The Development of Strategies for Solving Arithmetic Word Problems: Using Keywords or Diagrams?

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Solving simple arithmetic word problems is a major ability that children must acquire while in primary education. Yet, many students struggle with this maths task. The aim of the present contribution is to give an overview of how to effectively support problem-solving abilities in school-age children. For many decades a very popular problem-solving practice has been the keyword strategy. By describing difficulties that students encounter with inconsistent language problems (i.e. problems where the language featured in the problem text steers the student towards an inappropriate mathematical operation), we highlight the superficiality and inappropriateness of this approach. In future, practitioners should rather focus on teaching children to integrate the problem's textual information into an adequate mental representation, which is the basis for a successful solution strategy. As such, two methods that emphasize the use of visual representations, such as diagrams, are described.

Keywords: word problems, keywords, diagram, schema-based instruction, primary school

Introduction

The ability to solve arithmetic word problems is a crucial component of mathematics curricula in primary education (Daroczy et al. 2015). Arithmetic word problems can be defined as a particular type of arithmetic problems presented in a verbal rather than numerical formulation (Verschaffel, Greer, and Corte 2000). They represent a fundamental learning activity since they prepare students to connect mathematics to real-world situations (Depaepe, Corte, and Verschaffel 2010; Pongsakdi et al. 2020). It is thus not surprising

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that arithmetic word problem solving is a strong school-age predictor of future employment and quality of life (Batty, Kivimäki, and Deary 2010). Yet, challenges in mathematical word problem solving have long been documented and represent a common phenomenon worldwide (Carpenter et al. 1980; Cotič and Valenčič Zuljan 2009; Fuchs et al. 2020).

Why do children struggle when solving arithmetic word problems? What sort of instructional strategies can be provided to support children in such maths tasks? The aim of the present contribution is to give an overview of what is known (past) and what can be implemented (future) regarding the teaching and learning of arithmetic problem solving in school-age children. First, a brief description of the cognitive processes involved in word problem solving is given. Next, difficulties with inconsistent language problems (i.e. problems where the language featured in the problem text evokes an inappropriate mathematical operation) are analysed, highlighting the superficiality and inappropriateness of the popular keyword strategy. We conclude by describing two effective instructional methods for enhancing word problem solving that enable children to deeply understand the problem situation.

The Complexity of Arithmetic Word Problems

Consider the following two arithmetic problems: (1) 25 + 8, and (2) At Walmart, a black jacket costs 25 euros. This is 8 euros less than a black jacket at Target. How much will you pay for a black jacket at Target? Although both situations require the same computation (i.e. 25 + 8 = 33), children encounter more difficulties when solving the second one. Carpenter and colleagues (1980) found that pupils performed 10 to 30% worse on arithmetic word problems compared to similar numerical expressions. The discrepancy in performance on the numeric and verbal format clearly suggests that the latter is a complex maths task (Duque de Blas, Gómez-Veiga, and García-Madruga 2021) and that factors other than arithmetical skills are involved in the word problem solving process (Lin 2021).

Mayer (1992) argued that the problem solving process is primarily composed of two phases: problem representation and problem solution. Problem representation occurs when an individual seeks to understand the problem. Critical components of this phase include (1) translating (reading and comprehending the words and the sentences of the word problem), and (2) integrating (integrating and relating all solution-relevant elements into a coherent mental representation of the problem situation to derive the semantic structure of the problem). The problems solution phase occurs when the solver actually carries out the actions needed to solve the problem and to

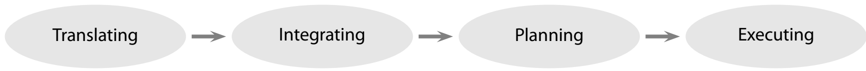

Figure 1 Components or Cognitive Processes of Word Problem Solving Proposed by Mayer (1992)

Table 1 Different Types of Arithmetic Word Problems Offered in Primary Education

Problem type	Example of word problem
Combine	Jenny has 14 red crayons and 5 blue crayons. How many crayons does she have altogether?
Change	Jenny has 14 crayons. Evelyn gives her 5 crayons. How many crayons does Jenny have now?
Compare	Jenny has 14 crayons. Evelyn has 5 crayons more than Jenny. How many crayons does Evelyn have?

reach the solution. This phase includes (1) planning (devising a plan for the solution), and (2) executing (executing the plan and the arithmetic computation).

