining optimal threshold points within a NARDL cointegration framework, its empirical application has been very limited and to the best of our knowledge, only the pioneering paper of Greenwood-Nimmo, Shin, and van Treeck (2011) has previously applied the T-NARDL model in the context of modelling 'non-zero' threshold points in the Canadian Phillips curve.

Theoretical Model

In this section of the paper we present the three-sector (i.e. government, households and production) endogenous growth model of Chen et al. (2017) which we adopt as our theoretical framework. Within the model, government size comprises both public consumption and public investment, which play substantially different roles in affecting steady-state growth. On one hand, public debt, alongside tax, is used to finance public investment which enters directly into the production function together with labour and private capital investment. On the other hand, public consumption enters directly into the consumption demand function alongside private or household consumption, which, in turn, is optimized by the household sector.

Within the model, public investment and public consumption can stimulate economic growth by being complementary (i.e. crowding-in effects) to private investment and private consumption, respectively, or can be harmful towards growth through crowding-out effects of investment and consumption (i.e. substitution effects). This induces nonlinear dynamics within the model in which public investment/consumption positively affects growth until an optimal level is reached, of which beyond this level, adverse effects begin to emerge. Similarly, public debt used in financing public investment will only be growth enhancing until a certain optimal debt threshold level. The model allows us to use optimization techniques to determine unique optimal levels of public debt and investment in the model which are then estimated using suitable econometric models.

GOVERNMENT SECTOR

The model assumes that government's total expenditure (G_t) is composed of government consumption ($G_{c,t}$) and government investment ($G_{k,t}$), i.e.

$$G_t = G_{c,t} + G_{k,t}, \tag{1}$$

where government investment is financed by net tax receipts (i.e. $\tau(Y_t - rD_t)$) and public debt ($\dot{D}_t + rD_t$), such that the government's capital accumulation function can be specified as:

$$\dot{G}_{k,t} = \phi G_t - \delta G_{k,t} = \phi[\tau(Y_t + rD_t) + \dot{D}_t - rD_t], \tag{2}$$

with τ denoting the tax rate, D denoting government debt, and r denoting the real interest rate.

Household Sector

The households within the model are assumed to face an infinite horizon economy and optimize the following intertemporal utility function:

$$U(C_t^*) = \int_0^\infty [\frac{(C_t^*)^{1-\sigma}}{1-\sigma}]e^{-pt}dt, \tag{3}$$

where p is a subjective discount factor, σ is the curvature parameter and C_t^* is the Keynesian effective demand function which incorporates both private and public consumption such that government directly affects the utility or welfare of households in the economy. Chen et al. (2017) propose that the demand function, C_t^*, be specified in Cobb-Douglas form to ensure that utility is increasing if public and private consumption are complementary, and that utility can decrease if private investment and public consumption are substitutes, i.e.

$$C_t^* = C_t^{\theta} G_{c,t}^{1-\theta}, \tag{4}$$

where C_t is private consumption. Further denoting W_t as household wealth, the household's budget constraint can be specified as:

$$W_{t+1} = (1 + r)W_t + Y_t + rD_t - C_t - G_{c,t}. \tag{5}$$

The household's dynamic optimization problem can be solved by setting the marginal utilities of (1) and (4) equal to each other (i.e. $\text{MU}(G_t) = \text{MU}(C_t^*)$, where MU denotes the marginal utility) and by extracting the following condition from the Lagrangian solution:

$$C_t = \frac{\theta G_{c,t}}{1 - \theta}. \tag{6}$$

PRODUCTIVE SECTOR

The production function is of augmented Cobb-Douglas form and specifies output (Y_t) as a function of household private capital, (K_t), labour (L_t) and government capital expenditure (i.e. $G_{c,t}$), i.e.

$$Y_t = AK_t^a L_t^{1-a-b} G_{c,t}^b, \quad 0 < a + b < 1, \tag{7}$$

where a and b are the elasticities of private and government investment, respectively.

In turn, private capital accumulation depends on private savings and capital depreciation:

$$\dot{K}_t = \phi(1 - \tau)(Y_t - rD_t) - C_t - K_t. \tag{8}$$

And by incorporating equation (6) into (8), we derive the following motion equation of capital accumulation:

$$\begin{aligned} \dot{K}_t = \{&(1 - \tau)(1 + \tau\frac{D_t}{Y_t}) \\ &- \frac{\tau}{1 - \tau}(1 - \phi)[\frac{D_t}{D_t} \cdot \frac{D_t}{Y_t} - (1 - \tau)\frac{D_t}{Y_t} + \tau]\}Y_t - \delta K_t. \end{aligned} \tag{9}$$

STEADY-STATE EQUILIBRIUM DYNAMICS AND OPTIMIZATION

To examine the steady-state equilibrium dynamics of the model, we take equations (2), (7) and (9) and re-specify them in per capita terms:

$$\dot{g}_{k,t} = \phi[\eta d_t - (1-\tau)rd_t + \tau]y_t - (\delta+n)g_{k,t}, \tag{10}$$

$$y_t = AK_t^a L_t^{1-a-b} G_{k,t}^b, \tag{11}$$

$$\dot{k}_{k,t} = [(1-\tau)(1-rd_t) - \frac{\theta}{1-\theta}(1-\phi)[\eta d_t - (1-\tau)rd_t + \tau]y_t - (\delta+n)k_t, \tag{12}$$

where $d_t = D_t/Y_t$ and $\eta = \dot{D}_t/D_t$. In the steady state, the growth rates of government expenditure, private capital and public capital are equal to zero (i.e. $\dot{g} = \dot{k} = \dot{g}_{k,t} = 0$) and the steady-state equilibrium per capita income, y^*, is solved as:

$$y^* = A[\phi(\eta d_t - (1-\tau)rd_t + \tau)]^{\frac{b}{1-a-b}}(\delta+n)^{\frac{b}{1-a-b}}\{(1-\tau)(1+rd_t) - \frac{\tau}{1-\tau}(1-\phi)[\eta d_t - (1-\tau)rd_t + \tau]\}^{\frac{b}{1-a-b}}. \tag{13}$$

From equation (13), the endogenous variables of interest are government investment and public debt as well as the elasticities of output with respect to private and public investment. A nonlinear relationship between output, public debt and public investment can be deduced, with the optimal level of public debt computed as the first derivative of steady state equilibrium output with respect to per capita debt, i.e.

$$\partial\frac{\frac{\dot{y}}{y}}{\partial d} = \frac{a + b(1-\phi)\tau\frac{\theta}{1-\theta} + \frac{(a+b)\tau - b(r-\eta)}{(1-\tau)r-\eta}(1-\tau)}{(a+b)\{\tau(1-\tau) + \frac{\theta}{1-\theta}(1-\phi)[(1-\tau)r-\eta]\}} = \frac{\partial^2(\frac{\dot{y}}{y})}{\partial^2 d} < 0, \tag{14}$$

whilst the optimal level of public investment is computed as the first derivative of steady state equilibrium output with respect to per capita debt, i.e.

$$\partial\frac{\frac{\dot{y}}{y}}{\partial d} = 0 = \frac{a}{a+b}[1 - \frac{1-\theta}{\theta}\frac{1}{\varpi}(1-\tau)(1+rd)] = \frac{\partial^2(\frac{\dot{y}}{y})}{\partial^2 d} < 0, \tag{15}$$

From equation (14), the optimal level of public debt is positively related with the output elasticity of public investment and total investment (private plus public capital) and with the intertemporal elasticity between public and private spending as well as the share of public investment in total government expenditure, but negatively related to the growth rate of public debt. This implies that governments characterized by larger (smaller) shares of public capital in total public expenditure, more (less) productive capital expenditure and slower (faster) growth rates of public debt will have higher (lower) debt tolerance or debt thresholds.

