Mathematical Literacy, Mathematical Modeling, and Realistic Mathematics Problems for Sustainability

Darjo Felda
Univerza na Primorskem
darjo.felda@pef.upr.si

Daniel Doz
Univerza na Primorskem
daniel.doz@pef.upr.si

Mara Cotič
Univerza na Primorskem
mara.cotic@pef.upr.si

This paper explores the pivotal role of mathematical literacy, mathematical modelling, and realistic mathematics problems in fostering sustainability education. The study emphasizes the need to move beyond traditional mathematics teaching methods, which often focus narrowly on abstract concepts and rote memorization, towards a more integrated approach that connects mathematics with real-world contexts. By highlighting the importance of horizontal and vertical mathematization, the paper argues for a curriculum that encourages students to apply mathematical concepts to realistic problems, thereby enhancing their problem-solving skills and critical thinking abilities. The paper also examines mathematical modelling as a crucial tool for understanding and solving complex sustainability challenges, illustrating how students can develop and refine their models to better reflect real-life situations. A problem-based learning approach not only improves mathematical competence but also equips students with the necessary skills to tackle environmental, social, and economic challenges.

Keywords: literacy, mathematics, problem-solving, realistic problems, sustainability

https://doi.org/10.26493/978-961-293-403-3.139-156

Introduction

The imperative of sustainability in the face of unprecedented global challenges such as climate change, environmental degradation, and resource scarcity demands innovative approaches grounded in rigorous scientific inquiry (Kuhlman & Farrington, 2010). At the heart of this endeavour lies mathematics – a discipline uniquely equipped to unravel the complexities of natural systems, optimize resource allocation, and inform evidence-based policies (Kuznetsova et al., 2021). In particular, mathematics should develop students' problem-solving abilities, which, in turn, help them address everyday, realistic problems and promote sustainable development (Akinmola, 2014; Putra et al., 2022; Szabo et al., 2020).

This paper explores the pivotal role of mathematics in advancing sustainability. Through mathematical modelling, simulation studies, and solving realistic mathematics problems, students can develop skills that enable them to understand, analyze, and critically think about sustainability. This includes shedding light on the dynamics of ecosystems, predicting the impacts of human activities on biodiversity, and designing resilient urban infrastructures. Mathematics enhances our understanding of environmental processes and empowers decision-makers with quantitative tools to assess trade-offs and devise strategies that balance ecological integrity with socio-economic development.

At its core, mathematics enables scientists and policymakers to quantify and predict the consequences of human actions on natural systems (Kuznetsova et al., 2021), assess risks, and optimize resource allocation strategies (Mordeson & Mathew, 2021). One key application of mathematics in sustainability is through mathematical modelling. Models capture the dynamics of ecological systems, climate patterns, and socio-economic interactions. For instance, differential equations are employed to describe population dynamics in ecosystems, helping researchers understand how changes in species populations affect biodiversity and ecosystem resilience (Doyen, 2018). Similarly, mathematical models of climate systems simulate future climate scenarios under different emissions trajectories, aiding policymakers in formulating climate mitigation and adaptation strategies.

The main aim of the present paper is to analyze the aforementioned methods of teaching mathematics, specifically through realistic problem-solving and modelling. Adopting these methods is paramount to promoting deeper learning, understanding, and the acquisition of skills that are critical to sustainable development.

Mathematical Literacy

The Programme for International Student Assessment (PISA) definition of mathematical literacy is the following (OECD, 2023):

> Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It helps individuals know the role that mathematics plays in the world

> and make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st Century citizens.

From this definition it is possible to understand that mathematical literacy is defined as an individual's ability to recognize and understand the role that mathematics plays in the world, to make well-founded decisions, and to use and engage with mathematics in ways that meet the needs of an individual's life as a constructive and reflective citizen (Repež et al., 2008). Based on the definition of mathematical literacy is it also possible to understand that it promotes and supports sustainable development skills (Nkuturum, 2023).

