8TH
EUROPEAN
CONGRESS OF
MATHEMATICS

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• BOOK OF ABSTRACTS

8th European Congress of Mathematics

20–26 June 2021 • Portorož, Slovenia

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WELCOME, FRIENDS OF MATHEMATICS!

When we first considered whether or not to apply for organising the 8th European Congress of Mathematics (8ECM), one of our prevailing aims was a commitment to make the Congress stand out from previous ones. Unfortunately, with the help of the Covid pandemic, we certainly succeeded. This is the first international mathematical congress that runs mainly over the internet.

After it became clear that the virus would not be conquered in time to keep the original dates for the 8ECM, we moved it forward a year, from 2020 to 2021. We remained cautiously optimistic that we would be able to hold a regular in-person congress this year, but when we followed the increase of numbers in a second wave of the virus through many parts of the world, and after some of us even had a first-hand experience of it ourselves, we had to make the tough decision to hold the 8ECM mostly on-line.

Despite the possibility of a worst-case scenario in which the 8ECM would have to be totally on-line, we wanted to pursue our original strategy and bring to the congress a large number of mathematicians. In this respect, there are two kinds of mathematicians: those who like big events covering all pertinent topics of mathematical science, and who enjoy learning how mathematics has advanced in recent years, and others who prefer smaller, more specialised workshops and conferences in which only topics from their own field of research are addressed. We wanted to reconcile both kinds of expectation, by designing a congress that would satisfy the needs of as many mathematicians as possible.

We wanted to combine the top-down approach, where committees appointed by the EMS select the best speakers and awardees, with the bottom-up approach where mathematicians propose topics for mini-symposia and then invite other colleagues to join them. We did not put any restrictions on the upper limit of the participants in a minisymposium. It seems that this strategy was quite a success.

There are 62 minisymposia (MS), with eight involving more than 20 talks, namely: Combinatorial Designs (MS-16, with 33 active participants), Operator Algebras (MS-14, with 30), Topological Methods in Differential Equations (MS-13, with 23), Graphs, Polynomials, Surfaces, and Knots (MS-49, with 22), Modeling roughness and long-range dependence with fractional processes (MS-18, with 22), Approximation Theory and Applications (MS-78, with 21), Harmonic Analysis and Partial Differential Equations (MS-28, with 21), and Recent Developments on Preservers (MS-38, with 21).

Also there are 19 countries with more than 20 participants: Slovenia (286), Italy (194), Germany (120), United Kingdom (109), Spain (94), United States (86), France (77), Russian Federation (56), Poland (48), Czech Republic (43), Croatia (41), Hungary (40), Austria (36), Ukraine (32), China (29), Switzerland (29), Canada (23), Belgium (21), and Romania (21). In total there are over 1000 contributions.

We are grateful to the Executive Committee of the EMS for supporting our approach to the 8ECM, resulting in such a great quantity, while at the same time maintaining high quality. Among the participants of the 8ECM there are three Fields medalists, an Abel Prize winner, the President of the ERC, and 50 ERC Grant holders.

We are very happy that the 8ECM has attained a truly all-European character, with substantial contributions by mathematicians from central Europe as well as the more traditional western

Europe. We are also proud that this congress has attracted mathematicians from all the populated continents of the world.

We should mention the courage of both the University of Primorska, which dared to make a bid for this Congress in 2015, and the Council of the EMS, which entrusted us with organisation of this endeavour. The EMS fully supported our preparation for this meeting via its Executive Committee, and we learned that the number of individual memberships of the European Mathematical Society increased in the years leading up to this event.

Our gratitude also goes to you, dear participants and friends of mathematics! Thanks to you, the number of registered participants grew to about 1700, breaking all records in the series of European Congresses so far. Your support for mathematics is invaluable! Special thanks go to our many sponsors and donors, and in particular, the Foundation Compositio Mathematica and the EMS Press, which both contributed significantly to the Prize Fund, and the Slovenian Insurance Association for serving as our primary sponsor. Last but not least, our thanks go to His Excellency, Mr Borut Pahor, the President of the Republic of Slovenia, for his honorary patronage of the 8ECM.

Tomaž Pisanski
Chair of the Organizing Committee of the 8th European Congress of Mathematics

Portorož, Slovenia, June 2021

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All abstracts appear in the form in which they were submitted by the author. The LaTeX source was modified only if an abstract corrupted the document layout (e.g. changing the font size or line spacing).

General Information

8ECM – The 8th European Congress of Mathematics
20 – 26 June 2021, Portorož, Slovenia

Organizers

The organization of the congress was entrusted to University of Primorska, in collaboration with all active mathematical institutions in Slovenia.

University of Primorska, Slovenia
Faculty of Mathematics, Natural Sciences and Information Technologies (UP FAMNIT)
Andrej Marušič Institute (UP IAM)

University of Ljubljana, Slovenia
Faculty of Mathematics and Physics (UL FMF)
Faculty of Education, Department of Mathematics and Computer Science (UL PeF)

University of Maribor, Slovenia
Faculty of Natural Sciences and Mathematics (UM FNM)

Institute of Mathematics, Physics and Mechanics, Slovenia (IMFM)

The Society of Mathematicians, Physicists and Astronomers, Slovenia (DMFA)

Slovenian Discrete and Applied Mathematics Society (SDAMS)

Abelium d.o.o., research and development – Slovenia
Abelium d.o.o. will provide full technical support before and for the duration of the Congress.

Jozef Stefan Institute (IJS)
IJS has an open access educational video lectures repository – Videolectures.net, which is the official 8ECM media partner.

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Special advisor to the organizing committee:

Wacław Marzantowicz, Adam Mickiewicz University

Scientific Committee

The Scientific Committee is appointed by the EMS and is responsible for the scientific content of the congress, including proposing and selecting Plenary and Invited speakers.

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The Prize Committee is appointed by the EMS to select 10 EMS Prize winners.

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Maria Chudnovsky, Princeton University
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Herbert Edelsbrunner, IST Austria
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Jan Philip Solovej, University of Copenhagen
Domokos Szász, Budapest University of Technology and Economics

Ragnar Winther, University of Oslo
Cem Yalcin Yildirim, Boğaziçi University

Local Scientific Committee

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Dragan Marušič (Chair), University of Primorska
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Josip Globevnik, University of Ljubljana
Klavdija Kutnar, University of Primorska
Tomaž Pisanski, University of Primorska
Marko Tadić, University of Zagreb

Plenary Speakers

- Statistical Learning: Causal-oriented and Robust, *Peter Bühlmann*
- Stable solutions to semilinear elliptic equations are smooth up to dimension 9, *Xavier Cabré*
- Minimal surfaces from a complex analytic viewpoint, *Franc Forstnerič*
- Bernoulli Random Matrices, *Alice Guionnet*
- The Mathematics of Deep Learning, *Gitta Kutyniok*
- Geometric Valuation Theory, *Monika Ludwig*
- Escaping the curse of dimensionality in combinatorics, *Janos Pach*
- The Beat of Math, *Alfio Quarteroni*
- Metric measure spaces and synthetic Ricci bounds, *Karl Theodor Sturm*
- Torsion in algebraic groups and problems which arise, *Umberto Zannier*

Statistical Learning: Causal-oriented and Robust

Peter Bühlmann, buehlmann@stat.math.ethz.ch
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Reliable, robust and interpretable machine learning is a big emerging theme in data science and artificial intelligence, complementing the development of pure black box prediction algorithms. Looking through the lens of statistical causality and exploiting a probabilistic invariance property opens up new paths and opportunities for enhanced robustness, with wide-ranging prospects for various applications.

Stable solutions to semilinear elliptic equations are smooth up to dimension 9

Xavier Cabré, xavier.cabre@upc.edu
ICREA and Universitat Politecnica de Catalunya, Spain

The regularity of stable solutions to semilinear elliptic PDEs has been studied since the 1970's. In dimensions 10 and higher, there exist stable energy solutions which are singular. In this talk I will describe a recent work in collaboration with Figalli, Ros-Oton, and Serra, where we prove that stable solutions are smooth up to the optimal dimension 9. This answers to an open problem posed by Brezis in the mid-nineties concerning the regularity of extremal solutions to Gelfand-type problems.

Minimal surfaces from a complex analytic viewpoint

Franc Forstnerič, franc.forstneric@fmf.uni-lj.si
University of Ljubljana, Slovenia

In this talk, I will describe some recent developments in the theory of minimal surfaces in Euclidean spaces which have been obtained by complex analytic methods. After a brief history and background of the subject, I will present a new Schwarz-Pick lemma for minimal surfaces, describe approximation and interpolation results for minimal surfaces, and discuss the current status of the Calabi-Yau problem on the existence of complete conformal minimal surfaces with Jordan boundaries.

Bernoulli Random Matrices

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The study of large random matrices, and in particular the properties of their eigenvalues and eigenvectors, has emerged from the applications, first in data analysis and later as statistical models for heavy-nuclei atoms. It now plays an important role in many other areas of mathematics such as operator algebra and number theory. Over the last thirty years, random matrix theory became a field on its own, borrowing tools from different branches of mathematics. The purpose of this lecture is to illustrate this theory by focusing on the special case of Bernoulli random matrices. Such matrices are particularly interesting as they represent the adjacency matrix of Erdos-Renyi graphs.

The Mathematics of Deep Learning

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Despite the outstanding success of deep neural networks in real-world applications, ranging from science to public life, most of the related research is empirically driven and a comprehensive mathematical foundation is still missing. At the same time, these methods have already shown their impressive potential in mathematical research areas such as imaging sciences, inverse problems, or numerical analysis of partial differential equations, sometimes by far outperforming classical mathematical approaches for particular problem classes.

The goal of this lecture is to first provide an introduction into this new vibrant research area. We will then survey recent advances in two directions, namely the development of a mathematical foundation of deep learning and the introduction of novel deep learning-based approaches to solve inverse problems and partial differential equations.

Geometric Valuation Theory

Monika Ludwig, monika.ludwig@tuwien.ac.at
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Valuations on compact convex sets in $\mathbb{R}^n$ play an active and prominent role in geometry. They were critical in Dehn's solution to Hilbert's Third Problem in 1901. They are defined as follows. A function Z whose domain is a collection of sets $\mathcal{S}$ and whose co-domain is an Abelian semigroup is called a *valuation* if

$$\mathrm{Z}(K) + \mathrm{Z}(L) = \mathrm{Z}(K \cup L) + \mathrm{Z}(K \cap L),$$

whenever $K, L, K \cup L, K \cap L \in \mathcal{S}$.

The first classification result for valuations on the space of compact convex sets, $\mathcal{K}^n$, in $\mathbb{R}^n$ (where $\mathcal{K}^n$ is equipped with the topology induced by the Hausdorff metric) was established by Blaschke.

Theorem 1 (Blaschke)**.** *A functional* $\mathrm{Z} : \mathcal{K}^n \to \mathbb{R}$ *is a continuous, translation and SL*(n) *invariant valuation if and only if there are* $c_0, c_n \in \mathbb{R}$ *such that*

$$\mathrm{Z}(K) = c_0 V_0(K) + c_n V_n(K)$$

for every $K \in \mathcal{K}^n$.

Probably the most famous result in the geometric theory of valuations is the Hadwiger characterization theorem.

Theorem 2 (Hadwiger)**.** *A functional* $\mathrm{Z} : \mathcal{K}^n \to \mathbb{R}$ *is a continuous and rigid motion invariant valuation if and only if there are* $c_0, \ldots, c_n \in \mathbb{R}$ *such that*

$$\mathrm{Z}(K) = c_0 V_0(K) + \cdots + c_n V_n(K)$$

for every $K \in \mathcal{K}^n$.

Here $V_0(K), \ldots, V_n(K)$ are the intrinsic volumes of $K \in \mathcal{K}^n$. In particular, $V_0(K)$ is the Euler characteristic of K, while $2\,V_{n-1}(K)$ is the surface are of K and $V_n(K)$ the n-dimensional volume of K. Hadwiger's theorem shows that the intrinsic volumes are the most basic functionals in Euclidean geometry. It finds powerful applications in Integral Geometry and Geometric Probability.

The fundamental results of Blaschke and Hadwiger have been the starting point of the development of Geometric Valuation Theory. Classification results for valuations invariant (or covariant) with respect to important groups are central questions. The talk will give an overview of such results including recent extensions to valuations on function spaces.

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Escaping the curse of dimensionality in combinatorics

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MIPT, Moscow, Russian Federation

We discuss some notoriously hard combinatorial problems for large classes of graphs and hypergraphs arising in geometric, algebraic, and practical applications. These structures escape the "curse of dimensionality": they can be embedded in a bounded-dimensional space, or they have small VC-dimension or a short algebraic description. What are the advantages of low dimensionality?

1. With the help of suitable topological and algebraic separator theorems, large families of geometric objects embedded in a fixed-dimensional space can be split into subfamilies of roughly the same size, and then the smaller families can be analyzed recursively.

2. Geometric objects in space can be compared by a number of naturally defined partial orders, so that one can utilize the theory of partially ordered sets.

3. Graphs and hypergraphs of bounded VC-dimension admit very small epsilon-nets and can be particularly well approximated by random sampling.

4. If the description complexity of a family of geometric objects is small, then the "combinatorial complexity" (number of various dimensional faces) of their arrangements is also small. This typically guarantees the existence of efficient algorithms to visualize, describe, control, and manipulate these arrangements.

The Beat of Math

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Mathematical models based on first principles are devised for the description of the blood motion in the human circulatory system, as well as for the simulation of the interaction between electrical, mechanical, and fluid-dynamical processes occurring in the heart. This is a classical environment where multi-physics and multi-scale processes have to be addressed.
Appropriate systems of nonlinear differential equations (either ordinary and partial) and efficient numerical strategies must be devised to allow for the analysis of both heart function and dysfunction, and the simulation, control, and optimization of therapy and surgery.
This presentation will address some of these issues and a few representative applications of clinical interest.

Acknowledgment: The work presented in this talk is part of the project iHEART that has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No 740132)

Metric measure spaces and synthetic Ricci bounds

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Metric measure spaces with synthetic Ricci bounds have attracted great interest in recent years, accompanied by spectacular breakthroughs and deep new insights. In this talk, I will provide a brief introduction to the concept of lower Ricci bounds as introduced by Lott-Villani and myself, and illustrate some of its geometric, analytic and probabilistic consequences, among them Li-Yau estimates, coupling properties for Brownian motions, sharp functional and isoperimetric inequalities, rigidity results, and structural properties like rectifiability and rectifiability of the boundary. In particular, I will explain its crucial interplay with the heat flow and its link to the curvature-dimension condition formulated in functional-analytic terms by Bakry-Émery. This equivalence between the Lagrangian and the Eulerian approach then will be further explored in various recent research directions: i) distribution-valued Ricci bounds which e.g. allow singular effects of non-convex boundaries to be taken into account, ii) time-dependent Ricci bounds which provide a link to (super-) Ricci flows for singular spaces, iii) upper curvature bounds.

Torsion in algebraic groups and problems which arise

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Scuola Normale Superiore, Italy

Since the investigations of Gauss on cyclotomy, the arithmetic of roots of unity and, more generally, of torsion points in (commutative) algebraic groups, has developed into rich and deep theories. Several problems were raised also concerning algebraic relations among torsion elements, as for instance in well-known (former) conjectures by Manin-Mumford and by Lang. In the talk we shall survey especially along this last topic, focusing on more recent finiteness results regarding torsion values attained by sections of families of abelian varieties. Finally, we shall briefly mention some applications.

Invited Speakers

- The dawn of formalized mathematics, *Andrej Bauer*
- Positive harmonic functions on the Heisenberg group, *Yves Benoist*
- From Kähler-Einstein metrics to zeros of zeta functions, *Robert Berman*
- Regularization methods in inverse problems and machine learning, *Martin Burger*
- From linear to nonlinear n-width : optimality in reduced modelling, *Albert Cohen*
- An invitation to Poisson Geometry, *Marius Crainic*
- Tackling discrete optimization problems by continuous methods, *Mirjam Dür*
- Modelling the genetics of spatially structured populations, *Alison Etheridge*
- Recent Results on Lieb-Thirring Inequalities, *Rupert Frank*
- Laplacians on infinite graphs, *Aleksey Kostenko*
- Exponential sums over finite fields, *Emmanuel Kowalski*
- Fast algorithms from low-rank updates, *Daniel Kressner*
- A proof of the Erdős-Faber-Lovász conjecture, *Daniela Kühn*
- Uniqueness results for discrete Schrodinger evolutions, *Eugenia Malinnikova*
- Some Recent Developments on the Geometry of Random Spherical Eigenfunctions, *Domenico Marinucci*
- Looking at Euler flows through a contact mirror: Universality and Turing completeness, *Eva Miranda*
- Bayesian inverse problems, Gaussian processes, and PDEs, *Richard Nickl*
- Topology of symplectic fillings of contact 3-manifolds, *Burak Özbağci*
- Nonstandard finite elements for wave problems, *Ilaria Perugia*
- Scaling Optimal Transport for High dimensional Learning, *Gabriel Peyré*
- Finite groups of birational transformations, *Yuri Prokhorov*
- Propositional Proof Complexity, *Alexander Razborov*
- Subset products and derangements, *Aner Shalev*
- Convex integration and synthetic turbulence, *László Székelyhidi Jr.*
- HMS categorical symmetries and hypergeometric systems, *Špela Špenko*
- Quadrature error estimates for layer potentials evaluated near curved surfaces in three dimensions, *Anna Karin Tornberg*
- AAA-least squares rational approximation and solution of Laplace problems, *Nick Trefethen*
- Structure and classification of simple amenable C^*-algebras, *Stuart White*

The dawn of formalized mathematics

Andrej Bauer, Andrej.Bauer@andrej.com
University of Ljubljana, Slovenia

When I was a student of mathematics I was told that someone had formalized an entire book on analysis just to put to rest the question whether mathematics could be completely formalized, so that mathematicians could proceed with business as usual. I subsequently learned that the book was Landau's "Grundlagen" [6], the someone was L. S. van Benthem Jutting [11], the tool of choice was Automath [12], and that popular accounts of history are rarely correct.

Formalized mathematics did not die out. Computer scientists spent many years developing proof assistants [1, 10, 8, 3] – programs that help create and verify formal proofs and constructions – until they became good enough to attract the attention of mathematicians who felt that formalization had a place in mathematical practice. The initial successes came slowly and took a great deal of effort. In the last decade, the complete formalizations of the odd-order theorem [4] and the solution of Kepler's conjecture [5] sparked an interest and provided further evidence of viability of formalized mathematics. The essential role of proof assistants in the development of homotopy type theory [9] and univalent mathematics [13] showed that formalization can be an inspiration rather than an afterthought to traditional mathematics. Today the community gathered around Lean [3], the newcomer among proof assistants, has tens of thousands of members and is growing very rapidly thanks to the miracle of social networking. The new generation is ushering in a new era of mathematics.

Formalized mathematics, in tandem with other forms of computerized mathematics [2], provides better management of mathematical knowledge, an opportunity to carry out ever more complex and larger projects, and hitherto unseen levels of precision. However, its transformative power runs still deeper. The practice of formalization teaches us that formal constructions and proofs are much more than pointless transliteration of mathematical ideas into dry symbolic form. Formal proofs have rich structure, worthy of attention by a mathematician as well as a logician; contrary to popular belief, they can directly and elegantly express mathematical insights and ideas; and by striving to make them slicker and more elegant, new mathematics can be discovered.

Formalized mathematics is changing the role of foundations of mathematics, too. A good century ago, a philosophical crisis necessitated the development of logic and set theory, which served as the bedrock upon which the 20th century mathematics was built safely. However, most proof assistants shun logic and set theory in favor of type theory, the original resolution of the crisis given by Bertrand Russell [14] and reformulated into its modern form by Per Martin-Löf [7]. The reasons for this phenomenon are yet to be fully understood, but we can speculate that type theory captures mathematical practice more faithfully because it directly expresses the structure and constructions of mathematical objects, whereas set theory provides plentiful raw material with little guidance on how mathematical objects are to be molded out if it.

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Positive harmonic functions on the Heisenberg group

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A harmonic function on a group is a function which is equal to the average of its translates, average with respect to a finitely supported measure. First, we will survey the history of this notion.

Then we will describe the extremal non-negative harmonic functions on the Heisenberg group. We will see that the classical partition function occurs as such a function and that this function is the only one beyond harmonic characters.

From Kähler-Einstein metrics to zeros of zeta functions

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While the existence of a unique Kähler-Einstein metrics on a canonically polarized manifold X was established already in the seventies there are very few explicit formulas available (even in the case of complex curves!). In this talk I will give a non-technical introduction to a probabilistic construction of Kähler-Einstein metrics, which, in particular, yields canonical approximations of the Kähler-Einstein metric on X. The approximating metrics in question are expressed as explicit period integrals and the conjectural extension to the case of a Fano variety leads to some intriguing connections to zeros of some Archimedean zeta functions.

Regularization methods in inverse problems and machine learning

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Regularization methods are at the heart of the solution of inverse problems and are of increasing importance in modern machine learning. In this talk we will discuss the modern theory of (nonlinear) regularization methods and some applications. We will put a particular focus on variational and iterative regularization methods and their connection with learning problems: we discuss the use of such regularization methods for learning problems on the one hand, but also the current route of learning regularization methods from data.

From linear to nonlinear n-width : optimality in reduced modelling

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The concept of n-width has been introduced by Kolmogorov as a way of measuring the size of compact sets in terms of their approximability by linear spaces. From a numerical perspective it may be thought as a benchmark for the performance of algorithms based on linear approximation. In recent years this concept has proved to be highly meaningful in the analysis of reduced modeling strategies for complex physical problems described by parametric problems. This lecture will first review two significant results in this area that concern (i) the practical construction of optimal spaces by greedy algorithms and (ii) the preservation of the rate of decay of widths under certain holomorphic transformation. It will then focus on recent attempts to propose non-linear version of n-widths, how these notions relate to metric entropies, and how they could be relevant to practical applications.

An invitation to Poisson Geometry

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Poisson structures originate in the work of Lagrange and Poisson on the motion of planets in the solar system; the process of understanding them was long and it prompted the discovery of several fundamental concepts in mathematics, such as: Jacobi identity, Maurer-Cartan equations, etc. Poisson Geometry (the geometric study of Poisson structures) can be traced back to the work of Lie and Kirilov; the first systematic studies are found in the work of Lichnerowicz in the 1970s and Weinstein in the 1980s. Its remarkable development over the last few decades was driven by several problems (such as integrability or Conn's linearization theorem) and led to surprising new connections with various other fields. All these led to the present-day understanding of Poisson Geometry: it is an amalgam of Lie Theory, Symplectic Geometry and Foliation Theory, offering the framework for exciting interactions between these theories, as well as others. In the talk I will try to expand this abstract.

Tackling discrete optimization problems by continuous methods

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Many NP-hard discrete and combinatorial optimization problems can be formulated with the help of quadratic expressions. These in turn can be linearized by lifting the problem from n-dimensional space to the space of n by n matrices. We show that this leads to a conic optimization problem, i.e., an optimization problem in matrix variables where a constraint requires the matrix to be in the cone of so called copositive or completely positive matrices. The complexity of the original problem is entirely shifted into the cone constraint. We discuss the pros and cons of this approach, and we review the state of the art in this area, covering both theory and numerical solution approaches.

Modelling the genetics of spatially structured populations

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The last century has seen remarkable developments in the nature and scale of genetic data, and the tools with which to interrogate them. Nonetheless fundamental questions remain unanswered. The genetic composition of a population can be changed by natural selection, mutation, mating, and other genetic and evolutionary mechanisms. The effect of these mechanisms, and the way in which they interact with one another, is also influenced by the spatial structure of the population. The hunt for adequate mathematical models with which to investigate the interaction between the forces of evolution and spatial structure is still very much in progress. Capturing stochastic effects in a biologically meangingful way is particularly challenging. In this talk we shall explore some of the progress that has been made, and some of the remaining challenges.

Recent Results on Lieb-Thirring Inequalities

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Lieb-Thirring inequalities are a mathematical expression of the uncertainty and exclusion principles in quantum mechanics. Since their discovery in 1975 they have played an important role in several areas of analysis and mathematical physics. We provide a gentle introduction to classical aspects of this subject and present some recent progress. Finally, we discuss extensions, in the spirit of Lieb-Thirring inequalities, of several inequalities in harmonic analysis to the setting of families of orthonormal functions.

Laplacians on infinite graphs

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There are two different notions of a Laplacian operator associated with graphs: discrete graph Laplacians and continuous Laplacians on metric graphs (widely known as quantum graphs). Both objects have a venerable history as they are related to several diverse branches of mathematics and mathematical physics.

The existing literature usually treats these two Laplacian operators separately. In this talk, I will focus on the relationship between them (spectral, parabolic and geometric properties). One of our main conceptual messages is that these two settings should be regarded as complementary (rather than opposite) and exactly their interplay leads to important further insight on both sides.