Similarly, from equation (15), the optimal level of public capital is positively related with the output elasticity of the share of private investment in total investment (public and private investment) as well as with the total government expenditure size such that economies with more (less) productive shares of private capital in total investment as well as those with larger (smaller) total government spending will have higher (lower) public investment thresholds.

Econometric Modelling

To econometrically capture the nonlinear 'debt-growth' and 'public investment-growth' dynamics presented in the theoretical model, we make use of the T-NARDL model of Greenwood-Nimmo, Shin, and van Treeck (2011) which is a generalization of the NARDL model of Shin, Yu, and Greenwood-Nimmo (2014) applied to the case of unknown threshold decompositions. By taking the logarithms of the production function and incorporating debt and public investment threshold effects, we propose the following two baseline NARDL regressions which are portioned by debt and public investment, respectively:

$$\frac{\dot{y}_t}{y_t} = \alpha_0 + \alpha_1 \left(\frac{K}{Y}\right)_t + \alpha_2 \frac{Y_t}{L_t} + \alpha_3 \frac{G_{k,t}}{Y_t} + \alpha_4^{(+)} \left(\frac{D}{Y}\right)_t^+ + \alpha_4^{(-)} \left(\frac{D}{Y}\right)_t^- + \xi_{1t}, \tag{16}$$

$$\frac{\dot{y}_t}{y_t} = \beta_0 + \beta_1 \left(\frac{K}{Y}\right)_t + \beta_2 \left(\frac{Y}{L}\right)_t + \beta_3 \left(\frac{K}{Y}\right)_t + \beta_4^{(+)} \left(\frac{G_k}{Y}\right)_t^+ + \beta_4^{(-)} \left(\frac{G_k}{Y}\right)_t^- + \xi_2 t \tag{17}$$

and define the associated unrestricted error correction representation of NARDL regressions (16) and (17) as follows:

$$\Delta\left(\frac{\dot{y}}{y}\right)_t = \sum_{j=1}^{p} \rho_i \left(\frac{y}{y}\right)_{t-1} + \psi_1 \left(\frac{K}{Y}\right)_t + \psi_2 \left(\frac{Y}{L}\right)_t + \psi_3 \left(\frac{G_k}{Y}\right)_t$$

$$+\psi_{4j}^{+}\left(\frac{D}{Y}\right)_t^{+}+\psi_{4j}^{-}\left(\frac{D}{Y}\right)_t^{-}+\sum_{j=1}^{p-1}\lambda_i\Delta\left(\frac{\dot{y}}{y}\right)_{t-j}$$
$$+\sum_{j=0}^{q-1}\left(\phi_{1j}\Delta\left(\frac{K}{Y}\right)_t+\phi_{2j}\Delta\left(\frac{L}{Y}\right)_t+\phi_{3j}\left(\frac{G_k}{Y}\right)_t\right.$$
$$\left.+\phi_{4j}^{+}\Delta\left(\frac{D}{Y}\right)_t^{+}+\phi_{4j}^{-}\Delta\left(\frac{D}{Y}\right)_t^{-}\right)+\xi_t, \tag{18}$$

$$\Delta\left(\frac{\dot{y}}{y}\right)_t=\sum_{j=1}^{p}\rho_i\left(\frac{\dot{y}}{y}\right)_{t-1}+\sigma_1\left(\frac{K}{Y}\right)_t+\sigma_2\left(\frac{Y}{L}\right)_t+\sigma_3\left(\frac{D}{Y}\right)_t$$
$$+\sigma_4^{+}\left(\frac{G_k}{Y}\right)_t^{+}+\sigma_4^{-}\left(\frac{G_k}{Y}\right)_t^{-}+\sum_{j=1}^{p-1}\lambda_i\Delta\left(\frac{\dot{y}}{y}\right)_{t-j}$$
$$+\sum_{j=0}^{q-1}\left(\theta_{1j}\Delta\left(\frac{K}{Y}\right)_t+\theta_{2j}\Delta\left(\frac{Y}{L}\right)_t+\theta_{3j}\Delta\left(\frac{D}{Y}\right)_t\right.$$
$$\left.+\ \theta_{4j}^{+}\Delta\left(\frac{G_k}{Y}\right)_t^{+}+\theta_{4j}^{-}\Delta\left(\frac{G_k}{Y}\right)_t^{-}\right)+\xi_t, \tag{19}$$

where defining

$$\xi_{1t}=\frac{\dot{y}_t}{y_t}-\alpha_1\left(\frac{K}{Y}\right)_t-\alpha_2\left(\frac{Y}{L}\right)_t-\alpha_3\left(\frac{G_k}{Y}\right)_t-\alpha_4^{(+)}\left(\frac{D}{Y}\right)_t^{+}$$
$$-\alpha_4^{(-)}\left(\frac{D}{Y}\right)_t^{-}\text{ and}$$
$$\xi_{2t}=\frac{\dot{y}_t}{y_t}-\beta_0-\beta_1\left(\frac{K}{Y}\right)_t-\beta_2\left(\frac{Y}{L}\right)_t-\beta_3\left(\frac{D}{Y}\right)_t-\beta_4^{(+)}\left(\frac{G_k}{Y}\right)_t^{+}$$
$$-\beta_4^{(-)}\left(\frac{G_k}{Y}\right)_t^{-}$$

the asymmetric error correction term in equations (18) and (19) can be computed as ξ_{1t-1} and ξ_{2t-1}, respectively, and the asymmetric long-run parameters are computed as $\alpha_1 = -(\psi_1/\rho)$, $\alpha_2 = -(\psi_2/\rho)$, $\alpha_3 = -(\psi_3/\rho)$, $\alpha_4^{+} = -(\psi_4^{+}/\rho)$, $\alpha_4^{-} = -(\psi_4^{-}/\rho)$ and $\beta_1 = -(\sigma_1/\rho)$, $\beta_2 = -(\sigma_2/\rho)$, $\beta_3 = -(\sigma_3/\rho)$, $\beta_4^{+} = -(\sigma_4^{+}/qrho)$, $\beta_4^{-} = -(\beta_4^{-}/\rho)$ for equations (18) and (19), respectively. To estimate the debt and public investment thresholds, we follow Greenwood-Nimmo, Shin, and van Treeck (2011) and decompose the 'debt' and 'public investment variables' into partial sum processes employing a non-zero threshold, γ, which we respectively define as:

$$\left(\frac{D}{Y}\right)^{+} = \sum_{j=1}^{i} \frac{D}{Y} I_{\cdot(D/Y<\gamma)} = \max\left(\frac{D}{Y}, \gamma\right),$$
$$\left(\frac{D}{Y}\right)^{-} = \sum_{j=1}^{i} \frac{D}{Y} I_{\cdot(\frac{D}{Y}>\gamma)} = \min\left(\frac{D}{Y}, \gamma\right), \tag{20}$$

$$\left(\frac{G_k}{Y}\right)^{+} = \sum_{j=1}^{i} \frac{G_k}{Y} I_{\cdot(G_k/Y<\gamma)} = \max\left(\frac{G_k}{Y}, \gamma\right),$$
$$\left(\frac{G_k}{Y}\right)^{-} = \sum_{j=1}^{i} \frac{G_k}{Y} I_{\cdot(G_k/Y>\gamma)} = \min\left(\frac{G_k}{Y}, \gamma\right), \tag{21}$$

where γ is an unknown threshold parameter responsible for regime-switching behaviour, which can be consistently estimated using the following minimization criterion, i.e.

$$\hat{\gamma} = \text{argmin}_{\gamma \in \mathrm{D}} Q(\gamma). \tag{22}$$

Once the optimal threshold level, $(\hat{\gamma})$, is obtained, the significance of the threshold estimate is verified by using the following likelihood ratio (LR) tests, i.e.

$$\text{LR} = \frac{\text{SSR}_0 - \text{SSR}_1(\hat{\gamma})}{\dot{\hat{\sigma}}^2}, \tag{23}$$

where SSR_0 and $\text{SSR}_1(\hat{\gamma})$ are the residuals from the linear ARDL model and the T-NARDL model, respectively, and $\dot{\hat{\sigma}}^2$ is the regression error variance. Note that the LR test is non-standard since the threshold parameter is unidentified under the null hypothesis of linearity. We therefore make use of the bootstrap method described by Hansen (2000) to simulate the asymptotic distribution of the LR statistic and construct p-values from the bootstrap that are asymptotically valid.

Besides the tests for LR threshold effects, there are three additional 'nonlinear cointegration' tests, suggested by Shin, Yu, and Greenwood-Nimmo (2014), to which we subject our T-NARDL model regressions. First, there is the F-test for adjustment asymmetry which evaluates the null hypothesis of $\rho = \psi = \psi_1 = \psi_2 = \psi_3 = \psi_4^+ = \psi_4^+ = 0$ (eq. 20) and $\rho = \sigma_1 = \sigma_2 = \sigma_3 = \sigma_4^+ = \sigma_4^+ = 0^-$ (eq. 21) against the alternative of $\rho \neq \psi_1 \neq \psi_2 \neq \psi_3 \neq \psi_4^+ \neq \psi_4^+ \neq 0$ (eq. 20) and

$\rho \neq \sigma_1 \neq \sigma_2 \neq \sigma_3 \neq \sigma_4^+ \neq \sigma_4^+ \neq 0$ (eq. 21). These hypotheses are evaluated using a test statistic denoted as F_{PSS}. Second, there are tests for long-run or reaction asymmetry which test the null of no long-asymmetry effects ($\alpha_4^+ = \alpha_4^-$ for eq. (18) and $\beta_4^+ = \beta_4^-$ in eq. (19)) against the alternative of significant long-run asymmetries ($\alpha_4^+ \neq \alpha_4^-$ for eq. (18) and $\beta_4^+ \neq \beta_4^-$ in eq. (19)) and the test statistic evaluating the null hypotheses is denoted as W_{LR}. Thirdly, there are tests for short-run asymmetries which test the null of no long-asymmetry effects ($\sum_{i=0}^{q-1} \phi_j^+ = \sum_{i=0}^{q-1} \phi_j^-$ for eq. (18) and $\sum_{i=0}^{q-1} \theta_j^+ = \theta_{i=0}^{q-1} \theta_j^-$ for eq. (19)) against the alternative of significant long-run asymmetries ($\sum_{i=0}^{q-1} \phi_j^+ \neq \sum_{i=0}^{q-1} \phi_j^-$ for eq. (18) and $\sum_{i=0}^{q-1} \theta_j^+ \neq \sum_{i=0}^{q-1} \theta_j^-$ for eq. (19)) and the test statistic evaluating the null hypotheses is denoted as W_{SR}.

Data, empirical analysis and diagnostics

DATA

The data used in this study has been solely sourced from the SARB online statistical database on a quarterly frequency over the period 1960:q2 to 2020:q2 and the span of our data is determined by the collective availability of the time series. We source 6 series from the database, namely, economic growth rate (KBP6006Z), gross fixed capital accumulation as a percentage of GDP (KBP6282L), labour productivity of non-agriculture (KBP7014L), total net public debt as a percentage of GDP (KBP4117), total public investment (KBP6006Z), and real GDP at constant prices (KBP6006Z). We use the last two times series to compute a measure of public investment as a percentage of GDP, i.e. public investment divided by GDP.

The summary statistics and the unit root tests are reported in tables 3 and 4, respectively. From table 3 we observe some stylized facts for South Africa such as the combination of low growth – low debt averages observed over the sample period. Moreover, judging from the statistics, growth and labour productivity have had relatively high volatility (compared to their averages) whilst debt and the remaining variables exhibit less volatility. Lastly, note that p-values Jarque-Bera (J-B) statistics obtained for all series indicate that the variables are non-normal, implying that their distributions of the individual series are nonlinear and hence justifying the use of nonlinear econometric methods to establish any debt-growth relationships. From table 4, we use the conventional ADF test and its more powerful alternative, the DF-GLS test, to establish the

TABLE 3 Summary Statistics of Time Series

Variable	Description	Mean	Sd	Min	Max	J-B*
$\partial Y/Y$	GDP growth rate	2.2627	5.5584	-51.7000	21.7000	0.0000
K/Y	Gross fixed capital formation as a % of GDP	21.4146	4.1998	15.0000	32.1000	0.0830
Y/L	Labour productivity	79.1508	21.9887	39.800	105.600	0.0463
D/Y	Total net debt of national government as a % of GDP	36.0203	8.2729	21.600	63.500	0.0120
GK/Y	Government investment capital as a % of GDP	3.9686	1.7111	2.0377	8.5865	0.0000

NOTES * p-value.