Mathematical Knowledge and Mathematical Literacy

A mathematically literate person must know some mathematics, but literacy is not entirely dependent on mathematical knowledge (De Lange, 2003). Someone who knows mathematics merely as a set of algorithms to memorize is certainly not mathematically literate (cf. Lin & Tai, 2015). Understanding that allows the application of mathematical ideas to similar examples and new situations is crucial (De Lange, 2003). Mathematical literacy is not directly proportional to the 'amount' of mathematics one knows but rather to how one can apply mathematics in everyday life (cf. OECD, 2023).

Mathematical literacy refers to the ability to use mathematical arguments in life situations, which are inherently diverse and usually more complex than most standard school examples or problems (OECD, 2023). Although mathematics instruction involves tools that could be useful for increasing mathematical literacy, we do not generally develop the appropriate skills and 'flexibility.' Students may be able to apply their mathematical knowledge in a mathematical context but are more or less adept at using this knowledge in everyday life.

Factors in Developing Mathematical Literacy

It is often emphasized that mathematics instruction should develop the following aspects of learning (Maslihah et al., 2020):

- investigation;
- problem-solving;
- creative thinking;
- data processing;
- logical reasoning;
- evaluation of results.

To make learning mathematics creative, students need to be involved in the practical solving of a realistic problem or another mathematical problem that has multiple solution paths (Schoenfeld & Sloane, 2016). Occasionally, students need to be challenged with so-called problem investigations that have multiple solutions and several possible strategies for solving them (Manfreda Kolar & Hodnik, 2021). Creative exploration and problem-solving is an excellent way to develop mathematical concepts and is often a useful tool for consolidating procedures. Through exploration and problem-solving, strategies are primarily honed, often referred to as thinking skills or logical reasoning in a mathematical context. Within this framework, students (Advisory Committee on Mathematics Education, 2008):

- Pose questions and hypothesize possible conclusions;
- Choose strategies and representations;
- Use their thinking skills;
- Prove or disprove claims;
- Critically review, check, and evaluate their work;
- Develop patience and persistence to reach a solution.

At the beginning of schooling, realistic problems that students solve must be very simple. Many studies have pointed out the difficulties students face when solving realistic problems (cf. Pratiwi et al., 2020). These difficulties are often in understanding the problem text and finding the appropriate mathematical content, as students operate quite randomly with the given data without considering their connection to the realistic context. Mistakes in solving are not due to a lack of experience, as it turns out that even the success of solving 'traditional' problems does not significantly improve, even if such problems are repeatedly solved (Renkl & Stern, 1994).

Realistic problems are most often solved by mathematizing a non-mathematical situation, which involves (Cotič & Felda, 2011) (Figure 1):

- Constructing a mathematical model based on the appropriate realistic situation or everyday life situation;
- Solving the constructed mathematical problem;
- Translating the solution of the mathematical problem, that corresponds to the mathematical model, back into the realistic environment.

The biggest obstacle in solving realistic problems is constructing the mathematical model, as it requires knowledge of the context of the realistic problem situation and a certain degree of creativity (Winter, 1994).

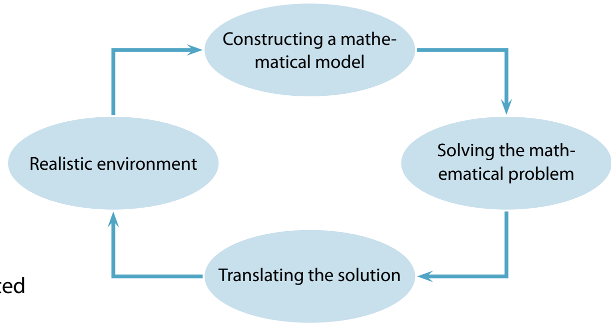

Figure 1

The Process of the Solution of Realistic Problems (adapted from Cotič & Felda, 2011)