A more comprehensive model of the cognitive processes involved in problem solving has been proposed by Passolunghi, Lonciari, and Cornoldi (1996) and Lucangeli, Tressoldi, and Cendron (1998). In this model the problem-solving process begins by text comprehension (stage 1); the next step is the representation stage (stage 2), in which the learner creates a mental model of the problem situation by integrating the linguistic and numerical information. Afterwards, the solver identifies the mathematical structure or mathematical problem model by recognizing the problem type (stage 3). Then, he or she creates a solution plan (stage 4) according to the identified type's solution method. Finally, after executing the mathematical operation(s) the learner checks the reasonableness of the mathematical outcome and evaluates the entire problem-solving process (stage 5).

Arithmetic Word Problem Types: Compare Problems

Several different types of arithmetic word problems are presented in school settings. Riley, Greeno, and Heller (1983) identified three types of simple word problems that are frequently offered in primary education: combine, change, and compare problems (see table 1). In the current contribution we will focus on compare problems, which have been demonstrated to be significantly more difficult than other problem types, although they all share a similar corresponding maths (Boonen and Jolles 2015; Schumacher and Fuchs 2012; Stern 1993).

Compare word problems contain a relational statement (e.g. more than, less than) that compares in a static manner the numerical values of two variables (Jitendra et al. 2007). Based on the semantic of the relational term, we can distinguish two subtypes of compare problems (Hegarty, Mayer, and Green 1992): consistent and inconsistent problems. In consistent problems, the relational term semantically aligns with the required mathematical operation. An example of a consistent problem is presented in table 1. We can notice that the relational term 'more than' present in the problem text is consistent with the arithmetic operation needed to solve the problem (e.g. addition). In contrast, in inconsistent problems the relational statement is inconsistent or incoherent with the required mathematical operation. To give an example of an inconsistent problem, consider the following problem: 'Jenny has 14 crayons. She has 5 crayons more than Evelyn. How many crayons does Evelyn have?' The featured adverb 'more' semantically evokes the concept of addition; however, the correct solution necessitates a subtraction (e.g., 14 – 5).

Several studies have documented that students make a higher number of errors and take a longer time to solve inconsistent problems compared to consistent ones (see Daroczy et al. 2015). We refer to this phenomenon as the *lexical consistency effect* (Hegarty, Mayer, and Monk 1995). Interestingly, the most frequent error in inconsistent problems is a *reversal error* in which the solver incorrectly applies the operation that is primed by the relational term (e.g. addition when the relational term is 'more than' and subtraction when the relational term is 'less than'), although the opposite operation is required.

The Role of the Mental Model

The lexical inconsistency effect could be related to the use of suboptimal solving strategies. According to Hegarty, Mayer, and Monk (1995), there are two solving procedures for arithmetic word problems: (1) the *direct-translation strategy,* a shortcut approach focused on 'grabbing numbers and keywords' and then applying the corresponding arithmetic operation(s), and (2) the *problem model strategy,* a meaningful approach in which the problem text is translated into a mental model of the problem situation in order to derive the mathematical event. The authors postulated that when confronted with an arithmetic word problem, unsuccessful problem-solvers rely on the direct-translation strategy, meaning that they search for numbers and keywords from the problem text and use that keyword to determine the operation needed to find the solution. In this respect they bypass the phase of creating a mental representation of the problem situation. In contrast, good problem-solvers are more likely to employ the problem model strategy: they begin by trying to construct a mental model of the situation described in the problem and plan their solution on the basis of this model (Pape 2003). This theory received support from several empirical studies. Hegarty and colleagues (1995) compared eye fixations of successful and unsuccessful problem-solvers when attempting to solve consistent and inconsistent problems and found that unsuccessful problem-solvers focused significantly more on relational terms and numbers compared to their successful peers. This finding from this eye-tracking study confirmed the idea that unsuccessful problem-solvers use the direct-translation strategy.

As far as consistent problems are concerned, the superficial direct-translation approach can still result in accurate solutions. Indeed, in consistent word problems the required mathematical operation can be derived straightforwardly from the keyword. There is no need to internally or externally represent the described problem situation to reach the solution (Koning et al. 2022).