TABLE 4 Unit Root Test Results (First Differences)

Variable	ADF		DF-GLS	
	Int	Int + trend	Int	Int + trend
$\partial Y/Y$	−15.0540(2)***	−15.0632(2)***	−14.8039(1)***	−5.3624 (1)***
K/Y	−16.7403(1)***	−16.7338(1)***	−16.7379(1)***	−6.7658(2)***
Y/L	−19.1220(2)***	−19.1187(2)***	−7.6505(1)***	−9.6126(1)***
D/Y	−4.5314(4)***	−5.0365(4)***	−3.4180(3)***	−3.2035(4)***
GK/Y	−20.4762(1)***	−20.4326(1)***	−3.6724(4)***	−9.6656(1)***

integration properties of the series. Note that we only present the unit root tests on the first differences of the series since the T-NARDL model is compatible with a mixture of I(0) and I(1) variables. The unit root tests confirm that none of the series are integrated of order I(2) which allows us to proceed with our empirical analysis. Our entire empirical analysis has been conducted in EViews 10.

GRID SEARCH FOR DEBT AND PUBLIC INVESTMENT THRESHOLDS

Following the modelling process prescribed by Greenwood-Nimmo, Shin, and van Treeck (2011), we begin our analysis by performing the grid search for the threshold level of debt and the associated threshold level of public investment. To recall, this involves estimating NARDL regressions for all possible values of the threshold and reporting the SSR obtained from each estimated regression. For the debt threshold regression, we estimate regressions over threshold values of 25% and 60%, whilst for the associated public investment threshold, the grid search is performed/conducted over values of 2% and 8%. Note that increments of 1 percent and 0.1 percent are used in the grid search for optimal debt and public investment levels, respectively.

Figures 1 and 2 present scatterplots based on the findings from our grid search for the optimal threshold levels of debt and public investment, respectively. Note that in both plots, the x-axis presents the selected threshold point and the y-axis presents the corresponding RSS of the NARDL estimated at each threshold point. Further note that the optimal lag length for each estimated NARDL regression used in the grid search is obtained via a minimization of the modified SC information criterion. For public debt threshold we plot the RSS for 27 estimated regressions (figure 1) whilst for public investment thresholds we plot the RSS for 58 estimated regressions (figure 2) and it is clear to see that the RSS is minimized at a threshold level of 47% for public debt and 2.8% for public investment, respectively.

From figure 1, we observe that our estimated debt threshold of 47% differs from most of the threshold estimates obtained in previous South African-related literature. Eberhardt and Presbitero (2015) find the optimal debt threshold to lie between 60% and 90% for 118 countries inclusive of South Africa using the threshold autoregressive distributive lag (TARDL) model. Using similar methodology applied to 38 African countries, Mensah et al. (2019) find the optimal debt threshold to lie at a lower range of 50% and 80%. Égert (2015) estimates a Panel Threshold Autoregressive (PTAR) model and finds a much lower threshold of 30% for a panel of 21 emerging economies inclusive of South Africa whilst Ndoricimpa (2017; 2020) finds two different thresholds of 92.78% and 74.3% for a sample of 29 African countries using Panel Threshold Autoregressive (PTAR) and Panel Smooth Transition Regression (PSTR) models, respectively. Law et al. (2021), more recently, estimate a debt threshold of 51% for a sample of 71 developing economies using the PTAR model.

Altogether, our estimated debt threshold of 47% is within the lower boundary of the range suggested by Mensah et al. (2019) and is comparable to the 51% threshold more recently estimated by Law et al. (2021), and yet differs significantly from the 30%, 93% and 74% estimated thresholds found in the studies of Eberhardt and Presbitero (2015) and Ndoricimpa (2017; 2020), respectively. As dictated by our endogenous growth model introduced in the third section of the paper, the optimal level of public debt is positively dependent on the output elasticity of public (and private) capital as well as on the elasticity of substitution between public

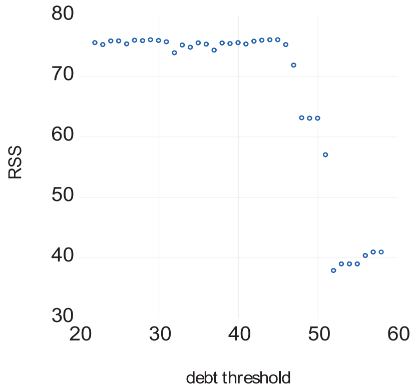

FIGURE 1 Grid Search for Optimal Public Debt Threshold

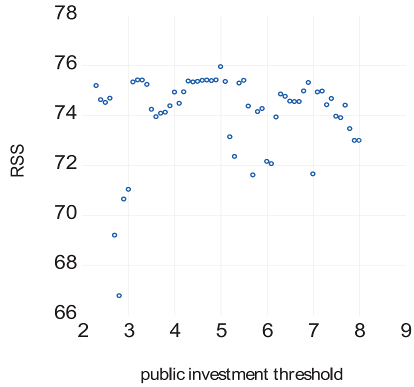

FIGURE 2 Grid Search for Optimal Public Investment Threshold

and private spending, and yet negatively dependent on the growth rate of public debt. The lower threshold estimate obtained for South Africa is thus expected since, as shown in the introduction, the growth rate of public debt over the last decade has been higher compared to other emerging and developed economies. Moreover, higher crowding effects of public capital on private capital, as documented by Biza, Kapingura, and Tsegaye (2015) and Makuyana and Odhiambo (2018) for South Africa, would reduce the elasticity, and hence productivity, of public and social (private plus public) capital, resulting in a lower tolerance or threshold debt level compared to those estimated in previous panel-based studies.

Similarly, in figure 2, we observe that the estimated public investment-to-GDP threshold of 3% is much lower than the threshold obtained in previous studies. For instance, Abounoori and Nademi (2010) estimate a public investment threshold of 8% for Iran using a TAR model. On the other hand, Fosu, Getachew, and Ziesemer (2016) estimate an optimal public capital threshold of 10% for 33 African countries inclusive of South Africa whilst Chen et al. (2017) make use of the PSTR model to estimate a public investment threshold of 16% for 65 economies inclusive of South Africa. As dictated by our endogenous growth model, the optimal level of public debt is dependent on the response of private capital to social capital as well as the ease of substitution between government and private capital. Our obtained lower public debt threshold implies that the South African economy is characterized by low responsiveness of private capital to total capital and low levels of substitutability between input factors in the public and private sector.