Mathematics Instruction and Mathematical Literacy

If we want to effectively foster literacy, we must consistently involve students in using mathematical concepts and algorithms in other subjects and continue with literacy efforts in subsequent stages of education, especially to build up the competencies related to sustainability. One of the negative aspects of literacy is the teaching or learning of mathematics as encountered in schools (cf. Fauzan & Arnawa, 2020). The foundation of mathematical literacy is recognizing and understanding the role of mathematics and its meaningful use in everyday life, which differs significantly from school experiences (OECD, 2023). Students focus on learning problem-solving procedures as demonstrated by the teacher in school and as they have 'practiced' on similar problems at home. Very few students seek general rules that simplify problem-solving, as they do not see the point in learning procedures by heart. Many students prefer memorizing procedures. Teachers encourage this by showing how each problem can be solved using a specific procedure, as if inserting individual values into an equation (Mann & Enderson, 2017).

Several studies have shown that students do not recognize the problem situation in word problems (cf. Verschaffel et al., 2002). They cannot meaningfully connect the final result with the problem text, and they uncritically accept the result obtained by strictly following the learned procedure as correct.

Ordinary mathematical problems or word problems encountered in mathematics are still quite 'rigid' (Cotič & Valenčič Zuljan, 2009). Word problems, as a reflection of real-life situations, should attract students to use their everyday experiences. In reality, students perceive them as a sort of nuisance where data appears in the text instead of being written as 'expected' in mathematics, that is, in the form of calculations, equations, or at least in an orderly column, so they can directly insert them into an appropriate formula. They do not take the text as a description of the problem situation, often failing to read it carefully enough to understand it. When practicing subtraction in school, it is already known in advance that subtraction will be necessary for the numbers appearing in the problem text. The problem text is quickly skimmed, the numbers are found, and a subtraction operation is written without focusing on the described problem situation and discussing it. The same is repeated in homework.

Mathematical Problems for Sustainability

Realistic Mathematical Problem

Problems encountered in everyday life, which we solve with the help of mathematical knowledge, are usually open-ended problems with open-ended goals. These are generally very complex problems with a lot of data, most of which is irrelevant, and it is not necessarily the case that we immediately find all the data needed to solve the problem. Typically, problems can be solved in different ways and have multiple solutions, and each individual chooses the solution or solutions that are most suitable or possible at the given moment. In mathematics teaching and during schooling, we encounter only such realistic problems that are sufficiently simplified reflections of the real problematic situation and thus adapted to the student's developmental stage.

Solving a Realistic Mathematical Problem

Solving a realistic mathematical problem requires additional steps occur due to the 'translation' of the realistic problem into mathematical language and the translation of the mathematical solution back into everyday language, which corresponds to the realistic problematic situation (Cotič & Felda, 2011).

Through mathematization or appropriate modelling, we should transition from the real situation into the mathematical domain, where the problem can be solved when translated into mathematical language. Then the mathematical solution is interpreted in the language of the real problematic situation (Müller & Wittmann, 1984). Students in the early years of schooling do not perceive mathematics as a tool for solving realistic problems. For them, mathematics represents a set of different things or objects, such as balls, blocks, sticks, etc., with numbers. Thus, the modelling situation is actually reversed for them, indicating a process of visualization or illustration (Peter-Koop, 2004).

By manipulating objects in a real situation, a new real situation is created that represents the result of the subtraction. The focus here is not on manipulating mathematical objects but on manipulating real objects. This means that 'standard' word problems are not the most suitable for developing mathematical modelling skills, as the text itself guides the selection of the appropriate mathematical operation or procedure.

Developing Mathematical Literacy

More complex problems, which challenge students, are suitable for developing mathematical literacy (Manfreda Kolar & Hodnik, 2021). At the beginning of schooling, it is better for students to solve these problems in small groups, as collaborative work helps them find suitable solving strategies and discuss critical points, searching for optimal solutions. This way, each individual gains experience and builds mathematical literacy (Hadiyanti et al., 2021). Problems should be sufficiently 'open' so that the solving process requires students to make justified decisions based on the mathematical model they use (Peter-Koop, 2004).

Importance of Understanding the Problem

Understanding the problem is crucial for its solving (Polya, 1945). The problem's formulation must match the developmental stage of the students and consider their communication abilities and language knowledge. Solving realistic mathematical problems involves two language connections: one with the written or spoken text, which must be understood in everyday language terms, and the other with mathematical language as a set of symbols and mathematical concepts that must be mastered to perform the 'mathematical' part of the solving process.