On the other hand, in inconsistent word problems the required mathematical operation cannot be directly derived from the relational term because the language employed in the word problem evokes an inappropriate mathematical operation. Thus, the superficial direct-translation strategy is inadequate to accurately solve the exercise. Rather, solvers need to engage with the problem model strategy and construct a coherent mental representation of the problem situation. In doing so, the solver needs to engage in additional cognitive processing: integrating text information, selecting relevant information and excluding the irrelevant, inferring missing elements, and, most importantly, dealing with inconsistent language (Kintsch and Greeno 1985).

From a psychological point of view, the construction of the mental representation of the problem requires several cognitive abilities. Besides good reading comprehension skills necessary to properly understand the problem text (Fuchs et al. 2015), working memory seems to play a pivotal role (Andersson 2007). Working memory is defined as a limited capacity cognitive system that allows individuals to hold and simultaneously manipulate information over brief periods of time (Baddeley and Hitch 1974). Thus, it allows the solver to maintain, integrate and organize verbal and numerical information retrieved from the text into a mental representation. Additionally, several studies highlighted the importance of inhibition and updating in problem-solving and, specifically, in the construction of the mental model (Passolunghi et al. 2022). Inhibition is defined as the ability to suppress irrelevant information and to inhibit dominant or prepotent responses, whereas updating represents the ability to replace outdated and irrelevant information with new and correct information. In a recent study, Passolunghi and colleagues (2022) explored the role of inhibition and updating in solving one-step and two-step consistent and inconsistent problems in a sample of fourth and fifth graders. The authors found that inhibition was a significant predictor of performance in both consistent and inconsistent problems, even after controlling for students' reading comprehension and intelligence. Indeed, to successfully solve a word problem one must inhibit all the nontarget and irrelevant linguistic and numerical information extrapolated from the text and retain in memory only the solution-relevant elements. In contrast, solvers' inability to properly suppress irrelevant information may generate an inadequate representation of the problem and therefore reflect a higher number of errors. Consistent with these findings, Passolunghi and Siegel (2001, 2004) showed that children with poor problem-solving ability had an impairment in inhibitory processes. However, findings from Passolunghi et al. (2022) also showed that in more complex problems (two-step inconsistent problems) inhibition could lose its relevance in favour of updating abilities. Updating is a more complex cognitive skill since it involves both inhibition of no longer relevant information and its substitution with new information. The updating skills would be particularly important in inconsistent problems where the mathematical operation evoked by the relational term must be firstly processed, but then inhibited and replaced with the opposite operation. It could be therefore speculated that lexical inconsistency and the problem's difficulty may increase the demand on the solver's ability to update and integrate information in order to create a logical mental representation of the problem.

Taken together, constructing a coherent mental representation of the situation described in the problem is the heart of successful problem solving. If solvers do not sufficiently engage in the cognitive processes when solving an inconsistent problem (e.g. do not inhibit and update inconsistent language) or do not construct a mental representation at all (e.g. rely on the direct-translation approach), they will most likely solve the word problem incorrectly (Schumacher and Fuchs 2012).

How to Enhance Children's Word Problem Solving Skills?

Keyword Strategy

For many decades a very popular method to support children's problem-solving abilities has been the *keyword strategy* (Kwok et al. 2022). This method helps to link the language in word problems to the mathematical operation(s) by encouraging learners to circle, underline or highlight the keyword(s) in the problem text and then to perform the operation(s) evoked by the linguistic marker(s). Interestingly, this approach was the most popular problem solving strategy reported by teachers in 2013 (Pearce et al. 2013).

However, as previously mentioned, the keyword strategy is a misleading problem solving practice that often leads to an incorrect solution. In fact, this strategy seems to be quick and effective when solving simple and consistent word problems, but it is not appropriate for inconsistent problems (Koning et al. 2017) and, in general, more so for non-conventional word problems such as problems with no solution, problems containing an insufficient amount of data for solution and multiple-solution problems (Cotič and Valenčič Zuljan 2009). According to Van de Walle (2004), the keyword strategy sends a completely wrong message about doing mathematics. By employing this approach students overlook the meaning and semantic structure of the word problem. Carpenter and colleagues (1980) warned that a keyword-based problem solving strategy does not support the development of reasoning skills and creative thinking necessary for approaching more complicated and unfamiliar problems. Indeed, creativity and creative researching are fundamental elements of maths problem solving (Cotič and Felda 2011) that should be encouraged in order to develop better maths reasoning skills. In this respect, Cotič and Valečič Zuljan (2009) found that nine-year-old students who received a problem-based instruction focused on non-conventional word problems (e.g. problems with less data than needed for arriving at a solution, problems that included more data than needed for arriving at a solution, problems that could not be solved without a sketch, drawing or additional computation, and problems with more than one line long text) displayed greater ability in solving difficult word problems compared to the group that received conventional maths instruction.