TABLE 5 T-NARDL Estimates at Optimal Public Debt of 47%

Panel A: Long run estimates			Panel B: Short-run and ECM		
Coefficient	Estimate	t-statistic	Coefficient	Estimate	t-statistic
C	2.4318	3.2990***	$\Delta\partial Y/Y(-1)$	−0.1389	−1.5806
$(D/Y)^{-}$	0.1256	3.2990***	$\Delta\partial Y/Y(-2)$	0.0381	0.4656
$(D/Y)^{+}$	0.0524	1.6100	$\Delta\partial Y/Y(-3)$	0.1456	2.3749**
K/Y	2.0312	0.3349***	$\Delta(D/Y)^{-}$	−0.4384	−2.1087*
Y/L	0.1051	1.6198	$\Delta(D/Y)^{+}$	−0.4434	−2.1567
G_K/Y	−0.8447	−3.2929***	$\Delta K/Y$	0.1356	0.8781
C	2.4318	3.2990***	$\Delta K/Y(-1)$	−0.7282	−3.2239***
			$\Delta K/Y(-2)$	−0.3450	−2.2318**
			$\Delta Y/L$	0.0098	0.3205
			$\Delta Y/L(-1)$	−0.0665	−2.2241**
			$\Delta G_K/Y$	−0.0819	−1.0550
			$\Delta G_K/Y(-1)$	0.2368	2.7416***
			$Ect(-1)$	−0.9784	−10.0053***

NOTES ***, **, * denote the 1%, 5%, 10% critical levels, respectively. A Newey-West estimator is used to obtain HAC standard errors.

T-NARDL ESTIMATES

Having obtained our optimal debt and public investment thresholds, in this section of the paper we present the associated TARDL regression coefficient estimates. Tables 5 and 6 present the TARDL model estimates for the optimal debt and public investment thresholds, respectively, with panel A presenting the long-run regression estimates whilst panel B presents the short-run and error correction model estimates.

From table 5, the long-run partitioned coefficients of the debt variable dictate the form of nonlinearity in the debt-growth relationship. Note that the (D/Y)pos variable which accounts for the debt dynamics in the upper regime of the regression model, produces an insignificant estimate whilst the (D/Y)neg which captures the debt dynamics in the lower regime of the model produces a positive and 1 percent statistically significant estimate of 0.126. By interpretation, the results imply that when the debt-to-GDP ratio exceeds 51 percent, debt exerts no effect on economic growth, whereas when debt is below its threshold level, a unit decrease in debt is associated with a 0.126 percent improvement in growth.

We observe that the nonlinear debt-growth dynamics described by our

TABLE 6 T-NARDL Estimates at Optimal Public Investment of 2.8%

Panel A: Long run estimates			Panel B: Short-run and ECM		
Coefficient	Estimate	t-statistic	Coefficient	Estimate	t-statistic
C	1.3199	0.86878	$\Delta\partial Y/Y(-1)$	−0.1145	−1.012
$(G_K/Y)^-$	0.4466	2.2569**	$\Delta\partial Y/Y(-2)$	0.0296	0.3158
$(G_K/Y)^+$	0.2082	1.3646	$\Delta\partial Y/Y(-3)$	0.1253	1.9774*
K/Y	2.1101	6.7256***	$\Delta(G_K/Y)^-$	−5.0949	−5.7239***
Y/L	−0.0306	−0.7827	$\Delta(G_K/Y)^-(-1)$	−2.4896	−2.6363***
D/Y	4.6658	1.4226	$\Delta(G_K/Y)^+$	−4.8715	−5.8831***
			$\Delta(G_K/Y)^+(-1)$	−1.9834	−2.2389**
			$\Delta K/Y$	0.2432	1.5498
			$\Delta K/Y(-1)$	−0.6170	−25574**
			$\Delta K/Y(-2)$	−0.2987	−1.7914
			$\Delta D/Y$	−26.4400	−1.2460
			$\Delta D/Y(-1)$	−33.5100	−1.5634
			$Ect(-1)$	−1.0022	−7.9281***

NOTES ***, **, * denote the 1%, 5%, 10% critical levels, respectively. A Newey-West estimator is used to obtain HAC standard errors.

results presented in table 5 differ from those predicted in previous South African-related literature. On one hand, Ndoricimpa (2017; 2020) and Law et al. (2021) find debt to be insignificantly related with growth below their estimated debt threshold and negatively (significantly) related with growth above the threshold. On the other hand, Chen et al. (2017) find debt to harm growth below a 59% threshold whilst above this threshold debt significantly harms growth. Moreover, Eberhardt and Presbitero (2015), Chudik et al. (2017) and Mensah et al. (2019) find that a negative and significant relationship emerges only within a threshold range of 60–90%, 50–80% and 30–60%, respectively.

In linking the empirical debt dynamics reported in table 6 to the endogenous growth model dynamics introduced in the third section of the paper, we find that below the public debt threshold of 51%, public investment financed by debt is complementary to private investment and therefore growth enhancing, which is a finding consistent with the Keynesian view of large deficits being expansionary for the economy through effective public investment expenditure. However, once the threshold is crossed, public investment financed by increased debt begins to crowd out private investment and hence higher levels of debt become insignificant towards economic growth; this is reminiscent of the Ricardian-equivalence hypothesis described in Barro (1989).

We now turn our attention to the T-ARDL estimates for the public-investment threshold regression reported in table 6 and observe similar nonlinear dynamics for the public investment-growth relationship. Note that the $(G_K/Y)^-$ coefficient estimate of 0.44 is statistically significant at a 1 percent critical level whilst the $(G_K/Y)^+$ variable produces a statistically insignificant estimate and these results imply that only below the 2.7 percent threshold is public capital growth enhancing whilst above this level, public investment is insignificantly related with economic growth. We further note that these dynamics differ from those presented in related literature. For instance, Abounoori and Nademi (2010) estimate a threshold of 8% for Iran and find that public capital is insignificant below and above the threshold level. For a sample of 33 African countries, Fosu, Getachew, and Ziesemer (2016) find that the positive effect of public investment outweighs the negative taxation effect (i.e. from taxes used to finance public investment) below the threshold of 9–10% whilst the adverse tax burden outweighs the private capital effects above the threshold. Similarly, Chen et al. (2017) find that below the public investment threshold of 16%, public capital is positively related to growth through its crowding-in effects on private investment whereas above the optimal level, public capital is negatively related with growth through crowding out effects of private investment. Our results differ from this previous literature in that we do not find a negative relationship between public capital and growth in either of the regimes of the T-NARDL model and hence the relationship is not 'inverted U-shaped' as hypothesized by the nonlinear 'BARS' curve between government size and growth.