Difficulties students have in one or both areas, or presenting a realistic problem whose understanding and potential solving methods exceed students' competencies, lead to poorer performance and slower development of mathematical thinking.

Overcoming Deficiencies through Discussion

Discussions involving the realistic problem topic, modelling, finding a suitable strategy to solve the arising mathematical problem, and interpreting the result or its appropriate real representation are extremely beneficial for exchanging opinions and experiences and learning problem-solving strategies. The teacher must ensure that even students who often unsuccessfully tackle problems on their own are included in the discussion. Such students tend to remain passive, and their rare answers or suggestions are simple and often unclear.

Emphasizing Real Situations

For developing mathematical literacy, it is necessary to emphasize real situations rather than problems that have already been 'stripped of unnecessary situational elements.' Such problems are, in a sense, already mathematized. Their solving can be represented by a model like the one developed by Pollak (1979).

According to this model, there is no phase between the real situation and the mathematical model. The problem is actually ready for mathematical processing, so the mathematical solution of the problem is simultaneously a solution that fits the real situation.

Problems from everyday life usually require thorough consideration; relevant data must be extracted, and the right solving strategy that leads to the mathematization of the problem must be found. The mathematical solution then needs to be verified, as it does not necessarily align with the everyday life situation. Blum and Leiss (2007) illustrated the process of solving such a problem with a cycle model According to this model, the solver of a realistic problem first constructs a model of the situation. This model reflects the understanding of the real situation, indicating how the solver actually sees or experiences the real situation described in the problem. The model of the situation depends on the individual, their personal understanding of the real situation, and their experiences with similar real situations. It is not necessarily true that the model corresponds to the situation since it often happens that especially students at the beginning of schooling do not pay enough attention to all the circumstances indicated by the given realistic problem.

Usually, the real situation is complex, and the model is already a somewhat simplified version where the solver omits some information and focuses on those that are essential for solving the given realistic problem. The transition from the real situation to the model is not systematic-it is merely a kind of subjective experiential view of the situation, showing the individual's tendency to approach similar situations in a particular way.

The model of the situation forms the basis for creating a realistic model. The solver consciously idealizes and simplifies the model of the situation, which for them is actually the real situation or that part of the real situation they perceive as crucial for solving the given realistic problem. Knowledge and insights, not necessarily mathematical or not solely mathematical, lead the individual to a realistic model.

A realistic model can be presented with an appropriate picture or descriptively, relying more or less on the model of the situation. In the phase of mathematization, thought processes shift from the realistic model to mathematical content. Appropriate pictures or descriptions take on mathematical symbolism or language, resulting in a mathematical model of the real situation-a mathematical problem. By using a suitable problem-solving strategy and chosen mathematical procedures, the solver arrives at a mathematical solution or result.

By interpreting the mathematical result, which arises from the mathematical solution of the mathematized realistic model, we return to the realistic model, which has meanwhile transformed into a realistic solution. The interpretation process involves establishing connections between the mathematical solution and the model of the real situation's solution.

The solver of a realistic problem thus faces the important task of verifying and confirming the consistency of the realistic solution with the model of the situation, which actually represents their understanding of the given real problem situation. Many children often exhibit intuitive verification of consistency when they cannot explain why they have adopted or rejected a particular solution. They rely on their feelings or experiences. Of course, students must be encouraged to verify the realistic solution and confirm decisions based on an appropriate explanation for the development of mathematical literacy.

Justification, which is based on the model of the situation as created by the problem solver, is also important for the exchange of opinions with other solvers who have created different models of the situation. The exchange of opinions means presenting the solution to the initial problem situation by multiple solvers, who thus either confirm the identity or similarity of the solution or point out certain discrepancies that require further appropriate handling of the problem situation.