Based on joint work with Noema Nicolussi.

Exponential sums over finite fields

Emmanuel Kowalski, kowalski@math.ethz.ch
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Exponential sums are among the simplest mathematical objects that one can imagine, but also among the most remarkably useful and versatile in number theory.

This talk will survey the history, the mysteries and the surprises of such sums over finite fields, with a focus on questions related to the distribution of values of families of exponential sums. General principles and applications will be illustrated by concrete examples, where sums of two squares, Sidon sets, Larsen's Alternative, the variance of arithmetic functions over function fields and the lines on cubic threefolds will make appearances.

(Based on joint work with A. Forey and J. Fresán)

Fast algorithms from low-rank updates

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The development of efficient numerical algorithms for solving large-scale linear systems is one of the success stories of numerical linear algebra that has had a tremendous impact on our ability to perform complex numerical simulations and large-scale statistical computations. Many of these developments are based on multilevel and domain decomposition techniques, which are closely linked to Schur complements and low-rank updates of matrices. In this talk, we explain how these tools carry over to other important linear algebra problems, including matrix functions and matrix equations. Fast algorithms are derived from combining divide-and-conquer strategies with low-rank updates of matrix functions. The convergence analysis of these algorithms is built on a multivariate extension of the celebrated Crouzeix-Palencia result. The newly developed algorithms are capable of addressing a wide variety of matrix functions and matrix structures, including sparse matrices as well as matrices with hierarchical low rank and Toeplitz-like structures. Their versatility will be demonstrated with several applications and extensions. This talk is based on joint work with Bernhard Beckermann, Alice Cortinovis, Leonardo Robol, Stefano Massei, and Marcel Schweitzer.

A proof of the Erdős-Faber-Lovász conjecture

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Coauthors: Dong-Yeap Kang, Tom Kelly, Abhishek Methuku, Deryk Osthus

Graph and hypergraph colouring problems are central to combinatorics, with applications and connections to many other areas, such as geometry, algorithm design, and information theory. However, for hypergraphs even basic problems have turned out to be rather challenging: in particular, the famous Erdős-Faber-Lovász conjecture (posed in 1972) states that the chromatic index of any linear hypergraph on n vertices is at most n. (Here the chromatic index of a hypergraph H is the smallest number of colours needed to colour the edges of H so that any two edges that share a vertex have different colours.) There are also several other equivalent (dual) versions of this conjecture, e.g. in terms of colouring the vertices of nearly disjoint cliques. Erdős considered this to be one of his three most favorite combinatorial problems and offered $500 for the solution of the problem.

In joint work with Dong-yeap Kang, Tom Kelly, Abhishek Methuku and Deryk Osthus, we prove this conjecture for every large n. We also provide 'stability versions' of this result, which confirm a prediction of Kahn.

In my talk, I will discuss some background, some of the ideas behind the proof as well as some related open problems.

Uniqueness results for discrete Schrodinger evolutions

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I will give a survey of some recent results on uniqueness for (semi)-discrete Schrödinger equation and their connections to classical uncertainty principle.

Some Recent Developments on the Geometry of Random Spherical Eigenfunctions

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A lot of efforts have been devoted in the last decade to the investigation of the high-frequency behaviour of geometric functionals for the excursion sets of random spherical harmonics, i.e., Gaussian eigenfunctions for the spherical Laplacian Δ_{S^2}. In this talk we shall review some of these results, with particular reference to the asymptotic behaviour of variances, phase transitions in the nodal case (the Berry's Cancellation Phenomenon), the distribution of the fluctuations around the expected values, and the asymptotic correlation among different functionals. We shall also discuss some connections with the Gaussian Kinematic Formula, with Wiener-Chaos expansions and with recent developments in the derivation of Quantitative Central Limit Theorems (the so-called Stein-Malliavin approach).

Looking at Euler flows through a contact mirror: Universality and Turing completeness

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The dynamics of an inviscid and incompressible fluid flow on a Riemannian manifold is governed by the Euler equations. Recently, Tao [6, 7, 8] launched a programme to address the global existence problem for the Euler and Navier-Stokes equations based on the concept of universality. Inspired by this proposal, we show that the stationary Euler equations exhibit several universality features, in the sense that, any non-autonomous flow on a compact manifold can be extended to a smooth stationary solution of the Euler equations on some Riemannian manifold of possibly higher dimension [1]. A key point in the proof is looking at the h-principle in contact geometry through a contact mirror, unveiled by Etnyre and Ghrist in [4] more than two decades ago.

We end up this talk addressing a question raised by Moore in [5] : *"Is hydrodynamics capable of performing computations?"*. The universality result above yields the Turing completeness of the steady Euler flows on a 17-dimensional sphere. Can this result be improved? In [2] we construct a Turing complete Euler flow in dimension 3. Time permitting, we discuss this and other generalizations contained in [3].

This talk is based on several joint works with Cardona, Peralta-Salas and Presas.

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Bayesian inverse problems, Gaussian processes, and PDEs

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The Bayesian approach to inverse problems has become very popular in the last decade after seminal work by A. Stuart (2010). Particularly in nonlinear applications with PDEs and when using Gaussian process priors, this can leverage powerful MCMC algorithms to tackle difficult high dimensional and nonconvex inference problems, with associated *uncertainty quantification methodology*. We review the main ideas and then discuss recent progress on rigorous mathematical performance guarantees for such algorithms. We will touch upon issues such as how to prove posterior consistency theorems, how to objectively validate posterior based statistical uncertainty quantification, as well as the polynomial time computability of posterior measures in some nonconvex model examples arising with PDEs.

Topology of symplectic fillings of contact 3-manifolds

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Koç University, Turkey

Ever since Donaldson showed that every symplectic 4-manifold admits a Lefschetz pencil and Giroux proved that every contact 3-manifold admits an adapted open book decomposition, at the turn of the century, Lefschetz fibrations and open books have been used fruitfully to obtain interesting results about the topology of symplectic fillings of contact 3-manifolds. In this talk, I will present my contribution to the subject at hand based on joint work with several coauthors during the past 20 years.

Nonstandard finite elements for wave problems

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Finite elements are a powerful, flexible, and robust class of methods for the numerical approximation of solutions to partial differential equations. In their standard version, they are based on piecewise polynomial functions on a partition of the domain of interest. Continuity requirements are possibly dictated by the regularity of the exact solutions. By breaking these constraints, new methods that are specifically tailored to the problem at hand have been developed in order to better reproduce physical properties of the exact solutions, to enhance stability, and to improve accuracy vs. computational cost. Nonstandard finite element approximations of wave propagation problems based on the space-time paradigm will be the focus of this talk.

Scaling Optimal Transport for High dimensional Learning

Gabriel Peyré, gabriel.peyre@ens.fr
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Optimal transport (OT) has recently gained lot of interest in machine learning. It is a natural tool to compare in a geometrically faithful way probability distributions. It finds applications in both supervised learning (using geometric loss functions) and unsupervised learning (to perform generative model fitting). OT is however plagued by the curse of dimensionality, since it might require a number of samples which grows exponentially with the dimension. In this talk, I will explain how to leverage entropic regularization methods to define computationally efficient loss functions, approximating OT with a better sample complexity. More information and references can be found on the website of our book "Computational Optimal Transport".

Finite groups of birational transformations

Yuri Prokhorov, prokhoro@mi-ras.ru
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I survey the classification theory of finite groups of birational transformations of higher-dimensional algebraic varieties. This theory has been significantly developed during the last 10 years due to the success of the minimal model program. I concentrate on certain properties of these groups in arbitrary dimension. Also, I am going to discuss the three-dimensional case in more details.

Propositional Proof Complexity

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U Chicago, United States, and *Steklov Institute, Russian Federation*

Propositional proof complexity studies efficient provability of statements that can be expressed in quantifier-free form, in various proof systems and under various notions of "efficiency". Statements of interest come from a variety of sources that, besides traditional combinatorial principles and other mathematical theorems, include areas like combinatorial optimization, practical SAT solving and operation research. While many proof systems considered in the modern proof complexity are still traditional, in the sense that they are Hilbert-style and logic-based, a considerable amount of attention has been in recent years paid to systems modelling algebraic and semi-algebraic reasoning, including elementary convex geometry. In this talk I will attempt to convey some basic ideas underlying this vibrant area, including a necessarily based sample of illustrating examples.

Subset products and derangements

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Coauthors: Michael Larsen, Pham Tiep

In the past two decades there has been intense interest in products of subsets in finite groups. Two important examples are Gowers' theory of Quasi Random Groups and its applications by Nikolov, Pyber, Babai and others, and the theory of Approximate Groups and the Product Theorem of Breuillard-Green-Tao and Pyber-Szabo on growth in finite simple groups of Lie type of bounded rank, extending Helfgott's work.

These deep theories yield strong results on products of three subsets (covering, growth). What can be said about products of two subsets? I will discuss a recent joint work with Michael Larsen and Pham Tiep on this challenging problem, focusing on products of two normal subsets of finite simple groups, and deriving a number of applications.

Our main application concerns derangements (namely, fixed-point-free permutations), studied since the days of Jordan. We show that every element of a sufficiently large finite simple transitive permutation group is a product of two derangements. Related results and problems will also be discussed.

The proofs combine group theory, algebraic geometry and representation theory; it applies the proof by Fulman and Guralnick of the Boston-Shalev conjecture on the proportion of derangements.

Convex integration and synthetic turbulence

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In the past decade convex integration has been established as a powerful and versatile technique for the construction of weak solutions of various nonlinear systems of partial differential equations arising in fluid dynamics, including the Euler and Navier-Stokes equations. The existence theorems obtained in this way come at a high price: solutions are highly irregular, non-differentiable, and very much non-unique as there is usually infinitely many of them. Therefore this technique has often been thought of as a way to obtain mathematical counterexamples in the spirit of Weierstrass' non-differentiable function, rather than advancing physical theory; "pathological", "wild", "paradoxical", "counterintuitive" are some of the adjectives usually associated with solutions obtained via convex integration. In this lecture I would like to draw on some recent examples to show that there are many more sides to the story, and that, with proper usage and interpretation, the convex integration toolbox can indeed provide useful insights for problems in hydrodynamics.

HMS categorical symmetries and hypergeometric systems

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Hilbert's 21st problem asks about the existence of Fuchsian linear differential equations with a prescribed "monodromy representation" of the fundamental group. The first (slightly erroneous) solution was proposed by a Slovenian mathematician Plemelj. A suitably adapted version of this problem was solved, depending on the context, by Deligne, Mebkhout, Kashiwara-Kawai, Beilinson-Bernstein, ... The solution is now known as the Riemann-Hilbert correspondence.

Homological mirror symmetry predicts the existence of an action of the fundamental group of the "stringy Kähler moduli space" (SKMS) on the derived category of an algebraic variety. This prediction was established by Halpern-Leistner and Sam for certain toric varieties. The decategorification of the action found by HLS yields a representation of the fundamental group of the SKMS and in joint work with Michel Van den Bergh we show that it is given by the monodromy of an explicit hypergeometric system of differential equations.

Quadrature error estimates for layer potentials evaluated near curved surfaces in three dimensions

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Coauthors: Ludvig af Klinteberg, Chiara Sorgentone

When numerically solving PDEs reformulated as integral equations, so called layer potentials must be evaluated. The quadrature error associated with a regular quadrature rule for evaluation of a layer potential increases rapidly when the evaluation point approaches the surface and the integral becomes nearly singular. Error estimates are needed to determine when the accuracy is insufficient and a more costly special quadrature method should be utilized.

In this talk, we start by considering integrals over curves in the plane, using complex analysis involving contour integrals, residue calculus and branch cuts, to derive such error estimates. We first obtain error estimates for layer potentials in $\mathbb{R}^2$, for both complex and real formulations of layer potentials, both for the Gauss-Legendre and the trapezoidal rule. By complexifying the parameter plane, the theory can be used to derive estimates also for curves in in $\mathbb{R}^3$. These results are then used in the derivation of the estimates for integrals over surfaces. The estimates that we obtain have no unknown coefficients and can be efficiently evaluated given the discretization of the surface, invoking a local one-dimensional root-finding procedure.

Numerical examples are given to illustrate the performance of the quadrature error estimates. The estimates for integration over curves are in many cases remarkably precise, and the estimates for curved surfaces in $\mathbb{R}^3$ are also sufficiently precise, with sufficiently low computational cost, to be practically useful.

AAA-least squares rational approximation and solution of Laplace problems

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Oxford University, United Kingdom

In the past five years, computation with rational functions has advanced greatly with the introduction of the AAA algorithm for barycentric rational approximation and of lightning least-squares solvers for Laplace, Stokes, and Helmholtz problems. Here we combine these methods into a two-step method for solving planar Laplace problems. First, complex rational approximations to the boundary data are determined by AAA approximation, locally near each corner or other singularity. The poles of these approximations outside the problem domain are then collected and used for a global least-squares fit to the solution. Typical problems are solved in a second of laptop time to 8-digit accuracy, all the way up to the corners. This is joint work with Stefano Costa.

Structure and classification of simple amenable C^*-algebras

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In this talk I will give an overview of recent progress in the structure theory of simple amenable C^*-algebras and classification results. C^*-algebras are norm closed self-adjoint subalgebras of the bounded operators on a Hilbert space, with examples arising naturally from unitary representations of groups, and topological dynamics. They have a topological flavour, seen through the commutative algebras of continuous functions on locally compact Hausdorff spaces.

The classification of C^*-algebras has its spiritual origins in the powerful structure and classification theorems for von Neumann algebras of Connes in the '70s. However, in the topological setting of C^*-algebras, higher dimensional phenomena can obstruct classification in general. Progress over the last decade has seen the identification of abstract structural conditions which give the maximal family of algebras which can be classified by K-theory and traces. These conditions now have equivalent formulations of very different natures, which can be used to bring naturally occurring examples within the scope of classification.

The talk is based in part on joint works with Castillejos, Carrión, Evington, Gabe, Schafhauser, Tikuisis, and Winter.

ABEL LECTURE

Graph limits and Markov spaces

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Limit objects for sequences of finite structures, larger and largerin size but more and more similar in some sense, have been constructedsporadically, perhaps since von Neumann constructed continuous geometries, butthis research has become quite extensive in the last decade and a half. Graphsare perhaps the simplest structures, and accordingly, the limit theory ofgraphs has made the most progress. The theory of graph limits is only understood, to a somewhat satisfactorydegree, in the cases of bounded degree graphs (initiated by Benjamini andSchramm) and of dense graphs (initiated by Borgs, Chayes, Lovász, Szegedy, Sósand Vesztergombi). More recently there is a lot of interest in the intermediatecases. It appears that the most important constituents of graph limits in the general case will be Markov spaces (Markov chains on measurable spaces with astationary distribution). Several important theorems can be extended from finite graphs to Markov spacesor, more generally, to measurable spaces: flow theory, expanders and spectra,mixing of random walks, etc. In this talk we will give a glimpse into thisemerging theory, based on the work of Á. Backhaus, D. Kunszenti-Kovács, B.Szegedy and the speaker.

HIRZEBRUCH LECTURE

A mathematical journey through scales

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The tiny world of particles and atoms and the gigantic world of the entire universe are separated by scales spanning about forty orders of magnitudes. As we move from one to the other, the laws of nature can behave in drastically different ways, going from quantum physics to general relativity through Newton's classical mechanics, not to mention other intermediate "ad hoc" theories. Understanding the way in which the behaviour of mathematical models changes as we move from one scale to another is one of the great classical questions in mathematics and theoretical physics. The aim of this talk is to explore how these questions still inform and motivate interesting problems in probability theory and why so-called toy models, despite their superficially playful character, can sometimes lead to useful quantitative (and not just qualitative) predictions.

PUBLIC SPEAKERS

- Topological explorations in neuroscience, *Kathryn Hess*
- Crossing Numbers: From Art and Circuit Design to Knots and Number Theory, *Bojan Mohar*
- Lie theory without groups, *Andrei Okounkov*
- Mathematics: art or science?, *Stanislav Smirnov*
- European mathematics: a history in 200 stamps, *Robin Wilson*

Topological explorations in neuroscience

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EPFL, Switzerland

The brain of each and every one of us is composed of hundreds of billions of neurons – often called nerve cells – connected by hundreds of trillions of synapses, which transmit electrical signals from one neuron to another. In reaction to stimulus, waves of electrical activity traverse the network of neurons, processing the incoming information. Tools provided by the field of mathematics called algebraic topology enable us to detect and describe the rich structure hidden in this dynamic tapestry.

During this talk, I will guide you on a mathematical mystery tour of what topology has revealed about how the brain processes information.

Crossing Numbers: From Art and Circuit Design to Knots and Number Theory

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Simon Fraser University, Canada, and *IMFM, Slovenia*

In 1864, Sylvester asked what is the probability that four randomly chosen points in the plane form a convex quadrilateral. During World War II, Paul Turán asked about an optimal design of railroads connecting n factories with m warehouses. In 1950s, the British painter Anthony Hill asked how to draw a network of n interconnected nodes with fewest number of crossings. All these questions are still unresolved. The speaker will overview mathematical foundations of the common theme — the theory of crossing numbers of graphs — and will show some surprising relations with other branches of mathematics.

Lie theory without groups

Andrei Okounkov, okounkov@math.columbia.edu
Columbia, United States, and *Skoltech, Russian Federation,* and *HSE, Russian Federation*

Group theory and, in particular, Lie group theory is central to many areas of mathematics and to many applications. Lie theory is a masterpiece of a theory and it may feel nearly complete. Because of this, and also because of the demand from applications, many Lie theorists have been exploring new worlds, in which groups yield the center stage to other geometric and algebraic structures.

Mathematics: art or science?

Stanislav Smirnov, sksmirnov@gmail.com
Université de Genève, Switzerland, and *St Petersburg University, Russian Federation,* and *Skoltech, Russian Federation*

Mathematics is an amazing and mysterious science. Ever since the time of Plato, philosophers argue whether mathematical objects are imaginary, or whether they come from the real world, while mathematicians mostly prove theorems without even asking about their link to reality. On the other hand, the Pharaohs of Egypt and the Kings of Babylon had already grasped the practical power of mathematics, and of course the technological advances of the past two centuries are built on successful applications of our science.

We will discuss the mathematician's perspective on
- where does mathematics come from?
- why is the "imaginary" science so useful in real life?
- how we choose problems to work on, and why do we find our science so fascinating?

We will not be able to answer all these questions in our talk, but we will try to give a glimpse of how mathematicians work.

European mathematics: a history in 200 stamps

Robin Wilson, r.j.wilson@open.ac.uk
Open University, United Kingdom (retired)

In this talk I shall cover the entire history of European mathematics in around 50 minutes – illustrating my talk with around 200 informative (and sometimes amusing) postage stamps from around the world featuring mathematics and mathematicians. This talk is aimed at an interested general audience and assumes no mathematical knowledge.

EMS Prize Winners

- Lefschetz beyond positivity and its implications, *Karim Adiprasito*
- Reciprocity laws for torsion classes, *Ana Caraiani*
- Smooth compactifications of differential graded categories, *Alexander Efimov*
- Discrete monodromy groups and Hodge theory, *Simion Filip*
- Zero sets of Laplace eigenfunctions, *Alexandr Logunov*
- On primes, almost primes and the Möbius function in short intervals, *Kaisa Matomäki*
- Excitation spectrum of dilute trapped Bose gases, *Phan Thanh Nam*
- From branching singularities of minimal surfaces to non-smoothness points on an ice-water interface, *Joaquim Serra*
- Elliptic curves and modularity, *Jack Thorne*
- The magic of dimensions 8 and 24, *Maryna Viazovska*

Lefschetz beyond positivity and its implications

Karim Adiprasito, ka@math.ku.dk
University of Copenhagen, Denmark

I will present results of the Hard Lefschetz type that go beyond the classical case of Kaehler manifolds and projectivity, and survey their applications in combinatorics, geometry and topology.

Reciprocity laws for torsion classes

Ana Caraiani, a.caraiani@imperial.ac.uk
Imperial College London, United Kingdom

The Langlands program is a vast network of conjectures that connect many areas of pure mathematics, such as number theory, representation theory, and harmonic analysis. At its heart lies reciprocity, the conjectural relationship between Galois representations and modular, or automorphic forms.

A famous instance of reciprocity is the modularity of elliptic curves over the rational numbers: this was the key to Wiles's proof of Fermat's last theorem. I will give an overview of some recent progress in the Langlands program, with a focus on new reciprocity laws over imaginary quadratic fields.

Smooth compactifications of differential graded categories

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Steklov Mathematical Institute of RAS, Russian Federation

We will give an overview of results on smooth categorical compactifications, the questions of their existence and their construction. The notion of a categorical smooth compactification is a straightforward generalization of the corresponding usual notion for algebraic varieties.

First, we will explain the result on the existence of smooth compactifications of derived categories of coherent sheaves on separated schemes of finite type over a field of characteristic zero. Namely, such a derived category can be represented as a quotient of the derived category of a smooth projective variety, by a triangulated subcategory generated by a single object. Then we will give an example of a homotopically finite DG category which does not have a smooth compactification: a counterexample to one of the Kontsevich's conjectures on the generalized Hodge to de Rham degeneration.

If time permits, we will formulate a K-theoretic criterion for existence of a smooth categorical compactification, using a DG categorical analogue of Wall's finiteness obstruction from topology.

Discrete monodromy groups and Hodge theory

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University of Chicago, United States

The monodromy of differential equations has been studied for a long time, providing motivation for many of the concepts in complex analysis, algebraic geometry, and dynamical systems. After providing the necessary background, I will present some applications of ideas from dynamics to the study of monodromy groups of families of algebraic varieties, and explain how they relate to differential equations. I will also describe some connections between interesting classes of discrete subgroups of Lie groups and Hodge theory.

Zero sets of Laplace eigenfunctions

Alexandr Logunov, alogunov@princeton.edu
Princeton University, United States

In the beginning of 19th century Napoleon set a prize for the best mathematical explanation of Chladni's resonance experiments. Nodal geometry studies the zeroes of solutions to elliptic differential equations such as the visible curves that appear in Chladni's nodal portraits. We will discuss the geometrical and analytic properties of zero sets of harmonic functions and eigenfunctions of the Laplace operator. For harmonic functions on the plane there is an interesting relation between local length of the zero set and the growth of harmonic functions. The larger the zero set is, the faster the growth of harmonic function should be and vice versa. Zero sets of Laplace eigenfunctions on surfaces are unions of smooth curves with equiangular intersections. Topology of the zero set could be quite complicated, but Yau conjectured that the total length of the zero set is comparable to the square root of the eigenvalue for all eigenfunctions. We will start with open questions about spherical harmonics and explain some methods to study nodal sets, which are zero sets of solutions of elliptic PDE.

On primes, almost primes and the Möbius function in short intervals

Kaisa Matomäki, ksmato@utu.fi
University of Turku, Finland

In this talk I will introduce some fundamental concepts in analytic number theory such as primes, the Riemann zeta function and the Möbius function. I will discuss classical results concerning them and connecting them to each other.

Then I will move on to discussing the distribution of primes, almost primes and the Möbius function in short intervals. In particular I will describe some recent progress on the topic.

Excitation spectrum of dilute trapped Bose gases

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We will discuss the low-lying eigenvalues of trapped Bose gases in the Gross-Pitaevskii regime. In particular, we will derive a nonlinear correction to the Bogoliubov approximation, thus capturing precisely the two-body scattering process of the particles. This extends a result of Boccato, Brennecke, Cenatiempo and Schlein on the homogeneous Bose gas. The talk is based on joint work with Arnaud Triay.

From branching singularities of minimal surfaces to non-smoothness points on an ice-water interface

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Stefan's problem, dating back to the XIX century, aims to describe the evolution of a block of ice melting in water. Its mathematical analysis experienced few progress until the 1970's, when Duvaut reformulated it as the gradient flow of a nice convex functional. In 1977, Caffarelli proved that the ice-water interface is an smooth surface outside of a certain closed set: the so-called singular set. This was as huge breakthrough. However, methods available back in the 1970's did not allow for a fine description of the structure of the singular set.

During the following 20 years, Almgren developed his theory of branching singularities of minimal surfaces, and in 2008 these methods were applied to the not-so-well-known "thin obstacle problem". This induced, in recent years, a fruitful use of Almgren's methods to study singularities in Stefan's problem. But even with these powerful new tools in hand, we were only halfway to obtaining a fully satisfying description of the non-smoothness points on an ice-water interface...

Elliptic curves and modularity

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University of Cambridge, United Kingdom

The modularity conjecture for elliptic curves is most famous in its formulation for elliptic curves over the rational numbers – indeed, Wiles proved the modularity of semistable elliptic curves over the rationals as part of his proof of Fermat's Last Theorem.