Overall, the flexibility of the reduced form regression derived from the endogenous growth model allows for heterogeneity in estimating debt and public investment thresholds for different economies and in our case, the estimate thresholds of 51% and 3%, respectively obtained for South Africa, are much lower compared to the threshold estimates obtained in previous panel-based studies. Moreover, the nonlinear dynamics observed between (i) public debt and growth, and (ii) public investment and growth, differ from those of previous studies and yet remain in sync and complement each other. In this sense, we find significant and positive debt-growth and public capital-growth relationships below their estimate thresholds and yet, above these estimated thresholds, both relationships

TABLE 7 Optimal Public Investment

Panel A: Nonlinear coint. tests			Panel B: Residual diag. tests		
Coefficient	(1)	(2)	Coefficient	(1)	(2)
Threshold (LR test)	16.6023**	17.0549**	J-B	0.4512 (0.7981)	0.5287 (0.7677)
Ftest	13.9160***	8.7377***	SC	0.0484 (0.9528)	0.1639 (0.8493)
WLR	9.9736***	6.6977	ARCH	1.4694 (0.2304)	1.0669 (0.4050)
WSR	2.6474	2.3564	RESET	1.3614 (0.2261)	0.0457 (0.8315)

NOTES Column headings are as follows: (1) T-NARDL (debt threshold regression), (2) T-NARDL (public investment threshold regression). ***, **, * denote the 1%, 5%, 10% critical levels respectively.

simultaneously turn insignificant. Moreover, the sign and significance of the coefficients of the covariates do not differ, with physical capital being positively related with growth whilst human capital is insignificantly related with growth in both estimated regressions.

DIAGNOSTICS

So far, the empirical analysis has focused on identifying the threshold values of public debt and public investment and presented the corresponding T-NARDL model estimates for both models, but we have yet to test the validity of the estimated threshold points and associated threshold model regressions. Table 7 reports the tests statistics for nonlinear cointegration (panel A) as well as for the diagnostic tests performed on the residuals of the estimated T-NARDL regressions (panel B).

In panel A of table 7, we report the findings from our LR test for threshold effects, the bounds test statistic for nonlinear cointegration (FSYG), and the Wald statistic for long-run (WLR) and short-run (WSR) asymmetries, and based on the reported statistics for both T-NARDL models, we find evidence supporting significant threshold effects, significant nonlinear bounds cointegration effects and significant long-run asymmetries. Note that both models produce insignificant short-run asymmetric effects, implying that the nonlinearity between debt-growth and public investment-growth is strictly a long-run phenomenon.

In panel B of table 7, we present the residual diagnostic test for normality, autocorrelation and heteroscedasticity, and the reported Jarque-Bera

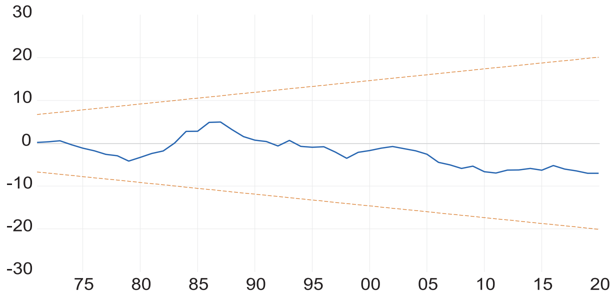

FIGURE 3
CUSUM – Debt Threshold (blue – CUSUM, orange – 5% significance)

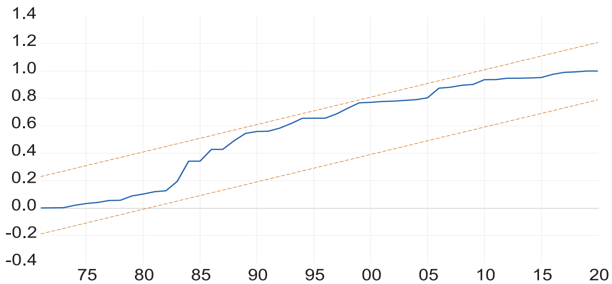

FIGURE 4
CUSUMSQ – Debt Threshold (blue – CUSUMSQ, orange – 5% significance)

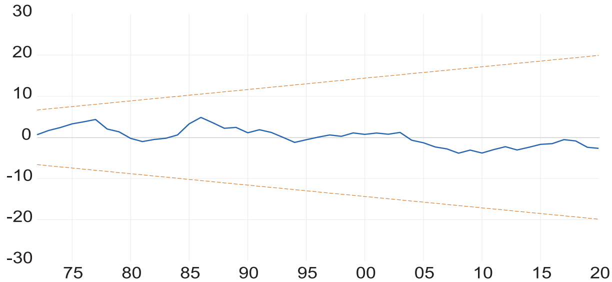

FIGURE 5
CUSUM – Public Investment Threshold (blue – CUSUM, orange – 5% significance)

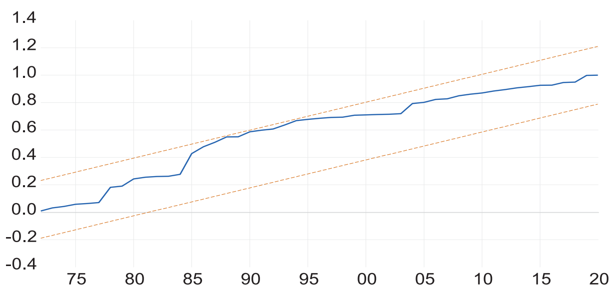

FIGURE 6
CUSUMSQ – Public Investment Threshold (blue – CUSUMSQ, orange – 5% significance)

(J-B), Breusch-Godfrey (χSC) and ARCH statistics (χARCH) evidence of well-behaved (with properties), homoscedastic regression errors in both estimated T-NARDL models. Moreover, we also report the tests statistics for correct functional form, and the reported RAMSEY test statistics (RE-

SET) provide evidence in favour of our T-NARDL regressions being the correct functional form and not requiring a higher-order functional form to fit the data. Lastly, we plot the CUSUM and CUSUMSQ plots for both regressions in figures 3–6, respectively, and both plots provide evidence of stable regressions at a 5 percent critical level.

Conclusions

The coronavirus pandemic has placed tremendous strain on the South African fiscal budget and debt-to-GDP levels are expected to reach historically high levels of 88% by 2022. Even though these levels are not as high as those experienced in other developed economies such as the US and Japan, there is much concern since South African fiscal authorities have acquired debt at a faster rate compared to other emerging and industrialized economies, particularly in the post 2007/08 financial crisis era. Recent empirical literature has speculated that the relationship between debt and growth is nonlinear such that debt acquired by fiscal governments is only harmful once it has crossed some optimal or tolerance level, when it starts to crowd out public investment. Moreover, new growth theory speculates that such tolerance levels of debt and public investment can depend on fiscal factors such as rate of acquired debt which would suggest that governments who acquire debt at faster (slower) rates have lower (higher) tolerance levels and there does not exist a universal debt or public investment threshold for different economies with different fiscal positions.

In our study, we provide country-specific debt and public investment threshold estimates for the South African economy based on reduced form econometric regressions derived from an endogenous growth model of public debt, public capital and economic growth. We make use of the TARDL model applied to quarterly time series collected between 1960:Q2 and 2021:Q2 to estimate the optimal threshold points as well as to capture the asymmetric debt-growth and private capital-growth relationships dictated by the theoretical model.