Finally, it must be noted that the repeated engagement of the shortcut approach may result in limited opportunities for representing the mental models. This may further result in difficulties in identifying different problem types and, thus, lower word problem solving performance. Therefore, we discourage educators and teachers from instructing children, especially those who manifest a mathematical learning disability, to use a strategy based on keywords selection.

The Use of Diagrams

Practitioners should rather focus on teaching children to integrate a problem's textual information into an adequate mental representation, which is the basis for a successful solution strategy. As such, a large number of studies in the past decades have emphasized that the use of visual and concrete representations, such as diagrams, is an effective method to lessen difficulties in arithmetic word problem solving (Ayabe et al. 2022; Boonen et al. 2014; Hembree 1992; Jitendra et al. 2007). Diagrams are visual tools that enable learners to identify the relevant information in the problem text, organize and integrate it into a coherent mental model, comprehend the semantic relationships between numerical variables, and recognize the underlying arithmetical operation needed for the solution (Jitendra 2002). Moreover, drawing diagrams facilitates self-explaining which in turn leads to deeper understanding of the problem situation (Ainsworth and Th Loizou 2003). Diagrams also provide children with strategies that reduce the cognitive demands involved in problem-solving (Fuchs et al. 2021).

Numerous types of diagrams exist and can be used in word problem-solving. However, it must be noted that not all diagrams are equal in helping to generate the correct solution (Ayabe et al. 2022). Indeed, each diagram provides a different 'representational guidance' (Suthers 2003). Below, we will introduce two types of diagrams particularly effective in solving compare problems: the model method and the schema-based instructions.

Model Method

The *model method* is a graphical approach for supporting word problem-solving that first originated in Singapore (Kho 1987; Ng and Lee 2009). Now it is increasingly used in various countries worldwide (Kaur 2019). The main aspect of the model method is to draw a bar diagram consisting of a series of rectangles that graphically depict the problem situation (i.e. the semantic or mathematical structure of the problem) (Kho 1987; Ng 2004). An example of the model method applied to an inconsistent compare problem is presented in figure 2.

Solving arithmetic word problems with the model method involves essentially three phases (Koning et al. 2022). The first phase is focused on the problem text: the solver reads the given word problem with the intention to identify the known as well as the unknown variables, quantities, and relations. For instance, if we consider the word problem presented in figure 2, the student must identify the names of the two girls (variables), the number of crayons (quantity), and relational term (relation). In the second phase, the learner represents the identified text information graphically using the bar diagram. Particularly, the solver draws a set of rectangles where each rectangle represents the quantity of a variable. The longer the rectangle, the larger the variable

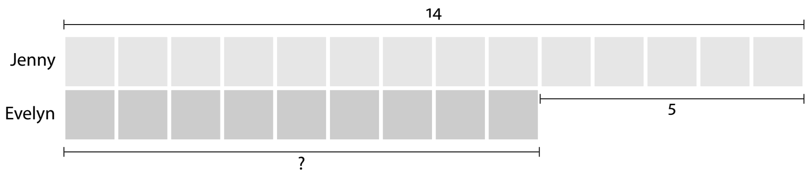

Figure 2 Model Method (Bar Diagram) Representing the Variables, Quantities, and Relations of an Inconsistent Problem