Recently it has become possible to attack the problem of modularity of elliptic curves over more general number fields. I will explain what this means, what we know, and what kinds of consequences we can expect for the solution of Diophantine equations.

The magic of dimensions 8 and 24

Maryna Viazovska, `viazovska@gmail.com`
Ecole Polytechnique Federale de Lausanne, Switzerland

This talk is about two exceptional mathematical objects: the E8 lattice and the Leech lattice. These two lattices are extraordinary tight and symmetric. They satisfy many extremal properties, in particular provide the densest sphere packings in their respective dimensions and are universally optimal. This lecture is an attempt to answer the question: what is so unusual about dimensions 8 and 24 and why they host these two "magic" lattices?

Felix Klein Prize Winner

Overcoming the curse of dimensionality: from nonlinear Monte Carlo to deep learning

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University of Münster, Germany, and *The Chinese University of Hong Kong, Shenzhen, China*

Partial differential equations (PDEs) are among the most universal tools used in modelling problems in nature and man-made complex systems. For example, stochastic PDEs are a fundamental ingredient in models for nonlinear filtering problems in chemical engineering and weather forecasting, deterministic Schroedinger PDEs describe the wave function in a quantum physical system, deterministic Hamiltonian-Jacobi-Bellman PDEs are employed in operations research to describe optimal control problems where companys aim to minimise their costs, and deterministic Black-Scholes-type PDEs are highly employed in portfolio optimization models as well as in state-of-the-art pricing and hedging models for financial derivatives. The PDEs appearing in such models are often high-dimensional as the number of dimensions, roughly speaking, corresponds to the number of all involved interacting substances, particles, resources, agents, or assets in the model. For instance, in the case of the above mentioned financial engineering models the dimensionality of the PDE often corresponds to the number of financial assets in the involved hedging portfolio. Such PDEs can typically not be solved explicitly and it is one of the most challenging tasks in applied mathematics to develop approximation algorithms which are able to approximatively compute solutions of high-dimensional PDEs. Nearly all approximation algorithms for PDEs in the literature suffer from the so-called "curse of dimensionality" in the sense that the number of required computational operations of the approximation algorithm to achieve a given approximation accuracy grows exponentially in the dimension of the considered PDE. With such algorithms it is impossible to approximatively compute solutions of high-dimensional PDEs even when the fastest currently available computers are used. In the case of linear parabolic PDEs and approximations at a fixed space-time point, the curse of dimensionality can be overcome by means of Monte Carlo approximation algorithms and the Feynman-Kac formula. In this talk we prove that suitable deep neural network approximations do indeed overcome the curse of dimensionality in the case of a general class of semilinear parabolic PDEs and we thereby prove, for the first time, that a general semilinear parabolic PDE can be solved approximatively without the curse of dimensionality.

Otto Neugebauer Prize Winner

On how mathematicians' historical and philosophical reflections have been essential to the advancement of mathematics: A historical perspective

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SPHERE, CNRS and Université de Paris, France

The presentation will focus on an episode in which the historical and philosophical work carried out by mathematicians played a decisive role in overcoming a mathematical difficulty and introducing an idea that had a major impact on future developments in mathematics. The episode in question is the introduction in 1845 by Ernst Eduard Kummer (1810-1893) of the "ideal factors" of what he called the "complex numbers". The first public presentation of this concept by Kummer in 1846 allows us to trace the impact on this breakthrough of the historical and philosophical reflections that Jean-Victor Poncelet (1788-1867) and Michel Chasles (1793-1880) developed while giving shape to what would become projective geometry. This episode suggests the benefits that could derive from a more systematic inclusion of historical and philosophical approaches in the practice of mathematics, as an integral part of it.

Minisymposia

Minisymposia in

ALGEBRA

- Computational aspects of commutative and noncommutative positive polynomials (MS-77)
- Noncommutative structures within order structures, semigroups and universal algebra (MS-67)

Minisymposium

COMPUTATIONAL ASPECTS OF COMMUTATIVE AND NONCOMMUTATIVE POSITIVE POLYNOMIALS (MS-77)

Organized by Victor Magron, *CNRS LAAS, France*
Coorganized by Igor Klep, *University of Ljubljana, Department of Mathematics, Slovenia*

- Non-commutative polynomial in quantum physics, *Antonio Acin*
- SOS relaxations for detecting quamtum entanglement, *Abhishek Bhardwaj*
- Sum-of-Squares proofs of logarithmic Sobolev inequalities on finite Markov chains, *Hamza Fawzi*
- Optimizing conditional entropies for quantum correlations, *Omar Fawzi*
- Fast Semidefinite Optimization with Latent Basis Learning, *Georgina Hall*
- Positive maps and trace polynomials from the symmetric group, *Felix Huber*
- Optimization over trace polynomials, *Victor Magron*
- Quantum Isomorphism of Graphs: an Overview, *Laura Mančinska*
- Quantum de Finetti theorems and Reznick's Positivstellensatz, *Ion Nechita*
- Tensor Decompositions on Simplicial Complexes: Existence and Applications, *Tim Netzer*
- Dual nonnegativity certificates and efficient algorithms for rational sum-of-squares decompositions, *Dávid Papp*
- Semidefinite relaxations in non-convex spaces, *Alejandro Pozas-Kerstjens*
- Quantum polynomial optimisation problems for dimension d variables, with symmetries, *Marc Olivier Renou*
- Hilbert's 17th problem for noncommutative rational functions, *Jurij Volčič*
- Efficient noncommutative polynomial optimization by exploiting sparsity, *Jie Wang*
- The tracial moment problem on quadratic varieties, *Aljaž Zalar*

Non-commutative polynomial in quantum physics

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Quantum physics is a natural source of problems involving the optimization of polynomials over non-commuting variables. The talk first introduces some of the most relevant problems, explains how they can be tackled using semi-definite programming hierarchies and concludes with some of the open problems.

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SOS relaxations for detecting quamtum entanglement

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Positive maps which are not completely positive (PNCP maps) are of importance in quantum information theory, in particular they can be used to identify entanglement in quantum states. PNCP maps can naturally be associated to non-negative polynomials which are not sums of squares (SOS). An algorithm for constructing such maps is given in “There are many more positive maps than completely positive maps" by Klep et al.

In this talk we will present a summary of their construction and discuss theoterical and numerical issues that arise in practice.

Sum-of-Squares proofs of logarithmic Sobolev inequalities on finite Markov chains

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Logarithmic Sobolev inequalities play an important role in understanding the mixing times of Markov chains on finite state spaces. It is typically not easy to determine, or indeed approximate, the optimal constant for which such inequalities hold. In this paper, we describe a semidefinite programming relaxation for the logarithmic Sobolev constant of a finite Markov chain. This relaxation gives certified lower bounds on the logarithmic Sobolev constant. Numerical experiments show that the solution to this relaxation is often very close to the true constant. Finally, we use this relaxation to obtain a sum-of-squares proof that the logarithmic Sobolev constant is equal to half the Poincaré constant for the specific case of a simple random walk on the odd n-cycle, with n in 5,7,...,21. Previously this was known only for n=5 and even n.

Optimizing conditional entropies for quantum correlations

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Coauthors: Peter Brown, Hamza Fawzi

The rates of quantum cryptographic protocols are usually expressed in terms of a conditional entropy minimized over a certain set of quantum states. In the so-called device-independent setting, the minimization is over all the quantum states of arbitrary dimension jointly held by the adversary and the parties that are consistent with the statistics that are seen by the parties. We introduce new quantum divergences and use techniques from noncommutative polynomial optimization to approximate such entropic quantities.
Based on https://arxiv.org/abs/2007.12575

Fast Semidefinite Optimization with Latent Basis Learning

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INSEAD, France

When faced with a semidefinite program (SDP), it is often the case that we do not need to solve just one specific SDP, but rather a family of very similar problems with varying data, for example, when solving matrix completion problems to obtain movie recommendations for users with similar preferences. In this talk, I will present Fast Semidefinite Optimization (FSDO), a data-driven method to quickly solve SDPs coming from the same family. Our method learns a shared latent basis representation across the family, which is then used as input to a second-order cone program, whose solution constitutes an approximate solution to the original SDP. The learning is done using neural networks and leverages recent advances in differentiable convex optimization.

Positive maps and trace polynomials from the symmetric group

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With techniques borrowed from quantum information theory, we develop a method to systematically obtain operator inequalities and identities in several matrix variables. These take the form of trace polynomials: polynomial-like expressions that involve matrix monomials $X_{\alpha_1} \cdots X_{\alpha_r}$ and their traces $\mathrm{tr}(X_{\alpha_1} \cdots X_{\alpha_r})$. Our method rests on translating the action of the symmetric

group on tensor product spaces into that of matrix multiplication. As a result, we extend the polarized Cayley-Hamilton identity to an operator inequality on the positive cone, characterize the set of multilinear equivariant positive maps in terms of Werner state witnesses, and construct permutation polynomials and tensor polynomial identities on tensor product spaces. We give connections to concepts in quantum information theory and invariant theory.

Optimization over trace polynomials

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LAAS CNRS & Institute of Mathematics from Toulouse, France
Coauthors: Igor Klep, Jurij Volčič

Motivated by recent progress in quantum information theory, this article aims at optimizing trace polynomials, i.e., polynomials in noncommuting variables and traces of their products. A novel Positivstellensatz certifying positivity of trace polynomials subject to trace constraints is presented, and a hierarchy of semidefinite relaxations converging monotonically to the optimum of a trace polynomial subject to tracial constraints is provided. This hierarchy can be seen as a tracial analog of the Pironio, Navascués and Acín scheme [New J. Phys., 2008] for optimization of noncommutative polynomials. The Gelfand-Naimark-Segal (GNS) construction is applied to extract optimizers of the trace optimization problem if flatness and extremality conditions are satisfied. These conditions are sufficient to obtain finite convergence of our hierarchy. The results obtained are applied to violations of polynomial Bell inequalities in quantum information theory. The main techniques used in this paper are inspired by real algebraic geometry, operator theory, and noncommutative algebra.

Quantum Isomorphism of Graphs: an Overview

Laura Mančinska, mancinska@math.ku.dk
University of Copenhagen, Denmark

In this talk I will introduce a quantum version of graph isomorphism. Our point of departure will be an interactive protocol (nonlocal game) where two provers try to convince a verifier that two graphs are isomorphic. Allowing provers to take advantage of shared quantum resources will then allow us to define quantum isomorphism as the ability of quantum players to win the corresponding game with certainty. We will see that quantum isomorphism can be naturally reformulated in the languages of quantum groups and counting complexity. In particular, we will see that two graphs are quantum isomorphic if and only if they have the same homomorphism counts from all planar graphs.

Quantum de Finetti theorems and Reznick's Positivstellensatz

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CNRS, Toulouse University, France
Coauthors: Alexander Müller-Hermes, David Reeb

We present a proof of Reznick's quantitative Positivstellensatz using ideas from Quantum Information Theory. This result gives tractable conditions for a positive polynomial to be written as a sum of squares. We relate such results to de Finetti theorems in Quantum Information.

Tensor Decompositions on Simplicial Complexes: Existence and Applications

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Coauthors: Gemma De las Cuevas, Matt Hoogsteder Riera, Andreas Klingler

Inspired by the tensor network approach from theoretical quantum physics, we develop a framework to define and analyze invariant decompositions of elements of tensor product spaces. We define an invariant decomposition with indices arranged on a simplicial complex, which is explicitly invariant under a given group action. Several versions of such decompositions also allow to cover positivity of the involved objects. We prove that these decompositions exists for all invariant/positive tensors, after possibly enriching the structure of the simplicial complex. The approach cannot only be applied to tensor products of matrices (as done in quantum physics), but to multivariate polynomials as well. This yields new types of decompositions and complexity measures for polynomials, containing certificates for positivity and symmetries.

Dual nonnegativity certificates and efficient algorithms for rational sum-of-squares decompositions

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North Carolina State University, United States
Coauthor: Maria L. Macaulay

We study the problem of computing rational weighted sum-of-squares (WSOS) certificates for positive polynomials over a compact semialgebraic set. In the first part of the talk, we introduce the concept of dual cone certificates, which allows us to interpret vectors from the dual of the WSOS cone as rigorous nonnegativity certificates. Every polynomial in the interior of the WSOS cone admits a full-dimensional cone of dual certificates; as a result, rational WSOS certificates can be constructed from numerically computed dual certificates at little additional cost. In the second part of the talk, we use this theory to develop an almost entirely numerical hybrid algorithm for computing the optimal WSOS lower bound of a given polynomial along with a rational dual certificate, with a polynomial-time computational cost per iteration and linear rate of convergence. As a special case, we obtain a new polynomial-time algorithm for certifying the nonnegativity of strictly positive polynomials over an interval.

Semidefinite relaxations in non-convex spaces

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Mathematical Analysis and Applied Mathematics Department, Universidad Complutense de Madrid, Spain

In analogy with Lassere's and Parillo's hierarchies of semidefinite relaxations for polynomial optimization problems, a semidefinite hierarchy exisits for non-commutative polynomial optimization problems of the form

$$\begin{array}{rl} p^* = \min\limits_{(\mathcal{H},\phi,X)} & \langle \phi, p(X)\phi \rangle \\ \text{s.t.} & q_i(X) \succeq 0 \end{array},$$

where ϕ is a normalized vector in the Hilbert space $\mathcal{H}$, $X = (X_1, \ldots, X_n)$ are the non-commuting variables in the problem, and $p(X)$ and $q_i(X)$ are polynomials in the variables X. This hierarchy, known as the Navascués-Pironio-Acín hierarchy, has encountered important applications in physics, where it has become a central tool in quantum information theory and in the certification of quantum phenomena.

Its success has motivated its application, within quantum information theory, in more complex scenarios where the relevant search spaces are not convex, and thus the characterizing constraints are in conflict with semidefinite formulations. The paradigmatic example is optimizing over probability distributions obtained by parties performing measurements on quantum systems where not all parties receive a share from every system available. The most illustrating consequence is that, in certain situations, marginalization over a selected number of parties makes the resulting probability distribution to factorize.

In this talk we address the problem of non-commutative polynomial optimization in non-convex search spaces and present two means of providing monotonically increasing lower bounds on its solution, that retain the characteristic that the newly formulated problems remain being semidefinite programs. The first directly addresses factorization-type constraints by adding auxiliary commuting variables that allow to encode (relaxations of) polynomial trace constraints, while the second considers multiple copies of the problem variables and constrains them by requiring the satisfaction of invariance under suitable permutations. In addition to presenting the methods and exemplifying their applicability in simple situations, we highlight the fundamental questions that remain open, mostly regarding to their convergence.

Quantum polynomial optimisation problems for dimension d variables, with symmetries

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Quantum Information Theory (QIT) involves quantum states and measurements, mathematically represented as non-commutative positive operators. A typical problem in QIT is to find the minimum of a polynomial expression in these operators. For instance, finding the maximum violation of a Bell inequality, or the ground state energy of a Hamiltonian are polynomial optimisation problems in non-commuting variables. The NPA hierarchy [New J. Phys. 10, 073013 (2008)], which can be viewed as the "eigenvalue" version of Lasserre's hierarchy, provides a converging hierarchy of SDP relaxation of a non-commutative polynomial optimisation problem involving variables of unbounded dimension. This hierarchy converges, it is one of QIT main technical tool.

Importantly, some QIT problems concern operators of bounded dimension d. The NPA hierarchy was extended into the NV hierarchy [Phys. Rev. Lett.115, 020501(2015)] to tackle this case. In this method, one first sample at random many dimension d operators satisfying the constraint, and compute the associated moment matrix. This first step discovers the moment matrix vector space, over which the relaxed SDP problem is solved in a second step. In this talk, we will first review this method. Then, based on [Phys. Rev. Lett. 122, 070501], we will show how one can reduce the computational requirements by several orders of magnitude, exploiting the eventual symmetries present in the optimization problem.

Hilbert's 17th problem for noncommutative rational functions

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One of the problems on Hilbert's 1900 list asked whether every positive rational function can be written as a sum of squares of rational functions. Its affirmative resolution by Artin in 1927 was a breakthrough for real algebraic geometry. This talk addresses the analog of this problem for noncommutative rational functions. More generally, a rational Positivstellensatz on matricial sets given by linear matrix inequalities is presented. The crucial intermediate step is an extension theorem on invertible evaluations of linear matrix pencils. A consequence of the Positivstellensatz is an algorithm for eigenvalue optimization of noncommutative rational functions. Lastly, the talk discusses some contrast between polynomial and rational Positivstellensätze.

Efficient noncommutative polynomial optimization by exploiting sparsity

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Coauthor: Victor Magron

Many problems arising from quantum information can be modelled as noncommutative polynomial optimization problems. The moment-SOHS hierarchy approximates the optimum of noncommutative polynomial optimization problems by solving a sequence of semidefinite programming relaxations with increasing sizes. In this talk, I will show how to exploit various sparsity patterns encoded in the problem data to improve scalability of the moment-SOHS hierarchy for eigenvalue and trace optimization problems.

The tracial moment problem on quadratic varieties

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University of Ljubljana, Slovenia

The truncated moment problem asks to characterize finite sequences of real numbers that are the moments of a positive Borel measure on $\mathbb{R}^n$. Its tracial analog is obtained by integrating traces of symmetric matrices. The solution of the bivariate quartic tracial moment problem with a nonsingular 7×7 moment matrix $\mathcal{M}_2$ whose columns are indexed by words of degree 2 was established by Burgdorf and Klep. In the talk we will present the solution of the singular bivariate tracial moment problem on quadratic varieties. In case the moment matrix $\mathcal{M}_n$ satisfies two quadratic column relations the existence of the measure can be completely characterized by certain numerical conditions, while in the presence of one quadratic column relation the existence of a representing measure is equivalent to the feasibility of certain linear matrix inequalities. The atoms in the representing measure are always pairs of 2×2 matrices. This is joint work with Abhishek Bhardwaj.

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Minisymposium

NONCOMMUTATIVE STRUCTURES WITHIN ORDER STRUCTURES, SEMIGROUPS AND UNIVERSAL ALGEBRA (MS-67)

Organized by Karin Cvetko-Vah, *University of Ljubljana, Slovenia*
Coorganized by
Joao Pita Costa, *Institute Jozef Stefan, Slovenia*
Michael Kinyon, *Denver University, United States*

- Duality for noncommutative frames, *Jens Hemelaer*
- Quasibands and nonassociative, noncommutative lattices, *Michael Kinyon*
- Covering skew lattices, *Jurij Kovič*
- Eilenberg-Moore, Kleisli and descent factorizations, *Fernando Lucatelli Nunes*
- On modular skew lattices and their coset structure, *Joao Pita Costa*
- Noncommutativity in an algebraic theory of clones, *Antonino Salibra*
- Skew lattices and set-theoretic solutions of the Yang-Baxter equation, *Charlotte Verwimp*

Duality for noncommutative frames

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Coauthors: Karin Cvetko-Vah, Lieven Le Bruyn

Noncommutative frames were introduced by Karin Cvetko-Vah as a noncommutative generalization of frames, similar to how skew lattices generalize lattices. The concept of a noncommutative frame was motivated by Lieven Le Bruyn's construction of a noncommutative topology on the points of the Arithmetic Site of Connes and Consani. In this talk, we will extend the duality between locales and frames to the noncommutative world. Our approach is inspired by an earlier paper of Bauer, Cvetko-Vah, Gehrke, van Gool and Kudryavtseva, that introduced a noncommutative Priestley duality.

The talk is based on joint work with Karin Cvetko-Vah and Lieven Le Bruyn.

Quasibands and nonassociative, noncommutative lattices

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University of Denver, United States

Nonassociative idempotent magmas arise naturally in various settings such as the faces of a building or chains in modular lattices. In this talk I will describe a variety of magmas we call *quasibands*, which arise as (sub)reducts of bands (idempotent semigroups): in a band $(B, \cdot)$, define a new operation $\circ$ by $x \circ y = xyx$. This is analogous to how quandles arise as subreducts of groups under the conjugation operations. The quasiband operation $\circ$ is sometimes used as a notational shorthand (especially in the theory of noncommutative lattices) or to characterize band properties. For instance, $(B, \circ)$ is associative, hence a left regular band, if and only if $(B, \cdot)$ is a regular band.

The variety of quasibands is defined by 4 identities. A main result is that this is precisely the variety of $\circ$-subreducts of bands. In addition, I will talk about the natural preorder and natural partial order on a quasiband, the center of a quasiband, the relationship between free quasibands and free bands, and some enumeration of quasibands for low orders.

In the case of a noncommutative lattice $(B, \wedge, \vee)$, the corresponding "double quasiband" $(B, \curlywedge, \curlyvee)$ can be viewed as a nonassociative, noncommutative lattice. I will discuss the relationship between the two structures for some of the more commonly studied classes of noncommutative lattices (quasilattices, paralattices, skew lattices, etc.)

This is joint work with Tomaž Pisanski (Ljubljana). My own work is partially supported by Simons Foundation Collaboration Grant 359872.

Covering skew lattices

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We introduce the concept of covering skew lattices, using the analogy with the covering graphs, and we explain some new discovered phenomena in different forms.

Eilenberg-Moore, Kleisli and descent factorizations

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Every functor that has a left adjoint has two well-known factorizations. The first one is through the category of Eilenberg-Moore algebras of the induced monad, while the second one is the factorization through the category of free coalgebras (co-Kelsili category) of the induced comonad. As usual in category theory, we also have the dual cases: a functor that has a right adjoint has a factorization through the category of Eilenberg-Moore coalgebras of the induced comonad, and other factorization through the Kleisli category of the induced monad.

More generally, if the functor has a codensity monad (resp. a op-codensity comonad), the functor has a factorization through the category of Eilenberg-Moore algebras (resp. the free coalgebras) (see, for instance, [5, Section 3]).

In [5], given a 2-category $\mathbb{A}$ satisfying suitable hypothesis, we show that every morphism inside a 2-category with opcomma objects (and pushouts) has a 2-dimensional cokernel diagram which, in the presence of the descent objects, induces a factorization of the morphism. We show that these factorizations generalize the usual Eilenberg-Moore and Kleisli factorizations.

The result is new even in the case $\mathbb{A} = \mathsf{Cat}$. In this case, we have that every functor has a factorization through the category of descent data of its 2-dimensional cokernel diagram. We show that, if a functor F has a left adjoint, this descent factorization coincides with the factorization through the category of algebras. Dually, if F has a right adjoint, this descent factorization coincides with the factorization through the category of coalgebras.

This specializes in a new connection between monadicity and descent theory, which can be seen as a counterpart account to the celebrated Bénabou-Roubaud Theorem (see [6] or, for instance, [4, Theorem 1.4] and [3, Section 4]). It also leads in particular to a (formal) monadicity theorem.

In this talk, we shall give a sketch of the ideas and constructions involved in this particular case of $\mathbb{A} = \mathsf{Cat}$.

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On modular skew lattices and their coset structure

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Institute Jozef Stefan, Slovenia

Modular lattices are of great importance in many branches of mathematics, from Algebra to Topology. There are many well-known examples of lattices that are modular but not distributive: the lattice of subspaces of a vector space, the modules over a ring, the normal subgroups of a group, and many others. Modularity is a lattice property of somewhat topological flavour where, in the finite case, maximal chains have the same size, and decomposition theorems as the Krull-Schmidt-Remak for modules over a ring are derived from the modularity properties of its subspace lattice. In this talk, we will discuss several approaches to the generalisation of modularity to the non-commutative context of skew lattices, relate it to the known properties of skew distributivity and skew cancellation, and derive topological-like properties from that generalised concept. We will also discuss aspects of the coset structure of skew lattices of this nature, and derive some of their combinatorial properties.

Noncommutativity in an algebraic theory of clones

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Coauthor: Antonio Bucciarelli

We introduce the notion of clone algebra, intended to found a one-sorted, purely algebraic theory of clones. Clone algebras are defined by true identities and thus form a variety in the sense of universal algebra. The most natural clone algebras, the ones the axioms are intended to characterise, are algebras of functions, called functional clone algebras. The universe of a functional clone algebra, called omega-clone, is a set of infinitary operations containing the projections and closed under finitary compositions. We show that there exists a bijective correspondence between clones (of finitary operations) and a suitable subclass of functional clone algebras, called block algebras. Given a clone, the corresponding block algebra is obtained by extending the operations of the clone by countably many dummy arguments. One of the main results is the general representation theorem, where it is shown that every clone algebra is isomorphic to a functional clone algebra. In another result we prove that the variety of clone algebras is generated by the class of block algebras. This implies that every omega-clone is algebraically generated by a suitable family of clones by using direct products, subalgebras and homomorphic images. We present three applications. In the first one, we use clone algebras to answer a classical question about the lattices of equational theories. The second application is to the study of the category VAR of all varieties. We introduce the category CA of all clone algebras (of arbitrary similarity type) with pure homomorphisms as arrows. We show that the category VAR is categorically isomorphic to a full subcategory of CA. We use this result to provide a generalisation of a classical theorem on independent varieties. In the third application we show how skew Boolean algebras are related to clone algebras.