There is often a slightly different depiction of solving a realistic problem, in which it is particularly emphasized that the preparation of the real model and the mathematization also involves the use of non-mathematical knowledge (Borromeo Ferri, 2006) if the given task requires it.

In this model, the first phase involves constructing a mental representation of the situation while understanding the given task. The problem-solving process concludes with verifying the realistic result and confirming its consistency with the mental representation of the situation, which reflects how the solver understood and envisioned the real situation. In our opinion, the connections between the mental representation of the situation and the actual real situation should also be addressed at the end. Several solvers encounter potential deviations due to different 'views' of the real situation or varying understandings of the task associated with the realistic problem. Individual solvers will identify potential missteps in certain phases or procedures of solving the realistic problem and focus on making appropriate corrections.

In our view, the depiction of solving a realistic problem is inadequate from the perspective of utilizing non-mathematical knowledge. It is true that, in addition to mathematical knowledge, other knowledge is required when transitioning from the mental representation of the situation to the realistic model and from the latter to the mathematical model. It is particularly important to emphasize the experiences and problem-solving strategies that the solver has already developed. Non-mathematical knowledge is also necessary and crucial when transitioning from the mathematical result to the realistic result and from the realistic result to the mental representation of the situation. When interpreting the mathematical result and verifying whether the realistic result aligns with the data and information from the mental representation, to complete the 'phase cycle,' we are no longer in the 'mathematical field' – we must rely on non-mathematical knowledge or everyday life insights.

It is also important to focus on the first phase of problem-solving. The mental representation cannot be merely a reflection or a mental image of the real situation. Before the solver moves towards creating a realistic model, they develop a mental representation that generally is not identical to the real situation. In the representation, those elements of the real situation that the solver believes are not crucial for solving the given realistic problem 'fade,' while those that will be included in the subsequent phases of the problem-solving process are 'strengthened.'

Thus, already in the first phase, the solver intervenes in the process and modifies the objective reality of the real situation. Here, their experiences, if they have encountered a similar situation before, or their beliefs are crucial. It is necessary to add a final phase, which is the connection between the mental representation of the situation or the model of the situation, as it was named in the previous depiction, and the real situation. The solution to the realistic problem, which the solver has accepted and incorporated into the mental representation, must be situated in the real situation and checked for consistency with all elements of that situation. The solver receives confirmation that they have developed an appropriate mental representation, which they can recall and suitably adjust for new parameters of a similar realistic problem in the future.

It is necessary to recognize that realistic problems are complex, and therefore students should be appropriately guided and directed when they first encounter them in the early stages of their education. To avoid exerting too much influence on their problem-solving process, which they should develop as independently as possible, we should offer students less complex problems from which they can more easily extract the necessary information and data to create an effective and sufficiently simple model of the situation. However, we must not prevent students from solving realistic problems on the grounds that their knowledge is not yet effective enough, as they need to continually develop and enhance appropriate strategies for solving realistic problems, incorporating their knowledge and skills.

Mathematical Modelling and Mathematical Literacy

Mathematical modelling is important because it helps in recognizing mathematical models in our everyday environment, thus (Blum & Borromeo Ferri, 2009):

- It helps children and adolescents understand the world;
- It supports learning mathematics in terms of motivation, concept formation, and understanding;
- It aids in developing various mathematical competencies and associated skills;
- It helps in understanding mathematics as a discipline.

Using modelling makes mathematics more meaningful to students because typical word problems are often just an artificial 'real-life' disguise for mathematical problems that present no challenge to the student – engaging with these tasks is extremely boring math-for-math-sake.

Mathematical modelling is also challenging for teachers, as it requires not only mathematical knowledge but also knowledge and experience gained from everyday life. Many teachers fear that students will not acquire the knowledge or achieve the appropriate learning objectives. According to their belief, only with 'traditional' teaching methods can they determine if the learning goal has been achieved. Modelling seems to them 'scattered' and time-consuming, beyond their control.