quantity; the shorter the rectangle, the smaller the variable quantity. If we again consider the word problem presented in figure 2, in this phase the child needs to draw a rectangle that represents the number of Jenny's crayons (known variable) and a rectangle for the number of crayons owned by Evelyn (unknown variable). Notably, to correctly draw the bar diagram there should be a constant coordination and integration between the first phase and the second phase. In fact, when the child depicts the first piece of information (e.g. the rectangle representing Jenny's crayons), then he or she must refer again to the text in order to draw the next piece of information (e.g. the rectangle representing Evelyn's crayons). By focusing on the relational statement, the learner reflects on the relation between the two variables (i.e. identifies which variable is bigger), which is an essential step to determine the length of the two rectangles. In the given example, the solver should understand that Evelyn has fewer crayons than Jenny and therefore draw a smaller rectangle for Evelyn. The second phase is completed when all pieces of information are combined into a series of rectangles (i.e. bar diagram). In the third and last phase, the visual representation of the bar diagram drawn in the previous phase helps the learner to decide which operation needs to be performed and to formulate the mathematical equation required to solve the problem. For instance, if we examine the bar diagram presented in figure 2, it is very clear that Evelyn has fewer crayons than Jenny. Specifically, from the visual-schematic representation we can infer that Evelyn has 5 crayons less than Jenny, who has 14 crayons. Thus, the equation 14 – 5 is established.

Prior research has shown the model method is an efficient graphical strategy to improve word problem solving performance in typically developing children (Ng and Lee 2005; Ng and Lee 2009), as well as students with learning disabilities (Sharp and Dennis 2017). Koning and colleagues (2022) tested the effectiveness of the model method in solving consistent and inconsistent word problems. The findings showed that drawing the bar diagram improved the performance on both consistent and inconsistent problems, but the strongest benefits were found for inconsistent ones. Nevertheless, it must be noted that the accuracy of the drawn diagram (i.e. accurate or inaccurate diagram) played an important role and was tightly related to the correctness of the performance (i.e. correct or incorrect answer, respectively), and this effect was again more pronounced for inconsistent problems. That is, drawing an accurate bar diagram led to a better word problem-solving performance compared to not drawing, whereas drawing an inaccurate bar diagram resulted in lower problem-solving performance than not drawing, especially for inconsistent problems. The results highlight the importance of drawing accurate diagrams. If students do not take drawing as an opportunity to analyse the word problem meaningfully (or are not successful in this), they will likely depict an incorrect relation in the bar diagram (e.g. making one bar longer instead of shorter than the other). The drawing might then visually reinforce the students' inadequate problem-solving approach, and be perceived by the solvers as a confirmation that they used an appropriate problem-solving method. Consequently, they will not engage in additional cognitive processes required to correctly solve the problem. Therefore, it is crucial to teach children to properly engage in constructing diagrams. It would thus be advisable to give students explicit instructions in how to draw an appropriate bar diagram and provide exercises that gradually develop their ability to automatize the process. For instance, teachers and educators could give students a word problem followed by a partially completed bar diagram, where pupils are simply required to insert the numerical quantity into the respective rectangles. Next, a more complex variant of a partially completed diagram could be given, where students are required to draw the second bar and determine whether this should be longer or shorter than the already given bar.

Schema-Based Instruction

Another effective method to help students better develop their ability to solve word problems is schema-based instruction (Cook et al. 2020). Generally, schemas refer to knowledge that is acquired and stored in long-term memory and can be applied to newly received information (Kalyuga 2008). In maths educational settings, repeated solution of similar word problems leads to the formation of a schema, which subsequently becomes part of the solver's repertoire. When encountering a new word problem, the solver analyses the features of a problem and relates them to an existing schema (Christou and Philippou 1999). In other words, relying on schema when solving word problems helps with the problem representation and recognition of the semantic structure of the problem.

In schema-based instruction, schema based on visual representation are

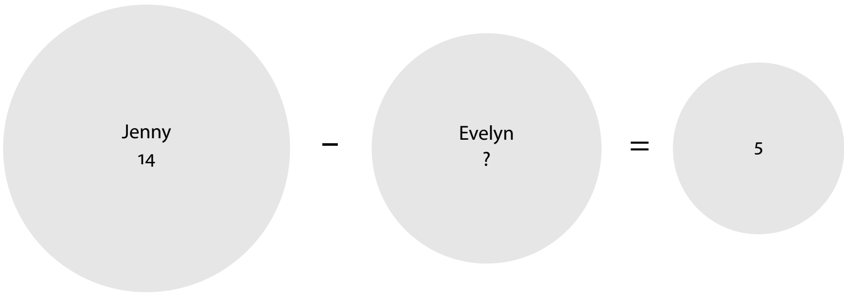

Figure 3 Schematic Diagram Representing the Variables, Quantities, and Relations of an Inconsistent Problem

used. In particular, students learn to solve word problems through four steps, namely: (1) identify the word problem type (i.e. change, combine or compare) and therefore the underlying semantic structure, (2) organize and place the relevant information from the word problem text into a visual schematic diagram, (3) plan the solution, and (4) use computational algorithms to solve for the unknown quantity (Jitendra 2019).