Skew lattices and set-theoretic solutions of the Yang-Baxter equation

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The Yang-Baxter equation originates from papers by Yang and Baxter on quantum and statistical mechanics, and the search for solutions has attracted numerous studies both in mathematical physics and pure mathematics. As the study of arbitrary solutions is complex, Drinfeld proposed in 1992 to focus on the class of set-theoretic solutions. The goal is simple, find all set-theoretic solutions of the Yang-Baxter equation.

Recently introduced algebraic structures, like braces, cycle sets and their generalizations, are related to special classes of set-theoretic solutions. Still, an algebraic structure that describes all set-theoretic solutions of the Yang-Baxter equation is not known. In this talk, we discuss set-theoretic solutions obtained from skew lattices, an algebraic structure that had not yet been related to the Yang-Baxter equation. Such solutions are degenerate in general, and thus different from solutions obtained from braces and other structures.

This talk is based on joint work with Karin Cvetko-Vah.

Minisymposia in

ALGEBRAIC AND COMPLEX GEOMETRY

- Arithmetic and Geometry of Algebraic Surfaces (MS-45)
- Complex Analysis and Geometry (MS-17)
- Topics in complex and quaternionic geometry (MS-74)

Minisymposium

Arithmetic and Geometry of Algebraic Surfaces (MS-45)

Organized by Francesco Polizzi, *Università della Calabria, Italy*
Coorganized by Matteo Penegini, *Università degli Studi di Genova, Italy*

- The Mumford–Tate conjecture for surfaces — state of the art, *Johan Commelin*
- Moduli space of stable (1,2)-surfaces, *Stephen Coughlan*
- Irregular covers of K3 surfaces, *Alice Garbagnati*
- Rigid complex manifolds via product quotients, *Christian Gleissner*
- Subbundles of the Hodge bundle, fibred surfaces and the Coleman-Oort conjecture, *Víctor González Alonso*
- Infinitesimal Torelli for elliptic surfaces revisited, *Remke Kloosterman*
- On a family of surfaces with $p_g = q = 2$ and $K^2 = 7$, *Matteo Penegini*
- New families of surfaces with canonical map of high degree, *Roberto Pignatelli*
- Diagonal double Kodaira fibrations with minimal signature, *Francesco Polizzi*
- Z/2-Godeaux surfaces, *Carlos Rito*
- Generalized Kummer surfaces and a configuration of conics in the plane, *Alessandra Sarti*
- Holomorphic one forms on projective surfaces and applications, *Sara Torelli*

The Mumford–Tate conjecture for surfaces — state of the art

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The Mumford–Tate conjecture (MTC) provides a bridge between the Hodge structure on the singular cohomology of an algebraic variety (over the complex numbers) and the Galois representation on the étale cohomology of a model of that same variety over a number field. Recently, there have been several new results on MTC for algebraic surfaces. In this talk I will give an overview of what is currently known, and explain some of the techniques used in the new results.

Moduli space of stable (1,2)-surfaces

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A (1,2)-surface is a surface of general type with geometric genus 2 and canonical degree 1. I describe ongoing work with several collaborators, aiming to understand the so-called KSBA compactification of the moduli space of (1,2)-surfaces. In particular, we are able to describe some irreducible components of the moduli space and some boundary strata too.

Irregular covers of K3 surfaces

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Coauthor: Matteo Penegini

The K3 surfaces are regular surfaces. They can be covered by irregular surfaces in several ways and the easiest example is given by the abelian surfaces: these often appear as the minimal model of Galois covers of K3 surfaces for several Galois group (e.g. all the Abelian surfaces are birational to the double cover of their Kummer surfaces).

Nevertheless is considerably more difficult to obtain an irregular cover of a K3 surface which is a surface of general type.

During the talk we discuss the construction of irregular covers of K3 surfaces, obtaining both surfaces with Kodaira dimension 1 and 2. In particular we focus our attention to the covers of order 2 and 3 (the latter not necessarly Galois).

Rigid complex manifolds via product quotients

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Product quotients give rise to a large class of varieties equipped with a lot of symmetry. They are particularly useful as a tool to provide examples of complex manifolds with special properties. In this talk I will explain how to construct in each dimension dimension n (at least three) an example of a rigid n-manifold of Kodaira dimension one, as a resolution of a singular product quotient. Their existence was conjectured by I. Bauer and F. Catanese and constructed in a joint project with I. Bauer.

Subbundles of the Hodge bundle, fibred surfaces and the Coleman-Oort conjecture

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The Hodge bundle of a semistable family of complex projective curves has two nested subbundles: the flat unitary subbundle (spanned by flat sections with respect to the Gauss-Manin connection), and the kernel of the Higgs field. The latter contains the flat subbundle, but it was not clear how strict the inclusion could be. In this talk I will present some techniques to estimate the ranks of both subbundles and to construct families where they are arbitrarily different. I will also discuss some implications of this result for the classification of fibred surfaces, as well as some connections with the Coleman-Oort conjecture on the (non-)existence of totally geodesic subvarieties in the Torelli locus of principally polarized abelian varieties. This is joint work with Sara Torelli.

Infinitesimal Torelli for elliptic surfaces revisited

Remke Kloosterman, klooster@math.unipd.it
University of Padova, Italy

In this talk we consider the infinitesimal Torelli problem for elliptic surfaces without multiple fibers.

We give a new proof for the case of elliptic surfaces without multiple fibers with Euler number at least 24 and nonconstant j-invariant and with Euler number at least 72 and constant j-invariant.

For all of the remaining cases we will indicate whether infinitesimal Torelli holds, does not hold or our methods are insufficient to decide.

This solve an issue raised by Atsushi Ikeda in a recent paper, in connection with his construction a counterexample to infinitesimal Torelli with $p_g = q = 1$.

On a family of surfaces with $p_g = q = 2$ and $K^2 = 7$

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Università di Genova, Italy
Coauthor: Roberto Pignatelli

In this talk we shall study a family of surfaces of general type with $p_g = q = 2$ and $K^2 = 7$, originally constructed by C. Rito. We provide an alternative construction of these surfaces, that allows us to describe their Albanese map and the corresponding locus $\mathcal{M}$ in the moduli space of the surfaces of general type. In particular we prove that $\mathcal{M}$ is an irreducible component, two dimensional and generically smooth.

New families of surfaces with canonical map of high degree

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Coauthor: Federico Fallucca

By a celebrated result of A. Beauville, if the dimension of the image of the canonical map of a complex surface is two, then the image is either of genus zero or a canonical surface. In both cases the degree of the map is uniformely bounded from above.

Until a few years ago, very few examples of surfaces with canonical map of high degree were known.

Thanks to the work of some collegues, new examples have been obtained in recent years, opening up new research questions and perspectives.

In this seminar we will present a new method for constructing surfaces with canonical map of high degree and some new examples produced with this method.

Finally, we will discuss the contribution our examples make to the aforementioned issues.

Diagonal double Kodaira fibrations with minimal signature

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We study some special systems of generators on finite groups, introduced in previous work by the first author and called *diagonal double Kodaira structures*, in order to investigate non-abelian, finite quotients of the pure braid group on two strands $\mathsf{P}_2(\Sigma_b)$, where Σ_b is a closed Riemann surface of genus b. In particular, we prove that, if G admits a diagonal double Kodaira structure, then $|G| \geq 32$, and equality holds if and only if G is extra-special. In the last part, as a geometrical application of our algebraic results, we construct two 3-dimensional families of double Kodaira fibrations having signature 16.

This is joint work with P. Sabatino.

Z/2-Godeaux surfaces

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University of Porto - CMUP, Portugal

I will report on work in progress in collaboration with Eduardo Dias towards the classification of Z/2-Godeaux surfaces.

Generalized Kummer surfaces and a configuration of conics in the plane

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A generalized Kummer surface is the minimal resolution of the quotient of an abelian surface by an automorphism of order three. The quotient surface contains nine cusps and this is a special example of a singular K3 surface. The converse is also true : the minimal resolution of a K3 surface containing nine cusps is a generalized Kummer surface. In this paper we study several configurations of nine cusps on the same K3 surface, which is the double cover of the plane ramified on a sextic with nine cusps. We get the new configurations by studying conics through the singular points of the sextic. The aim of the construction is to investigate the following question: is it possible that the K3 surface is the generalized Kummer surface associated to two non-isomorphic abelian surfaces ? This is a joint work with D. Kohl and X. Roulleau.

Holomorphic one forms on projective surfaces and applications

Sara Torelli, sara.torelli7@gmail.com
University of Hannover, Germany

In the talk I present a result, recently proven in collaboration with F.Favale and G.P.Pirola, that aims to recover holomorphic 1-forms on smooth projective surfaces from divisors contracted to points by big and semiample line bundles. I'll then discuss its application to investigate holomorphic forms on higher dimensional varieties (e.g. the moduli space of pointed curves $\mathcal{M}_{g,n}$) and related open problems.

Minisymposium

Complex Analysis and Geometry (MS-17)

Organized by Franc Forstnerič, *University of Ljubljana, Slovenia*
Coorganized by Bernhard Lamel, *University of Vienna*

- The Density Property for Calogero–Moser Spaces, *Rafael Andrist*
- “Levi core", Diederich-Fornaess index, and D’Angelo forms, *Gian Maria Dall’Ara*
- Metric geometry of domains in complex Euclidean spaces, *Hervé Gaussier*
- Fekete Configurations, Products of Vandermonde Determinants and Canonical Point Processes, *Jakob Hultgren*
- Images of CR manifolds, *Jiri Lebl*
- Strict density inequalities for interpolation in weighted spaces of holomorphic functions in several complex variables., *Joaquim Ortega Cerdà*
- Parabolic Fatou components with holes, *Josias Reppekus*
- Angular Derivatives and Boundary Values of H(b) Spaces of Unit Ball of $\mathbb{C}^n$, *Sibel Şahin*
- Seiberg-Witten equations and pseudoholomorphic curves, *Armen Sergeev*
- The CR Ahlfors derivative and a new invariant for spherically equivalent CR maps, *Duong Ngoc Son*
- Equivalence of neighborhoods of embedded compact complex manifolds and higher codimension foliations, *Laurent Stolovitch*
- Weighted-L^2 polynomial approximation in $\mathbb{C}$, *Jujie Wu*
- A criterion of Nakano positivity, *Xiangyu Zhou*

The Density Property for Calogero–Moser Spaces

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American University of Beirut, Lebanon

A Calogero–Moser space describes the (completed) phase space of a system of finitely many particles in classical physics. Since the past two decades, these spaces are also an object of ongoing study in pure mathematics. In particular, a Calogero–Moser space of n particles is known to be a smooth complex-affine variety, and to be diffeomorphic to the Hilbert scheme of n points in the affine plane. We establish the density property for the Calogero–Moser spaces and describe their group of holomorphic automorphisms.

"Levi core", Diederich-Fornaess index, and D'Angelo forms

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Coauthor: Samuele Mongodi

We introduce the notion of "Levi core" of a CR manifold M of hypersurface type, and prove that, whenever M is the boundary of a smoothly bounded pseudoconvex domain Ω, the Diederich-Fornaess index of Ω equals a numerical invariant of the Levi core. This line of research is connected and extends recent results of B. Liu, M. Adachi, and J. Yum.

Metric geometry of domains in complex Euclidean spaces

Hervé Gaussier, herve.gaussier@univ-grenoble-alpes.fr
Université Grenoble Alpes, France

The boundary geometry of a domain strongly constrains the asymptotic behaviour of invariant metrics and of their curvatures, as shown in important contributions. I will present recent results that, conversely, give metric characterizations of the boundary geometry of some domains.

Fekete Configurations, Products of Vandermonde Determinants and Canonical Point Processes

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University of Maryland, United States

The asymptotic behavior of Fekete configurations is a classical topic in complex analysis. By definition, Fekete configurations are arrays of points that maximize Vandermonde determinants. From a complex geometric perspective, any Hermitian ample line bundle over a compact Kähler manifold defines a sequence of Vandermonde determinants of increasing dimension and thus a notion of Fekete configurations. In this talk we will consider a collection of Hermitian ample line bundles over a fixed compact Kähler manifold. I will present two results. One regarding the product of the associated Vandermonde determinants and the asymptotic behavior of its maximizers and one regarding existence of sequences of point configurations which are asymp-

totically Fekete (a slightly weaker concept than Fekete) with respect to all Hermitian ample line bundles simultaneously. If time permits I will also outline a conjectural picture involving a related class of point processes and its connection to canonical metrics in complex geometry.

Images of CR manifolds

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Coauthors: Alan Noell, Sivaguru Ravisankar

We study CR singular submanifolds that have a removable CR singularity, that is, manifolds that are singular, but whose CR structure extends through the singularity. Bishop surfaces are trivial examples, but in higher CR dimension, generically the CR singularity is not removable. In particular, we study real codimension 2 submanifolds in $\mathbb{C}^3$. This is joint work with Alan Noell and Sivaguru Ravisankar.

Strict density inequalities for interpolation in weighted spaces of holomorphic functions in several complex variables.

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Coauthors: Karlheinz Gröchenig, Antti Haimi, José Luis Romero

I will present a joint work with Karlheinz Gröchenig, Antti Haimi and José Luis Romero were we solve a problem posed by Lindholm and prove strict density inequalities for sampling and interpolation in Fock spaces of entire functions in several complex variables defined by a plurisubharmonic weight.

In particular, these spaces do not admit a set that is simultaneously sampling and interpolating. To prove optimality of the density conditions, we construct sampling sets with a density arbitrarily close to the critical density.

Parabolic Fatou components with holes

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Many examples of invariant non-recurrent Fatou components of automorphisms F of $\mathbb{C}^2$ arise from local projections to one variable in which the dynamics are approximately parabolic. After constructing a local basin B, a critical step is to show that the associated completely invariant global basin Ω of all orbits ending up in B is not just a proper subset of a Fatou component.

I will present two types of examples in which the initial projection is non-linear and the resulting Fatou component Ω is biholomorphic to $\mathbb{C} \times \mathbb{C}^*$, i.e. “has a hole”. The constructions are based on the dynamics near a fixed point p on the boundary of U such that the eigenvalues of F at p are complex conjugate irrational rotations. For the first type, all orbits in Ω converge to p, whereas for the second type the orbits in Ω converge precisely to the points of an entire curve minus p.

Angular Derivatives and Boundary Values of $H(b)$ Spaces of Unit Ball of $\mathbb{C}^n$

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Mimar Sinan Fine Arts University, Turkey

In this talk we will consider a special subclass of the Hardy-Hilbert space $H^2(\mathbb{B}^n)$, namely deBranges-Rovnyak spaces $H(b)$, in the setting of the unit ball of $\mathbb{C}^n$. One of the main problems in the study of $H(b)$ functions is their representation and in the first part of this talk we will see how we can represent these classes through the Clark measure on S^n associated with b. In the second part we will give a characterization of admissible boundary limits in relation with finite angular derivatives and we will see the interplay between Clark measures and angular derivatives showing that Clark measure associated with b has an atom at a boundary point if and only if b has finite angular derivative at the same point. More detailed analysis of the concepts mentioned in this talk can be found in the following study:
Şahin, S. Angular Derivatives and Boundary Values of H(b) Spaces of Unit Ball of $\mathbb{C}^n$, Complex Varaibles and Elliptic Equations, doi:10.1080/17476933.2020.1715373, (2020).

Seiberg-Witten equations and pseudoholomorphic curves

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Seiberg–Witten equations (SW-equations for short) were proposed in order to produce a new kind of invariant for smooth 4-dimensional manifolds. These equations, opposite to the conformally invariant Yang–Mills equations, are not invariant under scale transformations. So to draw a useful information from these equations one should plug the scale parameter λ into them and take the limit $\lambda \to \infty$.

If we consider such limit in the case of 4-dimensional symplectic manifolds solutions of SW-equations will concentrate in a neighborhood of some pseudoholomorphic curve (more precisely, pseudoholomorphic divisor) while SW-equations reduce to some vortex equations in normal planes of the curve. The vortex equations are in fact static Ginzburg–Landau equations known in the superconductivity theory. So solutions of the limiting adiabatic SW-equations are given by families of vortices in the complex plane parameterized by the point z running along the limiting pseudoholomorphic curve. This parameter plays the role of complex time while the adiabatic SW-equations coincide with a nonlinear $\bar{\partial}$-equation with respect to this parameter.

The CR Ahlfors derivative and a new invariant for spherically equivalent CR maps

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Coauthor: Bernhard Lamel

In this talk, I will discuss a notion of the Ahlfors derivative for CR maps. This notion possesses several important properties similar to those of the conformal counterpart and provides a new invariant for spherical equivalent CR maps from strictly pseudoconvex CR manifolds into a sphere. The invariant can be computed easily and distinguishes many well-known equivalent classes of CR maps between spheres. In particular, it vanishes precisely when the map is spherical equivalent to the linear embedding. This is a joint work with Bernhard Lamel.

Equivalence of neighborhoods of embedded compact complex manifolds and higher codimension foliations

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Coauthor: Xianghong Gong

We consider an embedded n-dimensional compact complex manifold in $n+d$ dimensional complex manifolds. We are interested in the holomorphic classification of neighborhoods as part of Grauert's formal principle program. We will give conditions ensuring that a neighborhood of C_n in M_{n+d} is biholomorphic to a neighborhood of the zero section of its normal bundle. This extends Arnold's result about neighborhoods of a complex torus in a surface. We also prove the existence of a holomorphic foliation in M_{n+d} having C_n as a compact leaf, extending Ueda's theory to the high codimension case. Both problems appear as a kind linearization problem involving *small divisors condition* arising from solutions to their cohomological equations.

Weighted-L^2 polynomial approximation in $\mathbb{C}$

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We study the density of polynomials in $H^2(\Omega, e^{-\varphi})$, the space of square integrable holomorphic functions in a bounded domain Ω in $\mathbb{C}$, where φ is a subharmonic function. In particular, we prove that the density holds in Carathéodory domains for any subharmonic function φ in a neighborhood of $\overline{\Omega}$. In non-Carathéodory domains, we prove that the density depends on the weight function, giving examples. This is joint with Séverine Biard and John Erik Fornæss.

A criterion of Nakano positivity

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How to characterize Nakano positivity of holomorphic vector bundles is a difficult problem in complex geometry. In this talk, we'll give a criterion of Nakano positivity in terms of L^2 extensions. This is joint work with Fusheng Deng, Jiafu Ning, Zhiwei Wang.

Minisymposium

Topics in complex and quaternionic geometry (MS-74)

Organized by Jasna Prezelj, *University of Primorska, University of Ljubljana, IMFM, Slovenia*
Coorganized by
Simon Salamon, *King's College, London, UK*
Fabio Vlacci, *University of Triest, Italy*
Graziano Gentili, *University of Florence, Italy*

- Geometry of 3-(α, δ)-Sasaki manifolds and submersions over quaternionic Kähler spaces, *Ilka Agricola*
- Flags and Twistors, *Amedeo Altavilla*
- On cohomogeneity one Hermitian non-Kähler manifolds, *Daniele Angella*
- Slice Regular Quaternionic Manifolds., *Cinzia Bisi*
- Symplectic duality and implosion, *Andrew Dancer*
- Balanced Hermitian metrics, *Anna Fino*
- Slice-by-slice and global smoothness of slice regular functions, *Riccardo Ghiloni*
- On compact affine quaternionic curves and surfaces, *Anna Gori*
- Geometry of Kato manifolds, *Alexandra Iulia Otiman*
- Semisimple Lie algebras and special Hermitian metrics, *Fabio Podestà*
- On the continuation of quaternionic logarithm along curves and the winding number, *Jasna Prezelj*
- Stability of Einstein metrics, *Uwe Semmelmann*
- Analogues of the Dolbeault resolution in higher dimensions, *Vladimír Souček*
- Slice regular functions and orthogonal complex structures in eight dimensions, *Caterina Stoppato*
- Special geometries with torus symmetry, *Andrew Swann*
- $\overline{\partial}$-Harmonic forms on compact almost Hermitian manifolds, *Adriano Tomassini*
- On a construction of quaternionic and octonionic Riemann surfaces, *Fabio Vlacci*

Geometry of 3-(α, δ)-Sasaki manifolds and submersions over quaternionic Kähler spaces

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Coauthors: Leander Stecker, Giulia Dileo

We give a gentle introduction to the new class of 3-(α, δ)-Sasaki manifolds, which are a natural generalisation of 3-Sasaki manifolds. We prove that any such manifold admits a locally defined Riemannian submersion over a quaternionic Kähler manifold. In the non-degenerate case ($\delta \neq 0$) we describe all homogeneous 3-(α, δ)-Sasaki manifolds fibering over symmetric Wolf spaces and their noncompact dual symmetric spaces. In the compact base case, this yields a complete classification of homogeneous 3-(α, δ)-Sasaki manifolds, while for non-compact bases, we provide a general construction of homogeneous 3-(α, δ)-Sasaki manifolds fibering over nonsymmetric Alekseevsky spaces, the lowest possible dimension of such a manifold being 19.

References

[1] Ilka Agricola, Giulia Dileo, *Generalizations of 3-Sasakian manifolds and skew torsion*, Adv. Geom. 20 (2020), 331-374.

[2] Ilka Agricola, Giulia Dileo, Leander Stecker, *Homogeneous non-degenerate 3-(α, δ)-Sasaki manifolds and submersions over quaternionic Kähler spaces*, to appear in Ann. Global Anal. Geom.

Flags and Twistors

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Coauthors: Edoardo Ballico, Maria Chiara Brambilla, Simon Salamon

In this talk, I will present some first results on the geometry of the flag manifold $\mathbb{F}$ as twistor space of the complex projective plane. Firstly, I will present some general facts on low-degree curves and surfaces in the flag manifold. Afterward, I will introduce the twistor fibration associated with the standard Hermitian metric in $\mathbb{CP}^2$ and describe the set of twistor fibers. In the second part, I will give a description of the family of automorphisms of $\mathbb{F}$ that come from unitary automorphisms of $\mathbb{CP}^2$ and I will show a classification result for a family of algebraic surfaces in $\mathbb{F}$, up to such transformations. For a special sub-family of these surfaces, namely those which are j-invariant, I will give a deeper geometric description.

On cohomogeneity one Hermitian non-Kähler manifolds

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Coauthor: Francesco Pediconi

We study Hermitian manifolds with a compact Lie group action by holomorphic isometries with principal orbits of codimension one. In particular, we focus on a special class of these manifolds constructed by following Bérard-Bergery, which includes, among the others, the holomorphic line bundles on $\mathbb{C}P^{m-1}$, the linear Hopf manifolds and the Hirzebruch surfaces. We characterize their invariant special Hermitian metrics, such as balanced, Kähler-like, pluriclosed, locally conformally Kähler, Vaisman, Gauduchon. Furthermore, we construct new examples of cohomogeneity one Hermitian metrics solving the second-Chern-Einstein equation and the constant Chern-scalar curvature equation.

Slice Regular Quaternionic Manifolds.

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Coauthors: Graziano Gentili, Daniele Angella

In recent years, Slice Regular Quaternionic Manifolds have been introduced and studied by many authors, after the seminal definition of Slice Regular Function given by G. Gentili and D.C. Struppa in 2006-2007.
Slice Regular Quaternionic Manifolds have catched the attention because they have more structure than the real ones but they are less docile than the complex ones, due to the lack of commutativity in the product of two quaternionic numbers.
In this talk we will focus mainly on the two following examples:
1) Quaternionic Tori , their slice regular automorphisms group and their Moduli Space;
2) Quaternionic Hopf Surfaces , their slice regular automorphisms group and their deformations.
In both cases we will present the state of the art results and also some open problems.

Symplectic duality and implosion

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Oxford University, United Kingdom

We discuss symplectic duality between hyperkahler spaces and present candidates for the symplectic duals of various hyperkahler implosion spaces. (Joint work with A.Hanany, F.Kirwan, A.Bourget, Z.Zhong,J.Grimminger)

Balanced Hermitian metrics

Anna Fino, annamaria.fino@unito.it
Università di Torino, Italy

A Hermitian metric on a complex manifold is balanced if its fundamental form is co-closed. In the talk I will review some general results about balanced metrics and present new smooth solutions to the Hull-Strominger system, showing that the Fu-Yau solution on torus bundles over K3 surfaces can be generalized to torus bundles over K3 orbifolds.