Our empirical analysis points to threshold estimates of 47% for the debt-to-GDP ratio and 2.8% for the public investment-to-GDP ratio, and only below these thresholds are public debt and private capital growth enhancing. On one hand, the debt-to-GDP ratio crossed has been predominantly below its threshold during the entire sample period of 1960 to 2021, and only began to cross the threshold level of 47% in 2017 (see figure 7).

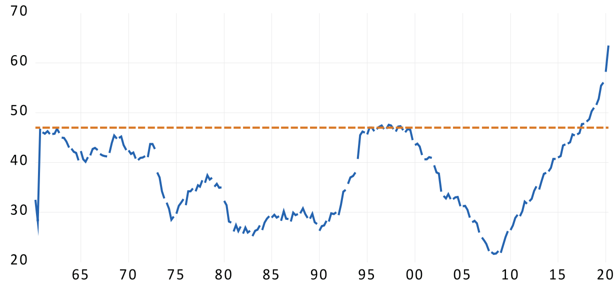

FIGURE 7

Performance of Public Debt Compared to Threshold Level (blue – net debt as 5% of GDP, orange – threshold level)

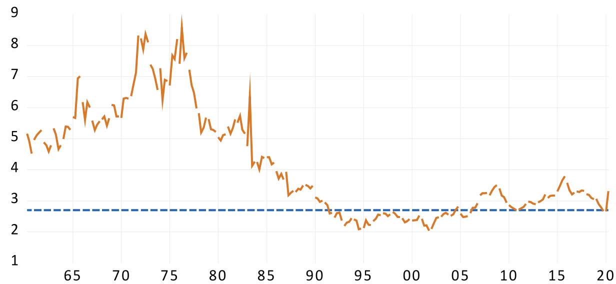

FIGURE 8

Performance of Public Investment Compared to Threshold Level (orange – government investment as 5% of GDP, blue – threshold level)

On the other hand, the public investment-to-GDP ratio has been predominantly above its 2.8% threshold from the 1960's to the early 1990's, and from the early 1990's to the mid-2000's public capital has been below its threshold. However, subsequent to the global financial crisis, public capital lies above its threshold level and since 2017 has maintained an upward trajectory (see figure 8).

To conclude our study, we provide key policy insights and recommendations derived from our empirical analysis. Firstly, our study finds that South Africa's debt woes began a few years prior to the start of the COVID-19 pandemic and previous related literature has over-estimated the country's tolerance level of debt. Secondly, our study recommends that fiscal authorities need to implement more rigorous policy measures than those which are currently proposed. For instance, the active approach to debt management and fiscal consolidation proposed by the national treasury in the 2020 Medium Term Budget Policy Statement (MTBPS) would need to include lower debt targets than those which are currently proposed. The aim of these policies should be to decrease the growth rate of debt levels to negative values, which in turn, would increase 'tolerance' levels of debt. Lastly, the government needs to find ways of balancing/stabilizing the public deficit by simultaneously reducing both the debt-to-GDP ratio and public investment-to-GDP ratio below their estimate thresholds of 47% and 2.8%, respectively. One of the current propositions from the government is to decrease the size of government expenditure by freezing the public wage bill. Based on our findings, the government needs to go beyond freezing the public wage bill and should further reduce the share of public capital in government expenditure and place more emphasis on private capital and non-investment government expenditure items. This could be achieved by spending less money on 'non-performing' State Owned Enterprises and increasing privatization in key economic sectors such as energy and transportation.

One main shortcoming with our study is that we use an aggregated measure of government debt. In practice, public debt is often classified into external debt and internal debt, and there may be different thresholds or tolerance levels associated with different disaggregated classifications of debt. Consequently, we suggest two directions for future studies. Firstly, growth theorists could consider developing dynamic growth models which can distinguish between external and internal public debt as well as their steady-state dynamics used to determine their optimal points. Secondly, researchers could consider expanding the two-regime framework to incorporate multiple regime dynamics. These models can be estimated using more advanced econometric techniques such as the multiple threshold nonlinear autoregressive distributive lag model.

Data Availability Statement

All data used in the study is from the SARB online database. Since this data is only accessible to South African residents, we have not obtained permission from the SARB to share their data. We therefore provide the source codes of the time series in our data description.

References

Abounoori, E., and Y. Nademi. 2010. 'Government Size Threshold and Economic Growth in Iran.' *International Journal of Business and Development Studies* 2 (1): 95–108.

Akanbi, O. 2016. 'External Debt Accumulation in Sub-Saharan African Countries: How Fast is Safe?' *International Journal of Sustainable Economy* 8 (2): 93–110.

Arčabić, V., J. Tica, J. Lee, and R. J. Sonora. 2018. 'Public Debt and Economic Growth Conundrum: Nonlinearity and Inter-Temporal Relationship.' *Studies in Nonlinear Dynamics and Econometrics* 22 (19). https://doi.org/10.1515/snde-2016-0086.

Arellano, C., Y. Bai, and G. P. Mihalache. 2020. 'Deadly Debt Crises: COVID-19 in Emerging Markets.' NBER Working Paper 27275, National Bureau of Economic Research.

Barro, R. 1989. 'The Ricardian Approach to Budget Deficits.' *Journal of Economic Perspectives* 3 (2): 37–54.

Baum, A., C. Checherita-Westphal, and P. Rother. 2013. 'Debt and Growth: New Evidence for the Euro Area.' *Journal of International Money and Finance* 32:809–21.

Bentour, E. M. 2021. 'On the Public Debt and Growth Threshold: One Size Does Not Necessarily Fit All.' *Applied Economics* 53 (11): 1280–99.

Bhorat, H., T. Köhler, M. Oosthuizen, B. Stanwix, F. Steenkamp, and A. Thornton. 2020. 'The Economics of COVID-19 in South Africa: Early Impressions.' DPRU Working Paper 202004, Development Policy Research Unit.

Bitar, N., A. Chakrabarti, and H. Zeaiter. 2018. 'Were Reinhart and Rogoff Right?' *International Review of Economics and Finance* 58:614–20.

Biza, R. A., F. Kapingura, and A. Tsegaye. 2015. 'Do Budget Deficits Crowd Out Private Investment? An Analysis of the South African Economy.' *International Journal of Economic Policy in Emerging Economies* 8 (1): 52–76.

Bouchrara, K., H. Rachdi, and K. Guesmi. 2020. 'The Non-Linear Relationship Between Economic Growth and Public Debt.' *Economics Bulletin* 40 (3): 2336–43.

Burger, P., and E. Calitz. 2021. 'Covid-19, Economic Growth and South African Fiscal Policy.' *South African Journal of Economics* 89 (1): 3–24.