As mentioned earlier, students have difficulties with modelling, as it requires complex cognitive processes. The very first step, when it is necessary to create a model of the situation, is a real challenge for students. If the realistic problem is presented as a word problem, they need to immerse themselves in the appropriate problem situation and apply knowledge and experience from everyday life. Particularly, students at the beginning of their education prefer to look for numerical data and use a randomly chosen operation, even if the resulting outcome makes no sense. This flawed pattern of solving word problems is common even among older students; successful solving of mathematical problems in a 'real-life' guise becomes dependent only on luck in choosing the right operation or procedure and handling the data correctly.

Realistic mathematics education focuses on guiding the student to 'invent' mathematics through mathematization, considering the student's informal strategies for solving realistic problems and their interpretations of the solution or the path to solving it. A realistic problem is not necessarily from everyday life but from a situation that is not new to the student and that they have experienced. This means it can also be a 'pure' mathematical problem – a problem from the abstract world of mathematics. Students capable of abstraction can expand and deepen their mathematical knowledge by facing suitable abstract mathematical problems, while others can deepen and expand it only with sufficiently tangible models or problems from the concrete world.

An important phase in realistic mathematics education is the so-called horizontal mathematization, where the student uses their informal knowledge to describe the situation and explain the solution to the problem. When informal strategies lead the student to solve the problem using mathematical language or an appropriate algorithm, it is vertical mathematization.

Mathematics teaching in our schools is too focused only on vertical mathematization. The teacher usually presents a certain mathematical content and some examples of applying the new knowledge, followed by exercises and homework. These exercises or tasks are intended more or less for the mechanical repetition of the procedure presented by the teacher. The learning process in such teaching lacks horizontal mathematization; students experience only formal vertical mathematization. This means that students do not use their knowledge and do not build a bridge between their already acquired knowledge and new knowledge; instead, they place the 'novelty' in their memory as just another piece of information. If they do not later recall this information from memory or recall it in connection with a problem that does not fit, they do not solve the new problem they are faced with.

We must recognize that mathematization is an activity through which acquired knowledge and skills are used to discover new rules, connections, and structures. With mathematization, we acquire new knowledge and concepts, refine language, and acquire and practice skills for successfully solving problems that can be set in a mathematical context or not. In particular, horizontal mathematization is a kind of schematization of the situation, allowing the introduction of mathematical tools. The activity that follows, related to mathematical processes or algorithms, solving the problem, generalizing the situation, and further formalization, is vertical mathematization. It is not always easy to determine where horizontal mathematization transitions into vertical mathematization (Treffers, 1987).

For successful learning of mathematics, both horizontal and vertical mathematization are necessary, as this enables the active participation of each individual in acquiring knowledge. Students need enough time to bridge the gap between their informal knowledge and formal mathematical knowledge. This is not a linear but a cyclical process – students should acquire the strategy of cyclically transitioning between horizontal and vertical mathematization to refine their understanding of mathematics.

Recommendations

Based on the findings of the research of Felda (2011) and Cotič and Felda (2011), policymakers and educators should thoroughly change the way mathematics is taught and learned, where so-called factual knowledge holds a central place, and even problem-solving is subordinated to the learning of procedures.

The National curriculum emphasizes the importance of problem-solving skills (Ministrstvo za šolstvo in sport in Zavod Republike Slovenije za šolstvo, 2011) and there have been increasing efforts for so-called subject integration, where mathematics is often meant to be 'connected' with other subjects (Krek et al., 2008), ultimately with the sustainability goals set by the United Nations. It seems that everyone remains confined to their 'field,' and any potential integration classes are treated merely as a necessary evil imposed 'from above.' Even primary school teachers, who impart knowledge from various 'fields,' do not recall any other during a specific 'field' lesson. In this way, students learn to 'compartmentalize' mathematical knowledge, whatever it may be, into a precisely defined slot that must be opened when mathematics is on the schedule.

In today's world, where we aim to convey much more than just routine knowledge to students, the desire and demand for teaching problem-solving skills in mathematics are even more prevalent. From the perspective of mathematics didactics, we can identify fundamental elements of problem-solving skills (metacognition, communication, heuristics, attitudes and prejudices, expertise, etc.), but it is not possible to definitively define how to teach these skills effectively (Cotič & Felda, 2011).