Slice-by-slice and global smoothness of slice regular functions

Riccardo Ghiloni, riccardo.ghiloni@unitn.it
University of Trento, Italy

The concept of slice regular function over the real algebra $\mathbb{H}$ of quaternions is a generalization of the notion of holomorphic function of a complex variable. Let Ω be an open subset of $\mathbb{H}$, which intersects $\mathbb{R}$ and is invariant under rotations of $\mathbb{H}$ around $\mathbb{R}$. A function $f : \Omega \to \mathbb{H}$ is slice regular if it is of class $\mathcal{C}^1$ and, for all complex planes $\mathbb{C}_I$ spanned by 1 and a quaternionic imaginary unit I ($\mathbb{C}_I$ is a 'complex slice' of $\mathbb{H}$), the restriction f_I of f to $\Omega_I = \Omega \cap \mathbb{C}_I$ satisfies the Cauchy-Riemann equations associated to I, i.e., $\overline{\partial}_I f_I = 0$ on Ω_I, where $\overline{\partial}_I = \frac{1}{2}\big(\frac{\partial}{\partial\alpha} + I\frac{\partial}{\partial\beta}\big)$.

We study the continuity and the differential regularity of slice regular functions viewed as solutions of the slice-by-slice differential equations $\overline{\partial}_I f_I = 0$ on Ω_I and as solutions of their global version $\overline{\vartheta} f = 0$ on $\Omega \setminus \mathbb{R}$.

Our results extend to the slice polyanalytic and monogenic cases.

On compact affine quaternionic curves and surfaces

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The introduction of Slice Regular functions in several variables in the recent years has led to a new definition of quaternionic manifolds. A class of manifolds appearing in this context is that of quaternionic affine ones. This talk is devoted to the study of affine quaternionic manifolds and to a possible classification of all compact affine quaternionic curves and surfaces. A direct result, based on the celebrated Kodaira Theorem states that the only compact affine quaternionic curves are the quaternionic tori. As for compact affine quaternionic surfaces, the study of their fundamental groups, together with the inspection of all nilpotent hypercomplex simply connected 8-dimensional Lie Groups, identifies a path towards their classification. This talk is based on a work in collaboration with Graziano Gentili and Giulia Sarfatti.

Geometry of Kato manifolds

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Coauthors: Nicolina Istrati, Massimiliano Pontecorvo, Matteo Ruggiero

Kato manifolds are compact complex manifolds containing a global spherical shell. Their modern study has been widely carried out in complex dimension 2 and originates in the seminal work of Inoue, Kato, Nakamura and Hirzebruch.

In this talk I plan to describe a special class of Kato manifolds in arbitrary complex dimension, whose construction arises from toric geometry. I will present several of their analytic and geometric properties, including existence of special complex submanifolds and partial results on their Dolbeault cohomology. Moreover, since they are compact complex manifolds of non-Kähler type, I will investigate what special Hermitian metrics they support.

Semisimple Lie algebras and special Hermitian metrics

Fabio Podestà, fabio.podesta@unifi.it
Università degli Studi di Firenze, Italy

In this talk I will very shortly review some facts about invariant complex structures on semisimple real non-compact Lie algebras and I will then discuss the existence and properties of special invariant Hermitian metrics (such as balanced metrics) on compact quotients of real simple non compact, even-dimensional groups.

On the continuation of quaternionic logarithm along curves and the winding number

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We present a continuation of the quaternionic logarithm along quaternionic curves. More precisely, given a continuous curve $\gamma : [0,1] \to \mathbb{H} \setminus \{0\}$, we determine the geometric properties of γ, which ensure that there exists a continuous curve $\tilde{\gamma} : [0,1] \to \mathbb{H}$ with $\exp(\tilde{\gamma}(t)) = \gamma(t)$, $t \in [0,1]$. We denote the continuation by $\operatorname{Log}\gamma := \tilde{\gamma}$. When $\operatorname{Log}\gamma$ exists, and γ is closed, we define the winding number of the curve γ.

Stability of Einstein metrics

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Einstein metrics can be characterised as critical points of the (normalised) total scalar curvature functional. They are always saddle points. However, there are Einstein metrics which are local maxima of the functional restricted to metrics of fixed volume and constant scalar curvature.

These are by definition stable Einstein metrics. Stability can equivalently be characterised by a spectral condition for the Lichnerowicz Laplacian on divergence- and trace-free symmetric 2-tensors, i.e. on so-called tt-tensors: an Einstein metric is stable if twice the Einstein constant is a lower bound for this operator. Stability is also related to Perelman's ν entropy and dynamical stability with respect to the Ricci flow.

In my talk I will discuss the stability condition. I will present a recent result obtained with G. Weingart, which completes the work of Koiso on the classification of stable compact symmetric spaces. Moreover, I will describe an interesting relation between instability and the existence of harmonic forms. This is done in the case of nearly Kähler, Einstein-Sasaki and nearly G_2 manifolds. If time permits I will also explain the instability of the Berger space $SO(5)/SO(3)$, which is a homology sphere. In this case instability surprisingly is related to the existence of Killing tensors. The latter results are contained in joint work with M. Wang and C. Wang.

Analogues of the Dolbeault resolution in higher dimensions

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Higher dimensional analogues of complex function theory are based on a suitable choice of an elliptic system of PDEs of the first order. The most developed case is theory of several complex variables. The old one-variable example is the Fueter equation for regular functions of a quaternionic variable, the new example of that sort is the Dirac equation for spinor-valued fields. Higher spin generalizations includes massless fields equations in dimension 4 and its higher dimensional analogues.

Theory of several complex variables includes the Dolbeault resolution as a proper generalization of the de Rham complex. In a similar way, function theories mentioned above can be considered in its several variables versions and the key tool here is a generalization of the Dolbeault complex. There is the general scheme of BGG sequences of invariant differential operators on manifolds with a given parabolic structure. They can be constructed for any regular infinitesimal character. Such sequences form complexes in homogeneous situation but in non-flat case, there is an obstruction given by nontrivial curvatures. Particularly nice examples are complexes of invariant differential operators on quaternionic manifolds introduced and studied by S. Salamon and R. Baston. They form complexes even in curved situation (for quaternionic manifolds) due to the fact that they correspond to singular infinitesimal character.

An understanding of constructions and properties of such complexes advanced a lot in case of Hermitian symmetric spaces mainly due to work of Enright and Shelton. Interest in cases outside this setting was iniciated by development of function theory of several Clifford variables. This corresponds to the case of $|2|$-graded parabolic geometry in singular infinitesimal character. Using methods of integral geometry (in particular of the Penrose transform), it was possible to construct analogues of the Dolbeault resolution in even dimensions and in the stable range.

The aim of the lecture is to introduce first main known results, to describe relations of resolutions with different type of symmetry (Clifford analysis in dimension 4 and several quaternionic variables) and to describe new results in stable as well as non stable range.

Slice regular functions and orthogonal complex structures in eight dimensions

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The theory of slice regular functions, introduced by Gentili and Struppa in 2006, is a successful quaternionic analog of the theory of holomorphic functions of one complex variable. It includes new interesting phenomena, due to the noncommutative setting.

This theory has been applied to the problem of classifying Orthogonal Complex Structures on open dense subsets $\mathbb{R}^4 \setminus \Lambda$ of $\mathbb{R}^4$. Traditionally, this problem had been addressed with a toolset limited to quaternionic linear fractional transformations: only the case when Λ has Hausdorff dimension less than 1 and the case when Λ is a circle or a straight line could be addressed. Then the class of injective quaternionic slice regular functions became available as a tool for classification, which made other cases approachable. The construction of this new toolset required a detailed study of the differential topology of quaternionic slice regular functions. This study was a joint work with Gentili and Salamon published in 2014.

The talk will look at the theory of *octonionic* slice regular functions, introduced by Gentili and Struppa in 2010, through the lens of differential topology. This study has an independent interest, because of the peculiar features of the nonassociative setting of octonions. It leads to a full-fledged version of the Open Mapping Theorem for octonionic slice regular functions. Moreover, it opens the path for a possible use of slice regular functions in the study of almost-complex structures in eight dimensions.

Special geometries with torus symmetry

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A survey of recent results on torus symmetry for metrics with special holonomy, particularly G_2 and Spin(7), and related geometries such as nearly Kähler six-manifolds. The known explicit examples of these geometries all have a large compact symmetry group and in particular an action of some torus. Studying the orbit structure and using ideas such as multi-moment maps, one gets pictures related to the Delzant description for toric manifolds, and/or certain geometric flows.

$\overline{\partial}$-Harmonic forms on compact almost Hermitian manifolds

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Let M be a smooth manifold of dimension $2n$ and let J be an almost-complex structure on M. Then, J induces on the space of forms $A^\bullet(M)$ a natural bigrading, namely

$$A^\bullet(M) = \bigoplus_{p+q=\bullet} A^{p,q}(M)\,.$$

Accordingly, the exterior derivative d splits into four operators

$$d : A^{p,q}(M) \to A^{p+2,q-1}(M) \oplus A^{p+1,q}(M) \oplus A^{p,q+1}(X) \oplus A^{p-1,q+2}(M)$$

$$d = \mu + \partial + \overline{\partial} + \bar{\mu}\,,$$

where μ and $\bar{\mu}$ are differential operators that are linear over functions.

Let g be a Hermitian metric on (M, J). Denote by

$$\Delta_{\overline{\partial}} := \overline{\partial}\,\overline{\partial}^* + \overline{\partial}^*\overline{\partial}$$

the $\overline{\partial}$-Laplacian. Then $\Delta_{\overline{\partial}}$ is an elliptic differential operator. We study the space of $\overline{\partial}$-harmonic forms on (M, J, g). Special results are obtained for $\dim_{\mathbb{R}} M = 4$. This a joint work with Nicoletta Tardini.

On a construction of quaternionic and octonionic Riemann surfaces

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We present an original way to introduce quaternionic and octonionic analogs of the classical Riemann surfaces.

The construction of these manifolds has nice peculiarities and the scrutiny of Bernhard Riemann approach to Riemann surfaces, mainly based on conformality, leads to the definition of slice conformal or slice isothermal parameterization of Riemann 4-manifold and 8-manifold. These new classes of manifolds include slice regular quaternionic and octonionic curves, graphs of slice regular functions, the 4 and 8 dimensional sphere and helicoidal and catenoidal manifolds.

This is a joint work with Graziano Gentili and Jasna Prezelj.

Minisymposia in

ANALYSIS AND ITS APPLICATIONS

- Approximation Theory and Applications (MS-78)
- Convex bodies - approximation and sections (MS-61)
- Current topics in Complex Analysis (MS-32)
- Differential equations, dynamical systems and applications (MS-52)
- Energy methods and their applications in material science (MS-25)
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- Recent Developments on Preservers (MS-38)
- Topological Methods in Differential Equations (MS-13)
- Variational and evolutionary models involving local/nonlocal interactions (MS-58)

Minisymposium

Approximation Theory and Applications (MS-78)

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Gianluca Vinti, *University of Perugia, Italy*
Leokadia Białas-Cież, *Jagiellonian University Cracow, Poland*

- Variation diminishing type estimates for generalized sampling operators and applications, *Laura Angeloni*
- Best Ulam constant of a linear difference equation, *Alina Ramona Baias*
- A generalization of extremal functions and polynomial inequalities, *Mirosław Baran*
- A complex funtion theory for Mellin Analysis and applications to sampling, *Carlo Bardaro*
- A generalization of a local form of the classical Markov inequality, *Tomasz Beberok*
- Metric Fourier approximation of set-valued functions of bounded variation, *Elena Berdysheva*
- Piecewise-regular approximation of maps into real algebraic sets, *Marcin Bilski*
- Discretisation of integrals on compact spaces using distance functions, *Martin Buhmann*
- Integral-type operators on mobile intervals, *Mirella Cappelletti Montano*
- Kernel-based approximation methods on graphs, *Wolfgang Erb*
- Strict positive definiteness of non-radial kernels on d-dimensional spheres, *Janin Jäger*
- Admissible meshes on algebraic sets, *Agnieszka Kowalska*
- An iso-geometric radial basis function partition of unity method for PDEs in thin structures, *Elisabeth Larsson*
- Bernstein-Chlodovsky operators preserving exponentials, *Vita Leonessa*
- Functional differential equations with maxima, via step by step contraction principle, *Diana Otrocol*
- Approximation by Durrmeyer-Sampling Type Operators in Functional Spaces, *Michele Piconi*
- Hölder continuity of the pluricomplex Green function, *Rafał Pierzchała*
- Best Ulam constant of a linear differential operator, *Dorian Popa*
- Modification of exponential type operators preserving exponential functions connected with x^3, *Carmen Violeta Popescu (Muraru)*
- A Hermite-Hadamard type inequality with applications to the estimation of moments of convex functions of random variables, *Pașca Raluca Ioana*
- Information potential for some probability distributions, *Ioan Rasa*
- Bounds for Several Statistical Indicators, *Augusta Ratiu*
- Sampling strategies for approximation in kernel spaces, *Gabriele Santin*
- Inequalities for Legendre polynomials and applications in information potential, *Daniel Florin Sofonea*
- Gauss-Lucas theorem in polynomial dynamics, *Margaret Stawiska Friedland*
- On derivative sampling using Kantorovich-type sampling operators, *Gert Tamberg*
- A general method to study the convergence of nonlinear operators in Orlicz spaces, *Luca Zampogni*

Variation diminishing type estimates for generalized sampling operators and applications

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The variation diminishing estimate is a classical result that is usually investigated working in BV spaces with some classes of operators: such result essentially ensures that the variation of the operator is not bigger than the variation of the function to which it is applied. We will present estimates of this kind, besides results about convergence in variation, for multivariate sampling-type operators. Differently from the one-dimensional frame, where variation diminishing type results are usually quite easy to be achieved, the multidimensional case is more delicate: nevertheless it is interesting, also from an applicative point of view, since it is connected to some problems of Digital Image Processing, in particular to smoothing procedures.

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Best Ulam constant of a linear difference equation

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An equation is called Ulam stable if for every approximate solution of it there exists an exact solution near it. We present some results on Ulam stability for some linear difference equations.

In a Banach space X the linear difference equation with constant coefficients $x_{n+p} = a_1x_{n+p-1} + \ldots + a_px_n$, is Ulam stable if and only if the roots r_k, $1 \leq k \leq p$, of its characteristic equation do not belong to the unit circle. If $|r_k| > 1$, $1 \leq k \leq p$, we prove that the best Ulam constant of this equation is $\frac{1}{|V|}\sum_{s=1}^{\infty}\left|\frac{V_1}{r_1^s}-\frac{V_2}{r_2^s}+\ldots+\frac{(-1)^{p+1}V_p}{r_p^s}\right|$, where $V = V(r_1, r_2, \ldots, r_p)$ and $V_k = V(r_1, \ldots, r_{k-1}, r_{k+1}, \ldots, r_p)$, $1 \le k \le p$, are Vandermonde determinants.

A generalization of extremal functions and polynomial inequalities

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Consider a normed space $(\mathcal{P}(\mathbb{C}^N), \mathcal{N})$ of polynomials of N variables equipped with a fixed norm $\mathcal{N}$, which can be arbitrary. We can define a *radial* version of a polynomial extremal function, which has a sense in a general situation. In the case of the supremum norm, $\mathcal{N}(P) = \sup\{|P(z)| : z \in E\}$ our extremal functions are a radial modification of the classical Siciak's extremal function $\Phi(E, z)$. In this special case we can also consider a local radialization of the Siciak's extremal function and its logarithm $V(E, z)$ (the pluricomplex Green function). We shall show connections between the behaviour of such extremal functions and polynomial inequalities of Markov's and Bernstein's type. In particular, there will obtained new results on Bernstein' inequality involving higher derivatives of polynomials at interior points of compact subsets of $\mathbb{R}^N$.

A complex funtion theory for Mellin Analysis and applications to sampling

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The aim of this research is to extend in a simple way the well-known Paley–Wiener theorem of Fourier Analysis, which characterizes the so-called bandlimited functions, to the setting of Mellin transform. In order to do that, we introduced in these last years a notion of polar analytic function [1], which provides a simple way of describing functions that are analytic on a part of the Riemann surface of the logarithm. Applications to various topics of Mellin Analysis are developed, in particular sampling type theorems and quadrature formulae (for a survey see [2]).

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A generalization of a local form of the classical Markov inequality

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In this talk we introduce a generalization to compact subsets of certain algebraic varieties of the classical Markov inequality on the derivatives of a polynomial in terms of its own values. We also introduce an extension to such sets of a local form of the classical Markov inequality, and show the equivalence of introduced Markov and local Markov inequalities.

Metric Fourier approximation of set-valued functions of bounded variation

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We study set-valued functions (SVFs) mapping a real interval to compact sets in $\mathbb{R}^d$. Older approaches to the approximation investigated almost exclusively SVFs with convex images (values), the standard methods suffer from convexification. In this talk I will describe a new construction that adopts the trigonometric Fourier series to set-valued functions with general (not necessarily convex) compact images. Our main result is analogous to the classical Dirichlet-Jordan Theorem for real functions. It states the pointwise convergence in the Hausdorff metric of the metric Fourier partial sums of a multifunction of bounded variation to a set determined by the values of the metric selections of the function. In particular, if the multifunction F is of bounded variation and continuous at a point x, then the metric Fourier partial sums of it at x converge to $F(x)$.

Piecewise-regular approximation of maps into real algebraic sets

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A real algebraic set W of dimension m is said to be uniformly rational if each of its points has a Zariski open neighborhood which is biregularly isomorphic to a Zariski open subset of $\mathbb{R}^m$. Let l be any nonnegative integer. It turns out that every map of class $\mathcal{C}^l$ from a compact subset of a real algebraic set into a uniformly rational real algebraic set can be approximated in the $\mathcal{C}^l$ topology by piecewise-regular maps of class $\mathcal{C}^k$, where k is an arbitrary integer greater than or equal to l.

Discretisation of integrals on compact spaces using distance functions

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For the purpose of partitioning compact sets, discretisation of integrals and finding quadrature rules on compact sets, it is important to have estimates for the ensuing error of the approximation. It is desirable to have estimates on the remainders that are independent of space dimension, and of course we wish the errors to decrease as fast as possible when the number of summands in the discretisation increases. In this joint work with Feng Dai and Yeli Niu (Edmonton) we find such error estimates using regular partitions with particular attention to (but not only to) discretisations on spheres. In fact, our estimates are quite general, they apply to compact path-connected metric spaces, and we are able to improve several earlier error estimates from the literature.

Integral-type operators on mobile intervals

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In this talk, we present a sequence $(C_n)_{n\geq 1}$ of positive linear operators, introduced in [1] and acting on spaces of continuous functions as well as on spaces of integrable functions on $[0,1]$. These operators represent a Kantorovich-type modification, on mobile intervals, of the ones discussed in [2].

We state some qualitative properties of the sequence $(C_n)_{n\geq 1}$ and we prove that it is an approximation process both in $C([0,1])$ and in $L^p([0,1])$, also providing some estimates of the rate of convergence. Moreover, we determine an asymptotic formula and we prove that suitable iterates of the operators C_n converge, both in $C([0,1])$ and, under suitable assumptions, in $L^p([0,1])$ to a limit semigroup. Finally, we compare our operators with other existing ones in the literature showing that they allow a lower approximating error estimate.

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Kernel-based approximation methods on graphs

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We study how the concept of positive definite functions can be transfered to a graph setting in order to approximate graph signals with generalized shifts of a graph basis function (GBF). This concept merges kernel-based approximation with spectral theory on graphs and can be regarded as a graph analog of radial basis function methods in euclidean spaces or on the sphere. We provide several descriptions of positive definite functions on graphs, the most relevant one is a Bochner- type characterization in terms of positive Fourier coefficients. These descriptions allow us to design GBF's and to study GBF approximation in more detail: we are able to characterize the native spaces of the interpolants, we give explicit estimates for the approximation error and provide ways on how to calculate the approximants in an efficient manner. As a final application, we show how GBFs can be used for classification tasks on graphs.

Strict positive definiteness of non-radial kernels on d-dimensional spheres

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Isotropic positive definite functions are used in approximation theory and are for example applied in geostatistics and physiology. They are also of importance in statistics where they occur as correlation functions of homogeneous random fields on spheres. We study a class of function applicable for interpolation of arbitrary scattered data on $\mathbb{S}^{d-1}$ by linear combinations of a kernel $K : \mathbb{S}^{d-1} \times \mathbb{S}^{d-1} \to \mathbb{C}$ evaluated at the interpolation points in the second argument. The isotropic kernels are a special case of this approach and we study kernels with more general properties like axial symmetry and invariance under parity.

A class of kernels for which the resulting interpolation problem is uniquely solvable for any distinct point set $\Xi \subset \mathbb{S}^d$ are known strict positive definite isotropic functions. Using recent results of Bonfim and Menegatto [2] and the famous representations of isotropic positive definite functions on $\mathbb{S}^{d-1}$ due to Schoenberg as starting point we derive new sufficient conditions for strict positive definiteness of axial symmetric and convolutional kernels. The results extend a necessary and sufficient characterisation of strict positive definite isotropic basis functions by Chen et al. proven in [1] to a non-radial kernel class.

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Admissible meshes on algebraic sets

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Admissible meshes were formally introduced by J. P. Calvi and N. Levenberg in 2008. Such meshes are nearly optimal for uniform least squares approximation and contain interpolation sets nearly as good as Fekete points of the domain. Optimal admissible meshes have been constructed on many polynomially determining compact sets, e.g., sections of discs, ball, convex bodies, sets with regular boundary, by different analytical and geometrical techniques. Regarding subsets of algebraic varieties admissible meshes are known only for a few compacts like sections of a sphere, a torus, a circle and curves in $\mathbb{C}$ with analytic parametrization. We construct polynomial weakly admissible meshes on compact subsets of algebraic hypersurfaces in $\mathbb{C}^{N+1}$. These meshes are optimal in some cases. We present also partial results for algebraic sets of codimension greater than one.

An iso-geometric radial basis function partition of unity method for PDEs in thin structures

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The application that motivates this work is numerical simulation of the biomechanics of the respiratory system. The main respiratory muscle is the diaphragm, which is a thin structure. There are several challenges associated with the geometry, including its representation. Here, we first use a radial basis function partition of unity method (RBF-PUM) to make a smooth reconstruction of the geometry from noisy medical image data. Then we use RBF-PUM to approximate the solution of a PDE problem posed in this geometry. In a PUM, the global approximation is expressed as a weighted combination of local approximations over patches that form a cover of the domain. A particular benefit of RBF-PUM is that we can adapt each local approximation to the local properties of the problem. For this thin, curved, non-trivial geometry, we can scale the local problems to ensure sufficient local resolution of the thickness dimension. We show results for a simple Poisson test problem and show that we can achieve high-order convergence with an appropriate choice of method parameters.

Bernstein-Chlodovsky operators preserving exponentials

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The aim of this talk is to illustrate a generalization of Bernstein-Chlodovsky operators, introduced and studied in [6, 4], that preserves the exponential function e^{-2x} ($x \geq 0$).

In particular, in [6] we studied its approximation properties in several function spaces, also evaluating the rate of convergence by means of suitable moduli of continuity. As a consequence, we proved better error estimation than the original operators on certain intervals.

In [4] we continued the study of such operators by proving some Voronovskaya type theorems and deducing saturation results. A comparison of this new class of operators with the classical Bernstein-Chlodovsky ones is also made, proving that the new operators provide better approximation results for certain functions on $[0, +\infty)$.

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Functional differential equations with maxima, via step by step contraction principle

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T.A. Burton presented in some examples of integral equations a notion of progressive contractions on $C([a, \infty[)$. In 2019, I.A. Rus formalized this notion (I.A. Rus, *Some variants of contraction principle in the case of operators with Volterra property: step by step contraction principle*, Advances in the Theory of Nonlinear Analysis and its Applications 3 (2019) No. 3, 111-120), put "step by step" instead of "progressive" in this notion, and give some variant of step by step contraction principle in the case of operators with Volterra property on $C([a, b], \mathbb{B})$ and $C([a, \infty[, \mathbb{B})$, where $\mathbb{B}$ is a Banach space. In this paper we use the abstract result given by I.A. Rus, to study some classes of functional differential equations with maxima.

Approximation by Durrmeyer-Sampling Type Operators in Functional Spaces

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Sampling-type operators have been introduced in order to give an approximate version of the celebrated classical sampling theorem. Among these operators, we have studied the Durrmeyer-Sampling type operators (DSO) [4], (see also [6, 2]), which represent a further generalization of the well-known Generalized [3] and Kantorovich-Sampling operators [1, 5].
In order to follow a unifying approach, we have provided a general result in terms of convergence, which is represented by a modular convergence theorem in Orlicz spaces. From the latter result, the convergence in L^p-spaces follows as particular case. This approximation result for DSO is important, in some particular case, from the applications point of view, e.g. in image processing, where we have to work with not-necessarily continuous signals.
For the sake of completeness of the theory, we have also studied the continuous case, providing a pointwise and uniform convergence theorem and quantitative estimates.
Moreover, all the above convergence results for DSO can also be extended in the multidimensional setting.