Imagine solving the inconsistent word problem presented in figure 3 with the schema-based approach. In the first step of the schema-based strategy, children identify the word problem as compare since it requires a comparison of Jenny's crayons to Evelyn's crayons. In Step 2, children are instructed to use the corresponding diagram (compare schematic diagram) to organize and represent the relevant information. In doing so, they carefully read the text and identify the variables that are being compared (e.g. Jenny and Evelyn); focusing on the comparison sentence they determine the identity of the bigger (e.g. Jenny) and smaller (e.g. Evelyn) variable and write them in the correct location in the diagram (e.g. Jenny in the first circle that represents the bigger set and Evelyn in the second circle which represents the smaller set). Students then refer to the text to identify the difference amount between the two variables (e.g. how many more crayons does Jenny have) and place the information in the diagram. Next, children read the problem to search for the quantities associated with the two variables (e.g. 14 for Jenny and unknown quantity for Evelyn) and write them in the diagram. Finally, pupils check the accuracy of the representation. In Step 3, solvers need to select the arithmetic operation to solve the unknown quantity. They learn that the bigger variable (e.g. Jenny's crayons) is the 'whole,' while the smaller variable and difference (e.g. Evelyn's crayons and 5 crayons) are 'parts' that make up the bigger variable (part-part-whole schema). Therefore, children learn that when solving for the smaller variable they need to do a subtraction, whereas when solving for the bigger quantity an addition must be applied. In Step

4, the children carry out the operation (e.g. 14 – 5) and check the answer.

Several studies (e.g. Fuchs et al. 2010; Jitendra et al. 2007; Zhang and Xin 2012) presented corroborating evidence for the efficacy of schema-based instruction for students with or at risk of mathematical learning disability. Overall, in the school context it would be useful to implement strategies based on graphical diagrams which seem to help students build a coherent mental model of the problem and recognize the underlying mathematical structure.

Conclusion

At the primary school level, the ability to solve arithmetic word problems is one of the most challenging mathematical skills to acquire. Over the years, various strategies for solving word problems have been taught, and, interestingly, the proposed methods have significantly evolved over time. In the past, the emphasis was often placed on teaching rote procedures and instructing students to rely on keywords to determine the required operation. However, evidence has shown that this approach is often inefficient and potentially detrimental. As a result, problem-solving strategies that promote a deeper comprehension of the problem and its underlying mathematical structure have emerged. In this regard, teaching students to use visual and concrete representations, such as bar diagrams and schemas, was found to substantially improve their proficiency in solving word problems as it helps them create a mental model of the problem. We argue that this shift in instructional strategies could be beneficial in promoting pupils' mathematical reasoning and critical thinking abilities, which are crucial for tackling both academic and real-world mathematical challenges.

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Razvoj strategij reševanja aritmetičnih besedilnih nalog: uporabiti ključne besede ali diagrame?

Reševanje aritmetičnih besedilnih nalog je pomembna spretnost, ki jo morajo otroci pridobiti med osnovnošolskim izobraževanjem. Kljub temu se veliko učencev sooča s težavami pri reševanju takšnih matematičnih nalog. Namen prispevka je analizirati, kako učinkovito podpirati sposobnost reševanja problemov pri učencih. Že več desetletij je eden izmed najbolj uporabljanih načinov reševanja besedilnih nalog t. i. strategija ključnih besed. V prispevku se izpostavi površnost in neprimernost tega pristopa ter opišejo težave, s katerimi se učenci srečujejo pri reševanju jezikovno nedoslednih besedilnih nalog (tj. nalog, kjer običajni pomen ključne besede ne sovpada z aritmetično operacijo, potrebno za pravilen rezultat). V prihodnosti bi se morali učitelji raje osredotočiti na strategije, ki temeljijo na razumevanju problemske situacije in na ustvarjanju notranje reprezentacije problema, saj sta ti dve vrsti osnova za uspešno reševanje. V prispevku se opišeta dve metodi, ki poudarjata uporabo vizualnih reprezentacij, kot so diagrami.

Ključne besede: besedilne naloge, ključne besede, diagram, sheme, osnovna šola