Based on research (Felda, 2011; Cotič & Felda, 2011), it is evident that students acquire higher-quality knowledge when they are actively involved in the learning process. This includes regular discussions with teachers and peers about their findings and concerns, and being open to different perspectives or justifications. Integrating new mathematical knowledge into students' cognitive frameworks enhances their level of mathematical literacy. It is natural for the applicability of mathematical knowledge to be evident in real-world situations. Thus, it is beneficial for students to experience realistic problem-solving scenarios while learning mathematics, as this helps them connect new knowledge with prior learning, considering informal knowledge and experiences.

In this teaching model, the authors introduced 'project sessions' where students faced problem situations and solved various real-world problems, extracting necessary data themselves or concluding that certain problems could not be solved due to insufficient data (Felda, 2011; Cotič & Felda, 2011). For effective mathematics teaching, it is essential to ensure that students acquire and retain practical mathematical knowledge, thereby continuously raising their level of mathematical literacy. This does not mean simplifying mathematics to a form of 'handicraft;' a well-educated individual should be able to effectively use their knowledge in school, work, and everyday life. Modern assessments focus on how well an individual can apply knowledge, logically connect it to practical use, and critically evaluate their solutions.

During schooling, students encounter multiple subjects, often treated as isolated worlds. It is illogical to claim that mathematics is omnipresent while teaching it solely in math classes. The above-mentioned teaching model emphasizes interdisciplinary connections, also with sustainability, enabling students to experience problem situations that integrate knowledge from different subjects, thus giving meaning to both mathematical and other subject knowledge while linking it to their experiences. Isolated knowledge segments from individual subjects are practically unusable, especially when learned just to be recited to a teacher. Hence, achieving the goals of implementing sustainability in regular classes requires proper interdisciplinary connections based on content relationships or planned student activities that foster lifelong learning skills.

A problem-based approach, where students encounter concepts in problem situations or directly through problem-solving, is preferable to a purely mathematical one. It is important that problem situations are tailored to the student, allowing them to use prior knowledge and experiences when learning new concepts and strategies. This approach acknowledges mathematics as an important tool for solving everyday problems and understanding it as a discipline. In this way, skills for sustainable development might be strengthened.

Students build their mathematical models based on their experiences and continuously refine and abstract them according to their capacity for abstract thinking. It is crucial that mathematics teaching is connected to real-world experiences. This way, all students can follow mathematics instruction by creating mental models based on real situations or objects, which they can recall and use for building further mathematical knowledge. It is important for students to be active participants in their learning, rather than being led to mechanistically accept concepts, rules, and procedures for practical use.

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Matematična pismenost, matematično modeliranje in realistični matematični problemi za trajnostnost

Prispevek razišče ključno vlogo matematične pismenosti, matematičnega modeliranja in reševanja realističnih matematičnih problemov pri spodbujanju izobraževanja za trajnostnost. V prispevku poudarimo potrebo po prehodu od tradicionalnih oblik poučevanja matematike, ki se pogosto osredotočajo zgolj na abstraktne pojme in učenje na pamet, k bolj integriranemu pristopu, ki matematiko povezuje z resničnimi konteksti. S poudarjanjem pomena horizontalne in vertikalne matematizacije prispevek zagovarja kurikulum, ki učence spodbuja k uporabi matematičnih konceptov pri reševanju realističnih problemov, s čimer izboljšujejo svoje sposobnosti reševanja problemov in kritičnega mišljenja. Prispevek prav tako obravnava matematično modeliranje kot ključno orodje za razumevanje in reševanje zapletenih trajnostnih izzivov ter prikazuje, kako lahko učenci razvijajo matematične modele, da bi bolje odražali resnične situacije. Pristop k učenju, ki temelji na problemih, ne le izboljšuje matematično pismenost, temveč tudi opremlja učence s potrebnimi veščinami za reševanje okoljskih, socialnih in ekonomskih izzivov.

Ključne besede: pismenost, matematika, reševanje problemov, realistični problemi, trajnostnost