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Hölder continuity of the pluricomplex Green function

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I will discuss the following problem of Pleśniak (1988). Let $h : U \to \mathbb{C}^{N'}$, where $U \subset \mathbb{C}^N$ is an open set, be a holomorphic map $(N, N' \in \mathbb{N})$. Assume that a compact set $\emptyset \neq K \subset \mathbb{C}^N$ has the HCP property (that is, the pluricomplex Green function V_K of K is Hölder continuous) and $\hat{K} \subset U$. Under what conditions does it happen that $h(K)$ has the HCP property?

Best Ulam constant of a linear differential operator

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The Ulam stability of an operator L acting in Banach spaces is equivalent with the stability of the associated equation $Lx = y$. An equation is called Ulam stable if for every approximate solution of it there exists an exact solution near it. We present some results on Ulam stability for some linear differential operators.

The linear differential operator with constant coefficients

$$D(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y, \quad y \in \mathcal{C}^n(\mathbb{R}, X)$$

acting in a Banach space X is Ulam stable if and only if its characteristic equation has no roots on the imaginary axis. We prove that if the characteristic equation of D has distinct roots r_k satisfying $\Re r_k > 0$, $1 \le k \le n$, then the best Ulam constant of D is $K_D = \frac{1}{|V|} \int_0^\infty \left| \sum_{k=1}^n (-1)^k V_k e^{-r_k x} \right| dx$, where $V = V(r_1, r_2, \ldots, r_n)$ and $V_k = V(r_1, \ldots, r_{k-1}, r_{k+1}, \ldots, r_n)$, $1 \le k \le n$, are Vandermonde determinants.

Modification of exponential type operators preserving exponential functions connected with x^3

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Coauthors: Ali Aral, Vijay Gupta

In the present paper, we propose modification of the exponential type operators, which are connected with x^3. Such operators are connected with exponential functions. We estimate moments and establish some direct results in terms of modulus of continuity.

A Hermite-Hadamard type inequality with applications to the estimation of moments of convex functions of random variables

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Coauthor: Gavrea Bogdan

In this paper we present a Hermite-Hadamard type inequality with applications to the estimation of moments for convex functions of several random variables. A particular case of the derived result can be applied to estimate the sum of moments for identically distributed random variables. We were motivated in our research by the applications of moment estimation in fields such as probability theory, mathematical statistics and statistical learning.

Information potential for some probability distributions

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Coauthor: Ana Maria Acu

This talk is devoted to some properties of the entropies of some discrete or continuous probability distributions. We will focus on the information potential (IP) associated with such distributions depending on a real parameter x. The Rényi entropy and the Tsallis entropy are naturally related to IP. Convexity properties and bounds of the associated IP, useful in Information Theoretic Learning, are discussed and translated as properties of the entropies. Several examples and numerical computations illustrate the general results as well as their relationship with positive linear operators.

Bounds for Several Statistical Indicators

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In this paper, we study the problem of finding new bounds for some statistical indicators that characterize a data series. Using a refinement of Aczél's inequality, we find other bounds for dispersion, standard deviation and coefficient of variation.

Sampling strategies for approximation in kernel spaces

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Kernel methods provide powerful and flexible techniques to approximate functions defined on general domains, with possible high-dimensional input and output dimension, and using samples at scattered locations.

In this context, the problem of choosing the location of the sampling points is of great interest, both from a practical and a theoretical viewpoint. On one hand, it is of theoretical interest to know the limit and benefits of the choice of optimal point location, and to design feasible algorithms to select them. On the other hand, several applications are described by large datasets, and it may be interesting to select a possibly small portion of the data that allows an accurate reconstruction of the full problem.

In this talk we will discuss some greedy methods and show that they are effective techniques in both scenarios.

In particular, we will first introduce some results on the general structure and theory of kernel-based greedy methods, and describe their efficient implementation. We will then show that, in certain circumstances, they may be proven to be worst-case optimal.

We will focus mainly on interpolation, and mention some application to quadrature. Moreover, we will discuss the use of these techniques on some real world applications.

Inequalities for Legendre polynomials and applications in information potential

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"Lucian Blaga" of Sibiu, Romania

Coauthor: Ioan Tincu

The classical orthogonal polynomials play an important role in applications of mathematical analysis, spectral method with applications in fluid dynamics and other areas of interest. This work is devoted to the orthogonal polynomials. Bounds of Legendre polynomials are obtained in terms of inequalities. Also, entropies associated with discrete probability distributions is a topic considered. Bounds of the entropies which improve some previously known results are obtained.

Gauss-Lucas theorem in polynomial dynamics

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Using versions of the Gauss-Lucas theorem adapted to dynamics, we prove that for every complex polynomial p of degree $d \geq 2$ the convex hull H_p of the Julia set J_p of p satisfies $p^{-1}(H_p) \subset H_p$. This settles positively a conjecture by P. Alexandersson. We also characterize the families of polynomials for which the equality $p^{-1}(H_p) = H_p$ is achieved.

On derivative sampling using Kantorovich-type sampling operators

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Coauthor: Olga Graf

For $f \in C(\mathbb{R})$ the generalized sampling operators are given by ($t \in \mathbb{R}$; $W > 0$)

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W}) s(Wt - k), \tag{1}$$

where s is a certain kernel function, i.e.

$$s \in L^1(\mathbb{R}), \quad \sum_{k \in \mathbb{Z}} s(u-k) = 1, \quad (u \in \mathbb{R}).$$

We show a connection between generalized sampling operators with averaged kernels and generalized Kantorovich-type sampling operators. Using this connection, we can estimate the order of approximation of derivatives.

A general method to study the convergence of nonlinear operators in Orlicz spaces

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Coauthor: Gianluca Vinti

We introduce a general setting in which we define nets of nonlinear operators whose domains are some set of functions defined in a locally compact topological group G. These nets assume the form

$$T_w f := z \mapsto \int_H \chi_w(z - h_w(t), L_{h_w(t)}(f)) d\mu_H(t), \; x > 0,$$

where H is a topological group with (left-invariant) Haar measure μ_H, $(\chi_w)_w$ is a net of Kernels functions defined on $G \times \mathbb{R}$, h_w are homeomorphism from H to G and $L_{h_w(t)} : L(G) \to \mathbb{R}$ are linear operators.

We analyze the behavior of such nets, and detect the fairest assumption which are needed for the nets to converge in Orlicz spaces. As a consequence, we give a result of convergence in a subspace of a Orlicz space.

Minisymposium

CONVEX BODIES - APPROXIMATION AND SECTIONS (MS-61)

Organized by Márton Naszódi, *Eotvos University, Budapest And Alfred Renyi Inst. of Mathematics, Budapest, Hungary*
Coorganized by Konrad Swanepoel, *London School of Economics and Political Science*

- Extremal sections and local optimization, *Gergely Ambrus*
- Cells in the box and a hyperplane, *Imre Bárány*
- A new look at the Blaschke-Leichtweiss theorem, *Károly Bezdek*
- The L_p Minkowski problem and polytopal approximation, *Karoly Boroczky*
- Strengthened inequalities for the mean width, *Ferenc Fodor*
- Colorful Helly-type Theorems for Ellipsoids, *Viktória Földvári*
- Coverings by homothets of a convex body, *Nora Frankl*
- The Golden ratio and high dimensional mean inequalities, *Bernardo González Merino*
- Functional John and Löwner Ellipsoids, *Grigory Ivanov*
- A solution to some problems of Conway and Guy on monostable polyhedra, *Zsolt Lángi*
- On the complex hypothesis of Banach, *Luis Montejano*
- Pea Bodies of Constant Width, *Deborah Oliveros*
- A cap covering theorem, *Alexandr Polyanskii*
- Rigidity of compact Fuchsian manifolds with convex boundary, *Roman Prosanov*
- On bodies floating in equalibrium in every direction, *Dmitry Ryabogin*
- Covering techniques in Integer & Lattice Programming, *Moritz Venzin*

Extremal sections and local optimization

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Alfréd Rényi Institute of Mathematics, Hungary

We will demonstrate how to apply local optimization methods in order to find, or, at least characterize, maximal or minimal central sections of convex bodies. We are mainly interested in the special cases when the convex body is the d-dimensional cube, or the d-dimensional regular simplex. Maximal sections of the former were determined by K. Ball in 1986, while monotinicity of the volume of diagonal central sections for $d \geq 3$ was proven by F. Bartha, F. Fodor and B. González Merino in 2021. On the other hand, finding minimal central sections of the regular simplex is still an open question. Among other results, we provide a geometric characterization for central sections of locally maximal volume of the d-dimensional cube.

Cells in the box and a hyperplane

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It is well known that a line can intersect at most $2n - 1$ cells of the $n \times n$ chessboard. We consider the high dimensional version: how many cells of the d-dimensional $n \times \ldots \times n$ box can a hyperplane intersect? We also prove the lattice analogue of the following well-known fact. If K, L are convex bodies in $\mathbb{R}^d$ and $K \subset L$, then the surface area of K is smaller than that of L. Joint work with Peter Frankl.

A new look at the Blaschke-Leichtweiss theorem

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The Blaschke-Leichtweiss theorem (Abh. Math. Sem. Univ. Hamburg 75: 257–284, 2005) states that the smallest area convex domain of constant width w in the 2-dimensional spherical space $\mathbb{S}^2$ is the spherical Reuleaux triangle for all $0 < w \leq \frac{\pi}{2}$. In this paper we extend this result to the family of wide r-disk domains of $\mathbb{S}^2$, where $0 < r \leq \frac{\pi}{2}$. Here a wide r-disk domain is an intersection of spherical disks of radius r with centers contained in their intersection. This gives a new and short proof for the Blaschke-Leichtweiss theorem. Furthermore, we investigate the higher dimensional analogue of wide r-disk domains called wide r-ball bodies. In particular, we determine their minimum spherical width (resp., inradius) in the spherical d-space $\mathbb{S}^d$ for all $d \geq 2$. Also, it is shown that any minimum volume wide r-ball body is of constant width r in $\mathbb{S}^d$, $d \geq 2$.

The L_p Minkowski problem and polytopal approximation

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Alfred Renyi Institute of Mathematics, Hungary

The L_p Minkowski problem , a Monge-Ampere type equation on the sphere, is a recent version of the classical Minkowski problem. I will review cases when one can reduce the equation to properties of polytopes, and then use polytopal approximation to solve the PDE in general.

Strengthened inequalities for the mean width

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According to a result of Barthe the regular simplex maximizes the mean width of convex bodies whose John ellipsoid is the Euclidean unit ball. The reverse statement that the regular simplex minimizes the mean width of convex bodies whose Löwner ellipsoid is the Euclidean unit ball is also true as proved by Schmuckenschläger. In this talk we prove strengthened stability versions of these theorems and also some related stability statements for the convex hull of the support of centered isotropic measures on the sphere. This is joint work with Károly J. Böröczky (Budapest, Hungary) and Daniel Hug (Karlsruhe, Germany).

Colorful Helly-type Theorems for Ellipsoids

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Coauthors: Gábor Damásdi, Márton Naszódi

Helly-type theorems have been widely studied and applied in discrete geometry. In this talk, I am going to give a brief overview of quantitative Helly-type theorems and introduce our joint result with Gábor Damásdi and Márton Naszódi, a colorful Helly-type theorem for ellipsoids.

Coverings by homothets of a convex body

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Coauthors: Janos Nagy, Marton Naszodi

Rogers proved that for any convex body K, we can cover R^d by translates of K of density roughly $d \log d$. We discuss several related results. First, we extend Roger's result by showing that, if we are given a family of positive homothets of K of infinite total volume, then we can find appropriate translation vectors for each given homothet to cover R^d with the same density. Second, we consider an extension to multiple coverings of space by translates of a convex body. Finally, we also prove a lower bound on the total volume of a family $\mathcal{F}$ of homothets of K that guarantees the existence of a covering of K by members of $\mathcal{F}$.

The Golden ratio and high dimensional mean inequalities

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Coauthors: René Brandenberg, Katherina von Dichter

The classical inequalities between means state that

$$\min\{a,b\} \leq \left(\frac{a^{-1}+b^{-1}}{2}\right)^{-1} \leq \sqrt{ab} \leq \frac{a+b}{2} \leq \max\{a,b\}, \tag{1}$$

for any $a, b > 0$, with equality if and only if $a = b$.

One can naturally extend (1) considering means of n-dimensional compact, convex sets. Notice that means of K and $-K$ are commonly known as symmetrizations of K. In this context, we will show that in even dimensions, if K has a large Minkowski asymmetry, then the corresponding inequalities between the symmetrizations of K can no longer be optimal. Especially in the planar case, we compute that the range of asymmetries of K for which the inequalities between the symmetrizations of K can be optimal is $[1, \phi]$, where ϕ is the Golden ratio. Indeed, we introduce the Golden House, (up to linear transformations) the only Minkowski centered set with asymmetry ϕ such that the symmetrizations of K are successively optimally contained in each other.

Functional John and Löwner Ellipsoids

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In this talk, we will speak about functional analogs of the John and Löwner ellipsoids for log-concave functions. We will discuss the existence and uniqueness results for these objects, their general properties such as volume ratio, containment, John's condition, etc. We will note the difference between the behavior of convex sets and log-concave functions concerning our problems, and highlight a number of open problems.

A solution to some problems of Conway and Guy on monostable polyhedra

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Budapest University of Technology and Economics, Hungary

A convex polyhedron is called monostable if it can rest in stable position only on one of its faces. In this talk we investigate three questions of Conway, regarding monostable polyhedra, from the open problem book of Croft, Falconer and Guy (Unsolved Problems in Geometry, Springer, New York, 1991), which first appeared in the literature in a 1969 paper. In this talk we answer two of these problems. The main tool of our proof is a general theorem describing approximations of smooth convex bodies by convex polyhedra in terms of their static equilibrium points.

On the complex hypothesis of Banach

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The following is known as the geometric hypothesis of Banach: let V be an m-dimensional Banach space (over the real or the complex numbers) with unit ball B and suppose all n-dimensional subspaces of V are isometric (all the n-sections of B are affinely equivalent). In 1932, Banach conjectured that under this hypothesis V is a Hilbert space (the boundary of B is an ellipsoid). Gromow proved in 1967 that the conjecture is true for n=even and Dvoretzky and V. Milman derived the same conclusion under the hypothesis n=infinity. We prove this conjecture for $n = 4k + 1$, with the possible exception of V a real Banach space and $n = 133$. [G.Bor, L.Hernandez-Lamoneda, V. Jiménez and L. Montejano. To appear Geometry $\&$ Topology] for the real case and [J. Bracho, L. Montejano, submitted to J. of Convex Analysis] for the complex case.

The ingredients of the proof are classical homotopic theory, irreducible representations of the orthogonal group and convex geometry. For the complex case, suppose B is a convex body contained in complex space C^{n+1}, with the property that all its complex n-sections through the origin are complex affinity equivalent to a fixed complex n-dimensional body K. Studying the topology of the complex fibre bundle $SU(n)- > SU(n-1)- > S^{2n+1}$, it is possible to prove that if n=even, then K must be a ball and using homotopical properties of the irreducible representations we prove that if $n = 4 + 1$ then K must be a body of revolution. Finally, we prove, using convex geometry and topology that, if this is the case, then there must be a section of B which is an complex ellipsoid and consequently B must be also a complex ellipsoid.

Pea Bodies of Constant Width

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Besides the two Meissner solids, the obvious constant width bodies of revolution, and the Meissner polyhedra, there are few concrete examples of bodies of constant width or a concrete finite procedure to construct them. In this talk, we will describe an infinite family of 3-dimensional bodies of constant width obtained from the Reuleaux Tetrahedron by replacing a small neighborhood of all six edges with sections of an envelope of spheres using the classical notion of confocal quadrics. This family includes Meissner solids as well as one with tetrahedral symmetry. (Joint work with I. Arelio and L. Montejano).

A cap covering theorem

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A *cap* of spherical radius α on a unit d-sphere S is the set of points within spherical distance α from a given point on the sphere. Let $\mathcal{F}$ be a finite set of caps lying on S. We prove that if no hyperplane through the center of S divides $\mathcal{F}$ into two non-empty subsets without intersecting any cap in $\mathcal{F}$, then there is a cap of radius equal to the sum of radii of all caps in $\mathcal{F}$ covering all caps of $\mathcal{F}$ provided that the sum of radii is less $\pi/2$.

This is the spherical analog of the so-called circle covering theorem by Goodman and Goodman and the strengthening of Fejes Tóth's zone conjecture proved by Jiang and the author.

Rigidity of compact Fuchsian manifolds with convex boundary

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It is known that convex bodies in the Euclidean 3-space are globally rigid, i.e., their shape is determined by the intrinsic geometry of the boundary. This story was developed separately in smooth and in polyhedral settings until in 50s it was unified by Pogorelov who proved the rigidity of general convex bodies without any assumptions on their boundaries except convexity. Later another approach was proposed by Volkov with the help of polyhedral approximation.

On the other hand, in 70s Thurston revolutionized the field of 3-dimensional topology by formulating his geometrization program culminated in the famous works of Perelman. In particular Thurston highlighted the abundance and the ubiquity of hyperbolic 3-manifolds. In the scope of this framework some amount of attention was directed towards hyperbolic 3-manifolds with convex boundary. It is conjectured that their shape is determined by the topology and the intrinsic geometry of the boundary. So far this was established only for smooth strictly convex boundaries by Schlenker.

In my recent work I obtained the general rigidity for a toy family of hyperbolic 3-manifolds with convex boundary. This was achieved by reviving the approach of Volkov.

On bodies floating in equalibrium in every direction

Dmitry Ryabogin, d.ryabogin@yahoo.com
Kent State University, United States

We give a negative answer to Ulam's Problem 19 from the Scottish Book asking *is a solid of uniform density which will float in water in every position a sphere?* Assuming that the density of water is 1, we show that there exists a strictly convex body of revolution $K \subset \mathbb{R}^3$ of uniform density $\frac{1}{2}$, which is not a Euclidean ball, yet floats in equilibrium in every direction. We prove an analogous result in all dimensions $d \geq 3$.

Covering techniques in Integer & Lattice Programming

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I will present two ideas based on high-dimensional coverings that yield some of the currently best approximation algorithms for Integer & Lattice Programming. These problems are as follows: Given some convex body $K \subseteq \mathbb{R}^n$, is there an integer point in K? This problem is central in the algorithmic geometry of numbers and has found applications in Integer Programming and plays a key role in proposed schemes for post-quantum Cryptography. The following two natural geometric considerations have led to improvements in the current state-of-the-art algorithms for Lattice Programming:

- Given some convex body $K \subseteq \mathbb{R}^n$, how many convex bodies Q_i are required to cover K so that, when scaled around their respective centroids by a factor of 2, these convex bodies are contained inside $(1+\epsilon) \cdot K$? This question was first considered by Eisenbrand et al. in the context of Integer Programming for $K = B_\infty^n$. In that case, a simple dyadic decomposition along the facets reveals that $O(1 + \log(1/\epsilon))^n$ convex bodies suffice. For general ℓ_p norm balls, exploiting the modulus of smoothness, $O(1+1/\epsilon)^{n/\min(p,2)}$ convex bodies suffice. This is tight for the Euclidean ball, but it is wide open whether for general K (even for the cross-polytope) there is an improvement over $O(1 + 1/\epsilon)^n$.

- Let K be some convex body and let $\mathcal{E}$ be some ellipsoid enclosing K. We consider the translative covering number $N(\mathcal{E}, K)$ but with a twist: For $\epsilon > 0$, is there some constant $C_\epsilon > 0$ so that $N(\mathcal{E}, C_\epsilon \cdot K) < 2^{\epsilon n}$? When $K = B_\infty^n$ and the ellipsoid is $\sqrt{n}B_2^n$, the answer is yes, i.e. for any $\epsilon > 0$, we can scale the cube by some constant and cover $\sqrt{n}B_2^n$ by fewer than $2^{\epsilon n}$ translates. Similarly, when K is the ℓ_p norm ball for $p \geq 2$ and the ellipsoid is an adequately scaled ball, the answer is yes, while for $p < 2$, the answer is no (compare the volume).

For both problems I will describe the techniques to obtain these results, briefly sketch their relevance to Lattice Programming and present some open questions. The talk is based on joint works with Márton Naszódi and Fritz Eisenbrand respectively.

Minisymposium

CURRENT TOPICS IN COMPLEX ANALYSIS (MS-32)

Organized by Stanislawa Kanas, *University of Rzeszow, Poland*

- Recent studies on the image domain of starlike functions, *Sahsene Altinkaya*
- Extremal decomposition of the complex plane, *Iryna Denega*
- Convex sums of biholomorphic mappings and Extension operators in $\mathbb{C}^n$, *Eduard Stefan Grigoriciuc*
- Geometry of planar domains and their applications in study of conformal and harmonic mappings, *Stanisława Kanas*
- Asymptotic estimates for one class of homeomorphisms, *Bogdan Klishchuk*
- Generalizations of Hardy type inequalities via new Green functions, *Kristina Krulić Himmelreich*
- Almost periodic functions revisited, *Juan Matías Sepulcre*
- Harmonic quasiconformal mappings and hyperbolic type metrics, *Vesna Todorčević*
- Some properties of functions from families of even holomorphic functions of several complex variables, *Edyta Trybucka*
- Sharp estimates of the Hankel determinant and of the coefficients for some classes of univalent functions, *Nikola Tuneski*

Recent studies on the image domain of starlike functions

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The geometric properties of image domain of starlike functions is of prominent significance to present a comprehensive study on starlike functions. For this purpose, by using a relation of subordination, we investigate a family of starlike functions in the open unit disc

$$\mathbf{U} = \{z \in \mathbb{C} : |z| < 1\}.$$

Subsequently, we discuss some interesting geometric properties, radius problems, general coefficients for this class. Further, we point out several recent studies related to image domain of starlike functions.

Extremal decomposition of the complex plane

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The talk is devoted to a few well-known an extremal problems in geometric function theory of a complex variable associated with estimates of the functionals defined on the systems of mutually non-overlapping domains [1–9]. An improved method is proposed for solving problems on extremal decomposition of the complex plane. And effective upper estimates for maximum of the products of inner radii of mutually non-overlapping domains for any fixed systems of points of the complex plane at all possible values of some parameter γ are obtained [6–9]. Also we established conditions under which the structure of points and domains is not important. In particular, we obtained full solution of an open problem about extremal decomposition of the complex plane with two free poles located on the unit circle [8].

Problem. [1] Consider the product

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k),$$

where B_0,..., B_n, $n \geqslant 2$, are pairwise non-overlapping domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$ and $\gamma \in (0, n]$ ($r(B, a)$ be an inner radius of the domain $B \subset \overline{\mathbb{C}}$ relative to a point $a \in B$). For all values of the parameter $\gamma \in (0, n]$ to show that it attains its maximum at a configuration of domains B_k and points a_k possessing rotational n-symmetry.

The proof is due to Dubinin for $\gamma = 1$ [1] and to Kuz'mina [2] for $0 < \gamma < 1$. Subsequently, Kovalev [3] solved this problem under the additional assumption that the angles between neighbouring line segments $[0, a_k]$ do not exceed $2\pi/\sqrt{\gamma}$.

Let

$$I_n^0(\gamma) = \left(\frac{4}{n}\right)^n \frac{\left(\frac{4\gamma}{n^2}\right)^{\frac{\gamma}{n}}}{\left(1 - \frac{\gamma}{n^2}\right)^{n+\frac{\gamma}{n}}} \left(\frac{1 - \frac{\sqrt{\gamma}}{n}}{1 + \frac{\sqrt{\gamma}}{n}}\right)^{2\sqrt{\gamma}}.$$

Theorem 1. [8] Let $\gamma \in (1, 2]$. Then, for any different points a_1 and a_2 of the unit circle and any mutually non-overlapping domains B_0, B_1, B_2, $a_1 \in B_1 \subset \overline{\mathbb{C}}$, $a_2 \in B_2 \subset \overline{\mathbb{C}}$, $a_0 = 0 \in$

$B_0 \subset \overline{\mathbb{C}}$, the inequality

$$r^{\gamma}\left(B_0, 0\right) r\left(B_1, a_1\right) r\left(B_2, a_2\right) \leqslant I_2^0(\gamma)\left(\frac{1}{2}\left|a_1-a_2\right|\right)^{2-\gamma}$$

is true. The sign of equality in this inequality is attained, when the points a_0, a_1, a_2 and the domains B_0, B_1, B_2 are, respectively, the poles and circular domains of the quadratic differential

$$Q(w) d w^2=-\frac{(4-\gamma) w^2+\gamma}{w^2\left(w^2-1\right)^2} d w^2 .$$

Theorem 2. [8] Let $n \in \mathbb{N}$, $n \geqslant 3$, $\gamma \in (1, n]$. Then, for any system of different points $\{a_k\}_{k=1}^n$ of the unit circle and for any collection of mutually non-overlapping domains B_0, B_k, $a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1, n}$, the following inequality holds

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k) \leqslant \left(\sin \frac{\pi}{n}\right)^{n-\gamma}\left(I_2^0\left(\frac{2\gamma}{n}\right)\right)^{\frac{n}{2}} .$$

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Convex sums of biholomorphic mappings and Extension operators in $\mathbb{C}^n$

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Let $\mathbb{B}^n$ be the Euclidean unit ball in $\mathbb{C}^n$ and let U be the unit disc in $\mathbb{C}$. The aim of this work is to study convex combinations of biholomorphic mappings on $\mathbb{B}^n$ starting from a result proved by Chichra and Singh in the case of one complex variable. They obtained the conditions in which a convex combination of the form $(1-\lambda)f + \lambda g$ is starlike on U, when f and g are starlike on the unit disc U and $\lambda \in [0,1]$. Using this ideea, we can construct a similar result for the case of several complex variables. Then, we use this result to characterize convex sums of biholomorphic starlike mappings on the Euclidean unit ball $\mathbb{B}^n$. Moreover, we obtain some remarks on convex sums of extension operators defined for locally univalent functions (for example, the Graham-Kohr extension operator).