Caner, M., T. Grennes, and F. Koehler-Geib. 2010. 'Finding the Tipping Point: When Sovereign Debt Turns Bad.' Policy Research Working Paper 5391, World Bank.

Casares, E. R. 2015. 'A Relationship Between External Public Debt and Economic Growth.' *Estudios Económicos* 30 (2): 219–43.

Cecchetti, S. G., M. S. Mohanty, and F. Zampolli. 2011. 'The Real Effects of Debt.' BIS Working Paper 352, Bank for International Settlements.

Chang, T., and G. Chiang. 2012. 'Transitional Behavior Of Government Debt Ratio on Growth: The Case of OECD Countries.' *Romanian Journal of Economic Forecasting* 15 (2): 24–37.

Checherita-Westphal, C., and P. Rother. 2012. 'The Impact of High Government Debt on Economic Growth and its Channels: An Empirical Investigation for the Euro Area.' *European Economic Review* 56 (7): 1392–405.

Checherita-Westphal, C., A. H. Hallett, and P. Rother. 2014. 'Fiscal Sustainability Using Growth-Maximizing Debt Targets.' *Applied Economics* 46 (6): 638–47.

Chen, S., and C. Lee. 2005, 'Government Size and Economic Growth in Taiwan: A Threshold Regression Approach.' *Journal of Policy Modelling* 27 (9): 1051–66.

Chen, C., S. Yao, P. Hu, and Y. Lin. 2017. 'Optimal Government Investment and Public Debt in an Economic Growth Model.' *China Economic Review* 45:257–78.

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi. 2017. 'Is There a Debt-Threshold Effect on Output Growth?' *The Review of Economics and Statistics* 99 (1): 135–50.

Cordella, T., L. A. Ricci, and M. Ruiz-Arranz. 2005. 'Debt Overhang or Debt Irrelevance? Revisiting the Debt-Growth Link.' IMF Working Paper 223, International Monetary Fund.

Dombi, Á., and I. Dedák. 2019. 'Public Debt and Economic Growth: What Do Neoclassical Growth Models Teach Us?' *Applied Economics* 51 (29): 3104–21.

Eberhardt, M., and A. F. Presbitero. 2015. 'Public Debt and Growth: Heterogeneity and Non-Linearity.' *Journal of International Economics* 97 (1): 45–58.

Égert, B. 2015. 'Public Debt, Economic and Nonlinear Effects: Myth or reality?' *Journal of Macroeconomics* 43:226–38.

Fosu, A. K., Y. Y. Getachew, and T. H. W. Ziesemer. 2016. 'Optimal Public Investment, Growth, and Consumption: Evidence from African Countries.' *Macroeconomic Dynamics* 20 (8): 1957–86.

Gómez-Puig, M., and S. Sosvilla-Rivero. 2017. 'Heterogeneity in the Debt-Growth Nexus: Evidence from EMU Countries.' *International Review of Economics & Finance* 5:470–86.

Greenwood-Nimmo, M., Y. Shin, and T. van Treeck. 2011. 'The Asymmetric ARDL Model with Multiple Unknown Threshold Decompositions: An Application to the Phillips Curve in Canada.' LUBS Working Paper Series, Leeds University Business School.

Hansen, B. E. 2000. 'Sample Splitting and Threshold Estimation.' *Econometrica* 68 (3): 575–603.

Herndon, T., M. Ash, and R. Pollin. 2014. 'Does High Public Debt Consistently Stifle Economic Growth? A Critique of Reinhart and Rogoff.' *Cambridge Journal of Economics* 38 (2): 257–79.

Kamiguchi, A., and T. Tamai. 2019. 'Public Investment, Public Debt, and Population Aging Under the Golden Rule of Public Finance.' *Journal of Macroeconomics* 60:110–22.

Law, S. H., C. H. Ng, A. M. Kutan, and Z. K. Law. 2021. 'Public Debt and Economic Growth in Developing Countries: Nonlinearity and Threshold Analysis.' *Economic Modelling* 98:26–40.

Liu, Z., and J. Lyu. 2021. 'Public Debt and Economic Growth: Threshold

Effect and its Influence Factors.' *Applied Economic Letters* 28 (3): 208–12.

Makuyana, G., and N. M. Odhiambo. 2018. 'Public and Private Investment and Economic Growth: An Empirical Investigation.' *Studia Universitatis Babes-Bolyai Oeconomica* 63 (2): 87–106.

Mensah, L., D. Allotey, E. Sarpong-Kumankoma, and W. Coffie. 2019. 'What Debt Threshold Hampers Economic Growth in Africa?' *International Journal of Development Issues* 19 (1): 25–42.

Mhlaba, N., and A. Phiri. 2019. 'Is Public Debt Harmful Towards Economic Growth? New Evidence from South Africa.' *Cogent Economics & Finance* 7 (1). https://doi.org/10.1080/23322039.2019.1603653.

Ncanywa, T., and M. M. Masoga. 2018. 'Can Public Debt Stimulate Public Investment and Economic Growth in South Africa?' *Cogent Economics & Finance* 6 (1). https://doi.org/10.1080/23322039.2018.1516483.

Ndoricimpa, A. 2017. 'Threshold Effects of Debt on Economic Growth in Africa.' *African Development Review* 29 (3): 471–84.

———. 2020. 'Threshold Effects of Public Debt on Economic Growth in Africa: A New Evidence.' *Journal of Economics and Development* 22 (2): 187–207.

Pattillo, C., H. Poirson, and I. Ricci. 2002. 'External Debt and Growth.' IMF Working Paper 69, International Monetary Fund.

Pescatori, A., D. Sandri, and J. Simon. 2014. 'Debt and Growth: Is There a Magic Threshold?' IMF Working Paper 34, International Monetary Fund.

Rahman, N. H. A., S. Ismail, and A. R. Ridzuan. 2019. 'How Does Public Debt Affect Economic Growth? A Systematic Review.' Cogent Business & Management 6 (1). https://doi.org/10.1080/23311975.2019.1701339.

Reinhart, C. M., and K. S. Rogoff. 2010. 'Growth in a Time of Debt.' *The American Economic Review* 100 (2): 573–8.

Shin, Y., B. Yu, and M. Greenwood-Nimmo. 2014. 'Modelling Asymmetric Cointegration and Dynamic Multipliers in a Nonlinear ARDL Framework.' In *The Festsschrift in Honor of Peter Schmidt: Econometric Methods and Applications,* edited by R. Sickels and W. Horace, 281–314. New York: Springer.

Smyth, D., and Y. Hsing. 1995. 'In Search of an Optimal Debt Ratio for Economic Growth.' *Contemporary Economic Policy* 13 (4): 51–59.

Yared, P. 2019. 'Rising Government Debt: Causes and Solutions for a Decades-Old Trend.' *Journal of Economic Perspectives* 33 (2): 115–40.