Geometry of planar domains and their applications in study of conformal and harmonic mappings

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The famous Riemann Mapping Theorem states that for every simply connected domain $\Omega \neq \mathbb{C}$ containing a point w_0 there exists a essentially unique univalent function f such that $f(0) = w_0$ and $f_z(0) > 0$, that maps the unit disk $\mathbb{D}$ onto Ω.

Open sense-preserving quasiconformal mappings of $\mathbb{D}$ arised as a solutions of linear elliptic partial differential equations of the form

$$\overline{f}_{\bar{z}}(z) = \omega(z) f_z(z), \quad z \in \mathbb{D},$$

where ω is an analytic function from $\mathbb{D}$ into itself, known as a dilatation of f and such that $|\omega(z)| < k < 1$.

A natural generalization of the classical class of normalized univalent functions on $\mathbb{D}$ is the class of sense-preserving univalent harmonic mappings on $\mathbb{D}$ of the form $f = h + \overline{g}$ normalized by $h(0) = g(0) = h'(0) - 1 = 0$.

In the context of univalent, quasiconformal and planar harmonic mappings a problem of convexity, linear convexity, starlikeness, etc. have been intensively studied in the past decades. Additional properties of a planar domains exhibits a very rich geometric and analytic properties.

We discuss behavior of the function f for which some functional are limited to the Both leminiscates, Pascal snail, hyperbola and conchoid of the Sluze. Some appropriate examples are demonstrated.

Asymptotic estimates for one class of homeomorphisms

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Let Γ be a family of curves γ in $\mathbb{R}^n$, $n \geqslant 2$. A Borel measurable function $\rho : \mathbb{R}^n \to [0,\infty]$ is called *admissible* for Γ, (abbr. $\rho \in \operatorname{adm}\Gamma$), if

$$\int\limits_{\gamma} \rho(x)\, ds \ \geqslant\ 1$$

for any curve $\gamma \in \Gamma$. Let $p \in (1,\infty)$.

The quantity

$$M_p(\Gamma) \ = \inf_{\rho\in\operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)$$

is called *p–modulus* of the family Γ.

For arbitrary sets E, F and G of $\mathbb{R}^n$ we denote by $\Delta(E,F,G)$ a set of all continuous curves $\gamma : [a,b] \to \mathbb{R}^n$ that connect E and F in G, i.å., such that $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in G$ for $a < t < b$.

Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, $x_0 \in D$ and $d_0 = \operatorname{dist}(x_0, \partial D)$. Set

$$\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}\,,$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n :\ |x - x_0| = r_i\}\,, \quad i = 1,\, 2\,.$$

Let a function $Q : D \to [0,\infty]$ be Lebesgue measurable. We say that a homeomorphism $f : D \to \mathbb{R}^n$ is ring Q-homeomorphism with respect to p-modulus at $x_0 \in D$ if the relation

$$M_p(\Delta(fS_1, fS_2, fD)) \ \leqslant \int\limits_{\mathbb{A}} Q(x)\, \eta^p(|x - x_0|)\, dm(x)$$

holds for any ring $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)\,,\ 0 < r_1 < r_2 < d_0$, $d_0 = \operatorname{dist}(x_0, \partial D)$ and for any measurable function $\eta : (r_1, r_2) \to [0,\infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr = 1\,.$$

Let

$$L(x_0, f, R) = \sup_{|x-x_0|\leqslant R} |f(x) - f(x_0)|\,.$$

Theorem. *Suppose that $f : \mathbb{R}^n \to \mathbb{R}^n$ is a ring Q-homeomorphism with respect to p-modulus at a point x_0 with $p > n$ where x_0 is some point in $\mathbb{R}^n$ and for some numbers $c > 0$, $\kappa \leqslant p$, $r_0 > 0$ the condition*

$$\int\limits_{\mathbb{A}(x_0, r_0, R)} Q(x)\, \psi^p(|x - x_0|)\, dm(x) \ \leqslant\ c\, I^{\kappa}(r_0, R) \quad \forall\, R > r_0\,,$$

holds, where $\psi(t)$ *is a nonnegative measurable function on* $(0,+\infty)$ *such that*

$$0 < I(r_0, R) = \int_{r_0}^{R} \psi(t)dt < \infty \quad \forall\, R > r_0\,,$$

then

$$\varliminf_{R\to\infty} L(x_0, f, R)\, I^{\frac{\kappa-p}{p-n}}(r_0, R) \geqslant \omega_{n-1}^{\frac{1}{n-p}} \left(\frac{p-n}{p-1}\right)^{\frac{p-1}{p-n}} c^{\frac{1}{n-p}}\,,$$

where ω_{n-1} *is an area of the unit sphere* $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$ *in* $\mathbb{R}^n$.

Generalizations of Hardy type inequalities via new Green functions

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This talk deals with Hardy inequality and its famous generalizations, extensions and refinements, i.e. with Hardy-type inequalities. The classical Hardy inequality reads:

$$\int_0^\infty \left(\frac{1}{x}\int_0^x f(t)\,dt\right)^p dx \le \left(\frac{p}{p-1}\right)^p \int_0^\infty f^p(x)\,dx,\ p>1, \tag{1}$$

where f is non negative function such that $f \in L^p(\mathbb{R}_+)$ and $\mathbb{R}_+ = (0,\infty)$. The constant $\left(\frac{p}{p-1}\right)^p$ is sharp. This inequality has been generalized and extended in several directions. In this talk the Hardy inequality is generalized by using new Green functions.

Almost periodic functions revisited

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In the beginnings of the twentieth century, Harald Bohr (1887-1951) gave important steps in the understanding of Dirichlet series and their regions of convergence, uniform convergence and absolute convergence. As a result of his investigations concerning those functions which could be represented by a Dirichlet series, he developed in its main features the theory of almost periodic functions (both for functions of a real variable and for the case of a complex variable). Based on a new equivalence relation on these classes of functions, in this talk we will refine Bochner's result that characterizes the property of almost periodicity in the Bohr's sense. Furthermore, we will present a thorough extension of Bohr's equivalence theorem which states that two equivalent almost periodic functions take the same set of values on every open half-plane or open vertical strip included in their common region of almost periodicity.

Harmonic quasiconformal mappings and hyperbolic type metrics

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We are going to present results about connections between quasiconformal and harmonic quasiconformal mappings and some metrics of hyperbolic type.

Some properties of functions from families of even holomorphic functions of several complex variables

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In the presentation we will consider some properties of two families of holomorphic functions defined on bounded complete n-circular domain G of $\mathbb{C}^n$ by the evenness (see [3], [5]). For instance, we present some relationships between considered families and another families investigated by Bavrin (see [1]), the growth and distortion type theorems for functions belonging to this families and the estimates of G-balances of homogeneous polynomials.

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Sharp estimates of the Hankel determinant and of the coefficients for some classes of univalent functions

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In recent times, the problem of finding upper bound, preferably sharp, of the Hankel determinant for classes of univalent functions, is being rediscovered and attracts significant attention among the mathematicians working in the field. In that direction, this presentation will provide some new and improvements of existing results for various classes of univalent functions. Most of the results are sharp.

Minisymposium

Differential equations, dynamical systems and applications (MS-52)

Organized by Sandra Carillo, *University of Rome "La Sapienza", Italy*
Coorganized by
Galina Filipuk, *University of Warsaw, Faculty of Mathematics, Informatics and Mechanics*
Federico Zullo, *Università di Brescia, DICATAM, Italy*

- Decay estimates for parabolic equations with inhomogeneous density in manifolds, *Daniele Andreucci*
- A boundary value problem for system of differential equations with piecewise-constant argument of generalized type, *Anar Assanova*
- On Muirhead and Schur inequalities, *Abdulla Azamov*
- Thermoelastic Bresse system with dual-phase-lag model, *Ivana Bochicchio*
- Modelling instability via a generalized Rulkov map: bursts, synchronization and chaos regularization in financial markets, *Michele Bufalo*
- Fractional Cauchy problem on random snowflakes, *Raffaela Capitanelli*
- Materials with memory: an overview on admissible kernels in the integro-differential model equations, *Sandra Carillo*
- Ultracontractivity for p-fractional Robin-Venttsel' problems in extension domains, *Simone Creo*
- Aspects of nonlinear differential equations, *Galina Filipuk*
- Integrability and asymptotic behaviour of a matrix lattice equation, *Pilar R. Gordoa*
- On the mixed Cauchy-Dirichlet problem for equations with fractional time derivative, *Davide Guidetti*
- Evolution problems for fractional operators in irregular domains, *Maria Rosaria Lancia*
- On classes of solutions of the matrix second Painlevé hierarchy, *Andrew Pickering*
- Evolution problems with dynamical boundary conditions of Wentzell-type, *Silvia Romanelli*
- Analytic properties of the complete formal normal form for the Bogdanov–Takens singularity, *Ewa Stróżyna*
- Generalized concepts of almost periodicity, *Daniel Velinov*
- Invariants of group representations, dimension/degree duality and normal forms of vector fields, *Henryk Zoladek*
- Non-linear convecting radiating fins: solutions, efficiency, entropy, *Federico Zullo*

Decay estimates for parabolic equations with inhomogeneous density in manifolds

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Coauthor: Anatoli Tedeev

We consider the problem

$$\rho(x)u_t = \Delta_{p,m}(u)\,, \qquad x \in M\,, t > 0\,, \tag{1}$$
$$u(x,0) = u_0(x) \geq 0\,, \qquad x \in M\,. \tag{2}$$

in a suitable Riemannian manifold M, where $\Delta_{p,m}$ is the suitable analogue of the doubly nonlinear elliptic operator and ρ is a density function vanishing at infinity.

We obtain the asymptotic decay rate for positive solutions. We also discuss in this talk optimal Sobolev and Hardy inequalities in M under assumptions on the isoperimetric profile of the manifold.

A boundary value problem for system of differential equations with piecewise-constant argument of generalized type

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In present communication, at the interval $[0, T]$ we consider the two-point boundary value problem for the system of differential equations with piecewise-constant argument of generalized type

$$\frac{dx}{dt} = A(t)x(t) + D(t)x(\gamma(t)) + f(t), \tag{1}$$
$$Bx(0) + Cx(T) = d, \qquad d \in R^n, \tag{2}$$

where $x(t) = col(x_1(t), x_2(t), ..., x_n(t))$ is unknown vector function, the $(n \times n)$ matrices $A(t)$, $D(t)$ and n vector function $f(t)$ are continuous on $[0, T]$; $\gamma(t) = \zeta_j$ if $t \in [\theta_j, \theta_{j+1})$, $j = \overline{0, N-1}$; $\theta_j \leq \zeta_j \leq \theta_{j+1}$ for all $j = 0, 1, ..., N-1$; $0 = \theta_0 < \theta_1 < ... < \theta_{N-1} < \theta_N = T$; B, C are constant matrices, and d is constant vector.

A solution to problem (1), (2) is a vector function $x(t)$ is continuously differentiable on $[0, T]$, it satisfies the system (1) and boundary condition (2).

Differential equations with piecewise-constant argument of generalized type (DEPCAG) are more suitable for modeling and solving various application problems, including areas of neural networks, discontinuous dynamical systems, hybrid systems, etc.[1-2]. To date, the theory of DEPCAG on the entire axis has been developed and their applications have been implemented. However, the question of solvability of boundary value problems for systems of DEPCAG on a finite interval still remains open.

We are investigated the questions of existence and uniqueness of the solution to problem (1), (2). The parametrization method [3] is applied for the solving to problem (1), (2) and based on the construction and solving system of linear algebraic equations in arbitrary vectors of new general solutions [4]. By introducing additional parameters we reduce the original problem (1), (2) to an equivalent boundary value problem for system of differential equations with parameters. The algorithm is offered of findings of approximate solution studying problem also it is proved its convergence. Conditions of unique solvability to problem (1), (2) are established in the terms of initial data. The results can be used in the numerical solving of application problems.

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On Muirhead and Schur inequalities

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It is considered three and four term generalizations of the well-known Muirhead inequality. Let positive integers n and m be given, $n \geqslant 2$, $m \geqslant 1$. A vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ with nonnegative integer components is called a power if $\alpha_1 \geqslant \alpha_2 \geqslant \cdots \geqslant \alpha_n$ and $\alpha_1 + \alpha_2 + \cdots + \alpha_n = m$. The set of all the powers will be denoted by $A(n,m)$. Then

$$\mu_{\boldsymbol{\alpha}}(\boldsymbol{x}) = \frac{1}{n!} \sum_{\sigma \in S_n} x_{\sigma_1}^{\alpha_1} x_{\sigma_2}^{\alpha_2} \ldots x_{\sigma_n}^{\alpha_n}$$

is an elementary homogeneous symmetric polynomial of the vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}_+^n$ (where S_n is a family of all permutations of the set $\{1, 2, \ldots, n\}$).

The family $A(n,m)$ will be considered with the order $\boldsymbol{\alpha} \leqslant \boldsymbol{\beta}$, defined by the relations

$$\begin{aligned}
&\alpha_1 \leqslant \beta_1, \\
&\alpha_1 + \alpha_2 \leqslant \beta_1 + \beta_2, \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&\alpha_1 + \alpha_2 + \cdots + \alpha_{n-1} \leqslant \beta_1 + \beta_2 + \cdots + \beta_{n-1}.
\end{aligned}$$

The following statements hold:
a) $\left[\forall \boldsymbol{x} \in \mathbb{R}_+^n : \ \mu_{\boldsymbol{\alpha}}(\boldsymbol{x}) \leqslant \mu_{\boldsymbol{\beta}}(\boldsymbol{x})\right] \Leftrightarrow \boldsymbol{\alpha} \leqslant \boldsymbol{\beta}$ (Muirhead inequality) [1];
b) for $n = 3$ ($\alpha, \beta \in \mathbb{N}^+$)

$$\mu_{(\alpha+2\beta,\,0,\,0)}(\boldsymbol{x}) + \mu_{(\alpha,\,\beta,\,\beta)}(\boldsymbol{x}) \geqslant 2\mu_{(\alpha+\beta,\,\beta,\,0)}(\boldsymbol{x})$$

(Schur inequality [2]).

Theorem. Let $\alpha,\ \beta,\ \gamma \in \mathbb{N}^{+}$.

1. If $\alpha + \gamma \geqslant 2\beta$ then $\mu_{\alpha}(\boldsymbol{x}) + \mu_{\gamma}(\boldsymbol{x}) \geqslant 2\mu_{\beta}(\boldsymbol{x})$.

2. For $n = 2$,

$$\mu_{(m,\ m-\alpha)}(\boldsymbol{x}) + \mu_{(m,\ m-\gamma)}(\boldsymbol{x}) \geqslant 2\mu_{(m,\ m-\beta)}(\boldsymbol{x})$$

if and only if $\beta(m-\beta) \geqslant (\beta-\gamma)(\beta+\gamma-m)$.

3. For $n = 3$,

$$\mu_{(\alpha+2\beta+2\gamma,\,0,\,0)}(\boldsymbol{x}) + \mu_{(\alpha,\,2\beta,\,2\gamma)}(\boldsymbol{x}) \geqslant 2\mu_{(\alpha+\beta+\gamma,\,\beta+\gamma,\,0)}(\boldsymbol{x}).$$

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Thermoelastic Bresse system with dual-phase-lag model

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The thermoelastic system modeling longitudinal, vertical and angular motion is the so-called Bresse system. It describes the behavior of a thin curved beam and it was introduced in 1856 by Bresse. The Bresse system generalizes the well-known Timoshenko model, obtained in the particular case where the longitudinal displacement is not considered and supposing zero initial curvature. The Bresse model becomes more interesting when also the thermal case is taken into account. It is commonly accepted that the classical heat conduction theory based on the Fourier law implies the fact that the thermal perturbations at any point of the body are felt instantly anywhere. So this work is devoted to analyze a non isothermal Bresse system where the dual-phase-lag heat conduction theory is used to model the heat transfer. In particular, setting the equations in an abstract framework, an existence and uniqueness result is obtained by using the theory of linear semi groups. Furthermore, the polynomial stability and the exponential energy decay are investigated.

Modelling instability via a generalized Rulkov map: bursts, synchronization and chaos regularization in financial markets

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Coauthor: Giuseppe Orlando

The Rulkov map has been successfully employed for describing bursts, mutual synchronization and chaos regularization in biology. Then, its applications have been extended to physics, engineering, medicine, etc. In this work we find an application of a generalized Rulkov map to

finance where the impact of COVID-19 can be clearly seen in the considered dataset. We examine the Financial Stress Index as well as a number of time series diversified by asset classes (swaps, equity and bonds), market (emerging vs developed), issuer (corporate vs government bond), maturity (short vs long). We find that a calibration of a single Rulkov map or of coupled Rulkov maps could describe the alternation between calm periods and financial turmoil (including a black swan) as well the synchronizing mechanism operating across markets. This result is compared to the so-called Naive and the ARIMA-GARCH model to determine how a chaotic deterministic model stands with respect first to a simple model and, second, to an advanced stochastic model expressly designed for handling moving average, autoregression, cointegration and heteroscedastic volatility.
Finally, we give some theoretical considerations about the stability of the nonlinear system described by our generalized Rulkov map.

Fractional Cauchy problem on random snowflakes

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We consider random snowflakes which are domains whose boundary is constructed by mixtures of Koch curves with random scales. These domains are obtained as limit of domains with Lipschitz boundary, whereas for the limit object, the fractal given by the random Koch domain, the boundary has Hausdorff dimension between 1 and 2.

We study time fractional Cauchy problems on the pre-fractal boundary and we prove asymptotic results for the corresponding fractional diffusions.

Materials with memory: an overview on admissible kernels in the integro-differential model equations

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Materials with memory are modelled via integro-differential equations in which the integral term is introduced to take into account the past *history* of the material. The *classical* regularity requirements the kernel, which in viscoelasticity represents the relaxation function, is assumed to satisfy are considered. Aiming to model wider classes of materials, less restrictive assumptions are adopted.

The results presented are part of a joint research program with M. Chipot, V. Valente and G. Vergara Caffarelli.

Ultracontractivity for p-fractional Robin-Venttsel' problems in extension domains

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In this talk we consider a Robin-Venttsel' problem for the regional fractional p-Laplace operator on an extension domain Ω; such domains can have a highly irregular boundary, for example of fractal type.
We first consider the parabolic problem and, by using nonlinear semigroup theory, we prove that it admits a unique weak solution. We then prove that the nonlinear semigroup associated to this problem is ultracontractive; this is achieved by means of a fractional logarithmic Sobolev inequality, adapted to the problem at hand.
We also consider the elliptic problem and we prove that it admits a unique bounded weak solution.
These results are obtained in collaboration with M. R. Lancia.

Aspects of nonlinear differential equations

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Painlevé equations appear in many applications of mathematics and mathematical physics. In this talk I shall give an overview how recurrence coeffi

cients of semi-classical orthogonal polynomials are related to solutions of the Painlevé equations. I shall also show how discrete systems for the recurrence coefficients are related to Bäcklund transformations of the Painlevé equations.

Integrability and asymptotic behaviour of a matrix lattice equation

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In this talk we consider a matrix lattice equation, both in its autonomous and nonautonomous versions, and show integrability in both cases. Moreover, we explore the construction of Miura maps which relate these two lattices, though intermediate integrable lattice equations, to matrix analogues of autonomous and nonautonomous Volterra equations but in two matrix dependent variables which, in general, are subject to consistency conditions. For these last Volterra-type systems we consider certain special classes of matrices and derive coupled systems of integrable equations. We also consider asymptotic reductions to matrix partial differential equations. In addition, in the nonautonomous case, we construct a new nonisospectral matrix Volterra system together with its corresponding Lax pair. It is also interesting that in the nonautonomous case, the relation obtained between certain lattices even in the scalar case seems to be new.

On the mixed Cauchy-Dirichlet problem for equations with fractional time derivative

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We consider first maximal regularity results for linear equations. In order to be applicable to fully nonlinear equations, we need to work in spaces of little Hoelder continuous functions. We extend to spaces of this type a recent theorem of the author in spaces of Hoelder continuous functions. Next, we prove the existence of a unique "maximal solution" in a suitable sense for fully nonlinear equations.

Evolution problems for fractional operators in irregular domains

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We consider nonlocal diffusion processes in non smooth domains of fractal type as well as in the corresponding smoother approximating domains. Existence, uniqueness and regularity issues will be discussed. The asymptotic behaviour of the smoother solutions, if any, will be discussed.

These results are contained in joint papers with S.Creo and P. Vernole

On classes of solutions of the matrix second Painlevé hierarchy

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We consider the iterative construction of solutions of members of the matrix second Painlevé hierarchy. We use compositions of auto-Bäcklund transformations applied to certain classes of initial solution. Results for other related matrix equations are also briefly discussed.

Evolution problems with dynamical boundary conditions of Wentzell-type

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Different classes of evolution equations with dynamical boundary conditions will be considered. We will derive analyticity results for the operator semigroups generated by suitable uniformly elliptic operators with general Wentzell boundary conditions.

Analytic properties of the complete formal normal form for the Bogdanov–Takens singularity

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In our previous paper a complete formal normal forms for germs of 2–dimensional holomorphic vector fields with nilpotent singularity was obtained. That classification is quite nontrivial (7 cases), but it can be divided into general types like in the case of the elementary singularities. One could expect that also the analytic properties of the normal forms for the nilpotent singularities are analogous to the case of the elementary singularities. This is really true. In the cases analogous to the focus and the node the normal form is analytic. In the case analogous to the nonresonant saddle the normal form is often nonanalytic due to the small divisors phenomenon. In the cases analogous to the resonant saddles (including saddle–nodes) the normal form is nonanalytic due to properties of some homological operators associated with the first nontrivial term in the orbital normal form.

Generalized concepts of almost periodicity

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Coauthors: Marko Kostić, Stevan Pilipović

The main subject of this talk are semi-Bloch (p, k)-periodic functions and sequence and weighted almost (ω, c)-pseudo periodic functions as a generalization of most of the all previously developed concepts in theory of almost periodic functions. A various types of weighted almost (ω, c)-pseudo periodic composition results are given. Also, a qualitative analysis in this context of a semilinear (fractional) Cauchy inclusions is provided.

Invariants of group representations, dimension/degree duality and normal forms of vector fields

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We develop a constructive approach to the problem of polynomial first integrals for linear vector fields. As an application we obtain a new proof of the theorem of Wietzenbock about finiteness of the number of generators of the ring of constants of a linear derivation in the polynomial ring. Moreover, we propose an alternative approach to the analyticity property of the normal form reduction of a germ of vector field with nilpotent linear part in a case considered by Stolovich and Verstringe.

Non-linear convecting radiating fins: solutions, efficiency, entropy

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Università di Brescia, Italy

The second order, nonlinear, ordinary differential equation describing the dissipation of heat in a convecting-radiating longitudinal fin is analyzed. The profile of the fin is arbitrary. With the introduction of an auxiliry variable, we obtain new solutions for the linear, purely convective, case and explicit solutions for the non-linear convecting-radiating case. The efficiency of the fin is discussed in both cases and it is compared with a novel introductiuon of the efficiency based on the entropy rates of the convecting-radiating processes.

Minisymposium

ENERGY METHODS AND THEIR APPLICATIONS IN MATERIAL SCIENCE (MS-25)

Organized by Martin Jesenko, *Univerza v Ljubljani, Slovenia*

- Higher order far-field developments around lattice defects, *Julian Braun*
- Variational Modeling of Paperboard Delamination under Bending, *Patrick Dondl*
- Analysis of a perturbed Cahn-Hilliard model for Langmuir-Blodgett films, *Marco Morandotti*
- Asymptotic analysis of rigidity constraints modeling fiber-reinforced composites, *Antonella Ritorto*
- Fracture in random heterogeneous particle systems, *Anja Schlömerkemper*
- Geometric linearization of theories for incompressible elastic materials and applications, *Bernd Schmidt*
- A derivation of Griffith functionals from discrete finite-difference models, *Francesco Solombrino*

Higher order far-field developments around lattice defects

Julian Braun, j.braun@warwick.ac.uk
University of Warwick, United Kingdom
Coauthors: Thomas Hudson, Christoph Ortner

We examine the elastic deformation induced by lattice defects in crystalline materials. The deformation is given as a (local) minimizer of an atomistic energy. We give a novel far-field development of such minimizers based on continuum PDEs and multi-pol terms. This expansion is given up to arbitrary high-order. We also showcase how to use this expansion for very efficient atomistic simulations of defects.

Variational Modeling of Paperboard Delamination under Bending

Patrick Dondl, patrick.dondl@mathematik.uni-freiburg.de
Albert-Ludwigs-Universität Freiburg, Germany

Paperboard is an engineering material consisting of a number of separate sheets of paper, that have been bonded together. Experimental evidence shows that paperboard undergoing bending develops phenomenologically plastic hinges. We consider a nonlinearly elastic mathematical model for paperboard, allowing debonding of the sheets at a given cost per unit area. Analysis of our model predicts a number of different regimes, including some where bending is concentrated in delaminated hinges, where the mid-plane of each individual layer may deform isometrically. This is joint work with Sergio Conti (Bonn) and Julia Orlik (Kaiserslautern).

Analysis of a perturbed Cahn-Hilliard model for Langmuir-Blodgett films

Marco Morandotti, marco.morandotti@polito.it
Politecnico di Torino, Italy

A one-dimensional evolution equation including a transport term is considered; it models a process of thin films deposition. Existence and uniqueness of solutions, together with continuous dependence on the initial data and an energy equality are proved by combining a minimizing movement scheme with a fixed-point argument. Finally, it is shown that, when the contribution of the transport term is small, the equation possesses a global attractor and converges to a purely diffusive Cahn-Hilliard equation. This is joint work with Marco Bonacini and Elisa Davoli.

Asymptotic analysis of rigidity constraints modeling fiber-reinforced composites

Antonella Ritorto, antonella.ritorto@ku.de
Katholische Universität Eichstätt-Ingolstadt, Germany
Coauthors: Carolin Kreisbeck, Dominik Engl

In this talk, we discuss recent analytical results describing the effective deformation behavior of composite materials. Our model involves a rigidity constraint acting on long fibers that are

semi-periodically distributed in a material body.

The main result characterizes the weak limits of sequences of Sobolev maps whose gradients on the fibers correspond to rotations. Such limits exhibit a restrictive anisotropic geometry in the sense that they locally preserve the length in the fiber direction. We illustrate the results with examples of attainable macroscopic deformations. Under additional regularization in terms of second-order derivatives in the directions orthogonal to the fibers, the model is less flexible, and only rigid body motions can occur.

Joint work with Dominik Engl (Utrecht University) and Carolin Kreisbeck (KU Eichstätt-Ingolstadt).

Fracture in random heterogeneous particle systems

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Coauthors: Laura Lauerbach, Stefan Neukamm, Mathias Schäffner

To understand the onset of fracture better, we study energy functionals for one-dimensional heterogenous particle systems with convex-concave interaction potentials that allow for fracture. We provide a notion of fracture in the discrete system and show that - in the continuum limit - this yields the same onset of fracture as corresponding energy functionals that are obtained by Γ-convergence methods.

Geometric linearization of theories for incompressible elastic materials and applications

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Universität Augsburg, Germany

Coauthor: Martin Jesenko

We derive geometrically linearized theories for incompressible materials from nonlinear elasticity theory in the small displacement regime. Our nonlinear stored energy densities may vary on the same (small) length scale as the typical displacements. This allows for applications to multiwell energies as, e.g., encountered in martensitic phases of shape memory alloys and models for nematic elastomers. Under natural assumptions on the asymptotic behavior of such densities we prove Gamma-convergence of the properly rescaled nonlinear energy functionals to the relaxation of an effective model. The resulting limiting theory is geometrically linearized in the sense that it acts on infinitesimal displacements rather than finite deformations, but will in general still have a limiting stored energy density that depends in a nonlinear way on the infinitesimal strains. Our results, in particular, establish a rigorous link of existing finite and infinitesimal theories for incompressible nematic elastomers.

A derivation of Griffith functionals from discrete finite-difference models

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Università di Napoli "Federico II", Italy
Coauthors: Vito Crismale, Giovanni Scilla

We analyse a finite difference approximation of an Ambrosio-Tortorelli-like functional in brittle fracture, in the discrete-to-continuum limit. In a suitable regime between the competing scales, namely if the discretization step is smaller than the ellipticity parameter, we show the Γ-convergence of the model to the Griffith functional. The limit analysis combines a blow-up procedure with suitable adaptions to the discrete setting of recent compactness and lower semicontinuity results in spaces of generalised functions of bounded deformation. In particular, we do not need to add any artificial L^p fidelity term, which would be unnatural in fracture mechanics.

Minisymposium

Nonlinear analysis for continuum mechanics (MS-33)

Organized by Nicolas Van Goethem, *Faculdade de Ciências da Universidade de Lisboa, Portugal*
Coorganized by
Filipe Oliveira, *ISEG-University of Lisbon*
Riccardo Scala, *Università degli Studi di Napoli*

- Topological singularities in periodic media: Ginzburg-Landau and core-radius approaches, *Roberto Alicandro*
- Asymptotic limit of linear parabolic equations with spatio-temporal degenerated potentials, *Pablo Alvarez Caudevilla*
- The Hopf-Oleinik Lemma for the divergence-type equations, *Daria Apushkinskaya*
- Characterization of oscillation and mass concentration patterns occurring along sequences of A-free functions, *Adolfo Arroyo Rabasa*
- Critical semilinear fractional elliptic problems involving an inverse fractional operator, *Eduardo Colorado*
- Phase-field approximation for a class of cohesive energies with an activation threshold, *Vito Crismale*
- A variational approach to edge dislocations in the triangular lattice, *Lucia De Luca*
- A mass optimisation problem with convex cost, *Maria Stella Gelli*
- Optimal control for rate-independent systems, *Dorothee Knees*
- Nonlocal variational problems: Structure-preservation during relaxation?, *Carolin Kreisbeck*
- Crack growth by vanishing viscosity in planar elasticity, *Ilaria Lucardesi*
- Upscaling and spatial localization of non-local energies with applications to crystal plasticity, *José Matias*
- Dynamics of visco-elastic bodies with a cohesive interface, *Matteo Negri*
- Crystallization results in mechanics and collective behaviour, *Marcello Ponsiglione*
- Shape optimization of light structures and the vanishing mass conjecture, *Filip Rindler*
- On a Class of Nonlocal Evolutionary Problems with Gradient-type Constraints, *José Francisco Rodrigues*
- Microscopical justification of Winterbottom shape, *Igor Velčić*
- Optimal design problems with non-standard growth, *Elvira Zappale*

Topological singularities in periodic media: Ginzburg-Landau and core-radius approaches

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Coauthors: Andrea Braides, Marco Cicalese, Lucia De Luca, Andrey Piatnitski

I will describe the emergence of topological singularities in periodic media within the Ginzburg-Landau model and the core-radius approach. In particular I will investigate, through a multi-scale analysis, the interaction between the oscillation period of the composite and the length scale parameter of the Ginzburg-Landau functionals.

Asymptotic limit of linear parabolic equations with spatio-temporal degenerated potentials

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University Carlos III Madrid, Spain

We observe how the heat equation in a non-cylindrical domain can arise as the asymptotic limit of a parabolic problem in a cylindrical domain, by adding a potential that vanishes outside the limit domain. We provide a strong convergence result for the solution by use of energetic methods and Γ-convergence technics. Then, we establish an exponential decay estimate coming from an adaptation of an argument due to B. Simon.

This analysis will be a crucial issue in getting the dynamical behaviour of several models of population dynamics with spatial or spatio-temporal heterogeneities. The geometrical distribution where this potential vanishes is related to the heterogeneous character of the nonlinear problem and will allow us to prove the existence and uniqueness of solutions as well as its long time behaviour.

The Hopf-Oleinik Lemma for the divergence-type equations

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Peoples' Friendship University of Russia, Russian Federation, and
St. Petersburg State University, Russian Federation
Coauthor: Alexander Nazarov

The Hopf-Oleinik lemma, known also as the"normal derivative lemma", is one of the important tools in qualitative analysis of partial differential equations.

This lemma states that *a supersolution of a partial differential equation with a minimum value at a boundary point, must increase linearly away from its boundary minimum provided the boundary is smooth enough.*

A major part of all known results on the normal derivative lemma concerns equations with nondivergence structure and strong solutions (see [1] and [2] for some recent results and the comprehensive historical review). The case of the divergence-type equations is less studied. It is well known that the normal derivative lemma fails for uniformly elliptic equations in divergence form with bounded and even continuous leading coefficients. Thus, one has to require more

smoothness of the leading coefficients.

The first result for elliptic equations with divergence structure was proved by R. Finn and D. Gilbarg (1957), who considered 2D bounded domains with $C^{1,\alpha}$-regular boundary, the Hölder continuous leading coefficients and continuous lower order coefficients. In our knowledge, the best result preceding our one was established by V. Kozlov and N. Kuznetsov (2018), who considered n-dimensional bounded $C^{1,\alpha}$-domains ($n \geqslant 3$) for equations with the lower-order coefficients belonging to the Lebesgue space L^q, $q > n$ and the same leading coefficients as before.

For the parabolic divergence-type equations we do not know such results. However, the normal derivative lemma for parabolic equations can be extracted from the lower bound estimates of the Green function for the corresponding operator.

We present several versions of the Hopf-Oleinik lemma for general elliptic and parabolic equations in divergence form under the sharp requirements on the coefficients of equations and on the boundary of a domain. All our assumptions are significantly weakened in comparison with the previous works. In fact, our requirements are close to the necessary ones.

The talk is based on the paper [3]. This work is supported by the German Research Foundation, grant no. AP 252/3-1, by the Russian Foundation of Basic Research (RFBR), grant no. 18-01-00472, and by the "RUDN University program 5-100".

References

[1] A.I. Nazarov, *A centennial of the Zaremba-Hopf-Oleinik lemma*, SIAM J. Math. Anal. **44** (2012), no. 1, 437–453.

[2] D.E. Apushkinskaya, A.I. Nazarov, *A counterexample to the Hopf-Oleinik lemma (elliptic case)*, Anal. PDE **9** (2016), no. 2, 439–458.

[3] D.E. Apushkinskaya, A.I. Nazarov, *On the Boundary Point Principle for divergence-type equations*, Rend. Lincei Mat. Appl. **30** (2019), 677–699.

Characterization of oscillation and mass concentration patterns occurring along sequences of A-free functions

Adolfo Arroyo Rabasa, adolforabasa@gmail.com

The University of Warwick, United Kingdom

I will establish a functional dichotomy between A-quasiconvex integrands, a notion of convexity associated to a constant coefficient differential operator "A(D)", and the many ways a sequence (u_j) of functions satisfying the differential constraint $A(D)u_j = 0$ may fail to converge strongly in L^1, due to oscillation and/or concentration effects. Concerning applications, I will discuss some interesting examples that showcase the failure of L1-compensated compactness when concentration of mass is allowed. For instance, I will explain why a sequence of divergence-free vector-fields may develop any type of oscillation/concentration pattern.

Critical semilinear fractional elliptic problems involving an inverse fractional operator

Eduardo Colorado, eduardo.colorado@uc3m.es
Universidad Carlos III de Madrid, Spain

In this talk we will study the existence of positive solutions for a problem related to a higher order fractional differential equation involving a nonlinear term depending on a fractional differential operator,

$$\begin{cases} (-\Delta)^{\alpha}u = \lambda u + (-\Delta)^{\beta}|u|^{p-1}u & \text{in } \Omega, \\ (-\Delta)^{j}u = 0 & \text{on } \partial\Omega, \text{ for } j \in \mathbb{Z},\, 0 \leq j < [\alpha], \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $0 < \beta < 1$, $\beta < \alpha < \beta + 1$ and $\lambda > 0$. In particular, we will show study the following fractional elliptic problem,

$$\begin{cases} (-\Delta)^{\alpha-\beta}u = \lambda(-\Delta)^{-\beta}u + |u|^{p-1}u & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

proving existence or nonexistence of positive solutions depending on the parameter $\lambda > 0$, up to the critical value of the exponent p, i.e., for $1 < p \leq 2^*_\mu - 1$ where $\mu := \alpha - \beta$ and $2^*_\mu = \frac{2N}{N-2\mu}$ is the critical exponent of the Sobolev embedding.

The results are mainly collected in the following paper,
Álvarez-Caudevilla, P.; Colorado, E.; Ortega, Alejandro. *Positive solutions for semilinear fractional elliptic problems involving an inverse fractional operator.* Nonlinear Anal. Real World Appl. 2020.

Phase-field approximation for a class of cohesive energies with an activation threshold

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Coauthor: Antonin Chambolle

I will present a work with A. Chambolle, in which we study the Gamma-limit of Ambrosio-Tortorelli-type functionals $D_\epsilon(u, v)$, whose dependence on the symmetrised gradient $e(u)$ is different in $\mathbb{A}u$ and in $e(u) - \mathbb{A}u$, for a $\mathbb{C}$-elliptic symmetric operator $\mathbb{A}$, in terms of the prefactor depending on the phase-field variable v. The limit energy depends both on the opening and on the surface of the crack, and is intermediate between the Griffith brittle fracture energy and the one considered by [Focardi-Iurlano, SIMA 2014]. In particular we prove that G(S)BD functions with bounded $\mathbb{A}$-variation are (S)BD.

A variational approach to edge dislocations in the triangular lattice

Lucia De Luca, lucia.deluca@cnr.it
Italian National Research Council, Italy

We discuss discrete and continuous models for edge dislocations in the framework of plane elasticity. We focus, in particular, on the elastic energy induced by a family of edge dislocations in the triangular lattice and we study, within the rigorous formalism of Gamma-convergence, its asymptotic behaviour as the lattice spacing vanishes.

A mass optimisation problem with convex cost

Maria Stella Gelli, maria.stella.gelli@unipi.it
University of Pisa, Italy

A mass optimisation problem consists in finding the best distribution of a given elastic material in order to achieve the minimal compliance with prescribed external loads. Here the unknown mass distribution is subject to a given convex cost depending pointwise both on the underlying design region and on the conductivity properties of the material under analysis. After providing a relaxed formulation of the problem we investigate the existence and structure of the associated minimal measures. This is joint work with D. Lučić (Pisa) and G. Buttazzo (Pisa).

Optimal control for rate-independent systems

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University of Kassel, Germany

Coauthor: Stephanie Thomas

Rate independent systems can be formulated based on an energy functional and a dissipation potential that is assumed to be convex, lower semicontinuous and positively homogeneous of degree one. Here, we will focus on the nonconvex case meaning that the energy is not convex. In this case, the solution typically is discontinuous in time. There exist several (in general not equivalent) notions of weak solutions. We focus on so-called balanced viscosity solutions, discuss the properties of solution sets and discuss the well posedness of an optimal control problem for such systems. If time permits we also address an existence result with discontinuous loads. This is joint work with Chiara Zanini (Torino).

Nonlocal variational problems: Structure-preservation during relaxation?

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KU Eichstätt-Ingolstadt, Germany

Coauthors: Antonella Ritorto, Elvira Zappale

Nonlocal variational problems arise in various applications, such as in continuum mechanics through peridynamics, the theory of phase transitions, or image processing. Naturally, the presence of nonlocalities leads to new effects, and the standard methods in the calculus of variations,

which tend to rely intrinsically on localization arguments, do not apply.

This talk addresses the relaxation of two classes of functionals – double-integrals and nonlocal supremals. Our focus lies on the question of whether the resulting relaxed functionals preserve their structure. We give an affirmative answer for nonlocal supremals in the scalar setting, along with a closed representation formula in terms of separate level convexification of a suitably diagonalized supremand, and discuss results in the vectorial case. As for double-integrals, a full understanding of the problem is still missing. We present the first counterexample showing that weak lower semicontinuous envelopes fail to be double-integrals in general. On a technical level, both findings rely on a characterization of the asymptotic behavior of (approximate) nonlocal inclusions via Young measures, a theoretical result of independent interest.

Crack growth by vanishing viscosity in planar elasticity

Ilaria Lucardesi, ilaria.lucardesi@univ-lorraine.fr

Université de Lorraine, France

In this talk we show existence of quasistatic evolutions in a fracture model for brittle materials by a vanishing viscosity approach, in the setting of planar linearized elasticity. Here the crack is not prescribed a priori and is selected in a class of (unions of) regular curves. To prove the result, it is crucial to analyze the properties of the energy release rate showing that it is independent of the crack extension. This is joint work with S. Almi (Wien) and G. Lazzaroni (Florence).

Upscaling and spatial localization of non-local energies with applications to crystal plasticity

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Instituto Superior Técnico, U. of Lisbon, Portugal

Coauthors: Marco Morandotti, David Owen, Elvira Zappale

An integral representation result is obtained for the asymptotics of energies including both local and non-local terms, in the context of structured deformations. Starting from an initial energy featuring a local bulk and interfacial contribution and a non-local measure of the jump discontinuities, an iterated limiting procedure is performed. First, the initial energy is relaxed to structured deformation, and then the measure of non-locality is sent to zero, with the effect of obtaining an explicit local energy in which the non-linear contribution of submacroscopic slips and separations is accounted for. Two terms, different in nature, emerge in the bulk part of the final energy: one coming from the initial bulk energy and one arising from the non-local contribution to the initial energy. This structure turns out to be particularly useful for studying mechanical phenomena such as yielding and hysteresis. Moreover, in the class of invertible structured deformations, applications to crystal plasticity are presented.

Dynamics of visco-elastic bodies with a cohesive interface

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Coauthor: Riccardo Scala

We consider the dynamics of elastic materials with a common cohesive interface (or a domain with a prescribed cohesive fracture). In the bulk, the evolution is provided by linearized elastodynamics with Kelvin-Voigt visco-elastic dissipation, while on the interface the evolution is governed by a system of Karush-Kuhn-Tucker depending on the crack opening and on the internal variable. The weak formulation reads

$$\begin{cases} \rho\ddot{u}(t) + \partial_u \mathcal{E}(u(t)) - \langle F(t), u\rangle + \partial_{[u]}\Psi(\xi(t), [u(t)]) + \partial_{\dot{u}}\mathcal{R}(\dot{u}) \ni 0, \\ \dot{\xi}(t)(\xi(t) - |[u(t)]|) = 0 \ \text{ and } \ |[u(t)]| \leq \xi(t), \\ u(0) = u_0, \ \dot{u}(0) = u_1, \end{cases}$$

where $\mathcal{E}$ is the elastic energy, F is the external force, $\mathcal{R}$ is Kelvin-Voigt visco-elastic dissipation, while Ψ is the interface cohesive potential, concave under loading and quadratic under unloading.

First we provide existence of a time-discrete evolution by means of incremental minimization problems (fully implicit in the displacement) and then its time-continuous limit, which satisfies the energy identity

$$\mathcal{E}(u(t)) + \Psi([u(t)], \xi(t)) + \mathcal{K}(\dot{u}(t)) = \mathcal{E}(u_0) + \Psi([u_0], \xi_0) + \mathcal{K}(v_0) + \\ + \int_0^t \langle F(s), \dot{u}(s)\rangle \, ds - \int_0^t \partial_v \mathcal{R}(\dot{u}(s))[\dot{u}(s)] \, ds.$$

Finally, we discuss the strong formulation of the system, with acceleration in L^2 and equilibrium of forces on the interface.

Crystallization results in mechanics and collective behaviour

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We will present variational approaches to crystallization in two dimensions. First, we will discuss classical pairwise interaction potentials leading to microscopic crystallization and hexagonal Wulff shapes. Then, we will show that potentials with non integrable tails may lead to round Wulff shapes minimizing fractional perimeters. Finally, we will discuss how this kind of analysis can be adapted to deal with problems arising in collective behaviour with possible applications to fish schooling .

Shape optimization of light structures and the vanishing mass conjecture

Filip Rindler, F.Rindler@warwick.ac.uk
University of Warwick, United Kingdom

It is a classical problem in the theory of shape optimization to find a shape with minimal (linear) elastic compliance (or, equivalently, maximal stiffness) for a given amount of mass and prescribed external forces. It is an intriguing question with a long history, going back to Michell's seminal 1904 work on trusses, to determine what happens in the limit of vanishing mass. Contrary to all previous results, which utilize a soft mass constraint by introducing a Lagrange multiplier, we here consider the hard mass constraint. Our results establish the convergence of approximately optimal shapes of (exact) size tending to zero to a limit generalized shape represented by a (possibly diffuse) probability measure. This limit generalized shape is a minimizer of the limit compliance, which involves a new integrand, namely the one conjectured by Bouchitte in 2001 and predicted heuristically before in works of Allaire-Kohn (80's) and Kohn-Strang (90's). This integrand gives the energy of the limit generalized shape understood as a fine oscillation of (optimal) lower-dimensional structures. Its appearance is surprising since the integrand in the original compliance is just a quadratic form and the non-convexity of the problem is not immediately obvious. I will also present connections to the theory of Michell trusses and show how our results can be interpreted as a rigorous justification of that theory on the level of functionals in both two and three dimensions. This is joint work with J.F. Babadjian (Paris-Saclay) and F. Iurlano (Paris-Sorbonne).

On a Class of Nonlocal Evolutionary Problems with Gradient-type Constraints

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We study the solutions of nonlocal problems with local and nonlocal gradient type constraints, the nonlocal gradient being the distributional Riesz fractional gradient. We show new existence results on the solution of an Monge-Kantorovich type equation in an open domain with Dirichlet boundary conditions. The solution is given by a couple, the transport density and the potential u, which solves a variational inequality with a nonlocal gradient constraint. Using a penalisation of the transport flux we extend to the case of nonlocal second order linear operators with only somable coefficients the existence of Lagrange multipliers, which includes the case of the classsical local anisotropic problem related to the elastoplastic torsion problem. This is a joint work with Assis Azevedo and Lisa Santos.

Microscopical justification of Winterbottom shape

Igor Velčić, igor.velcic@fer.hr
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Coauthor: Paolo Piovano

In this talk we will discuss the microscopical derivation of a continuum model for the Winterbottom problem, i.e., the problem of determining the equilibrium shapes for droplets attached to a wall. Our strategy consists in showing that properly defined atomistic energies of crystalline configurations Γ-converge, as the number of atoms grows, converge to a surface energy which is minimized by the Winterbottom shape. The work generalizes the results which deal with the equilibrium shape of particles in a free crystalline configuration (without a substrate) where it was that the limit minimizing configuration is the Wulff shape. This problem finds applications in the framework of growth of thin films over substrates.

Optimal design problems with non-standard growth

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Coauthor: Ana Cristina Barroso

We provide a measure representation for certain functionals arising in the context of optimal design problems under non-standard growth conditions and perimeter penalization. We also consider applications to modelling of thin structures.

The talk is based on joint works with Ana Cristina Barroso at University of Lisbon.

Minisymposium

OPERATOR ALGEBRAS (MS-14)

Organized by Stefaan Vaes, *KU Leuven, Department of Mathematics, Belgium*
Coorganized by Stuart White, *University of Glasgow, School of Mathematics and Statistics, United Kingdom*

- SU(2)-symmetries, subproduct systems, and exact sequences of C*-algebras, *Francesca Arici*
- A complete characterisation of algebraic number fields using Cartan pairs, *Chris Bruce*
- Uniform property Gamma, *Jorge Castillejos*
- KMS spectra for group actions on compact spaces, *Johannes Christensen*
- Nuclearity and generalized inductive limits, *Kristin Courtney*
- Toeplitz quotient C^*-algebras and ratio limits for random walks, *Adam Dor On*
- Refined moves for structure-preserving isomorphisms between graph C^*-algebras, *Søren Eilers*
- Bases, unitary group representations, and comparison of projections, *Ulrik Enstad*
- Nuclear Dimension of Simple C*-Algebras and Extensions, *Samuel Evington*
- Quantum groups in the heat, *Amaury Freslon*
- Nuclear dimension of crossed products attached to partial homeomorphisms, *Shirly Geffen*
- Noncommutative ergodic theory of higher-rank lattices, *Cyril Houdayer*
- Boundary quotient C*-algebras for product systems, *Evgenios Kakariadis*
- Elementary amenability and almost finiteness, *David Kerr*
- A duality theorem for non-unital operator systems, *Se Jin Kim*
- Positive trace polynomials, *Igor Klep*
- C*-algebras of right LCM monoids and their equilibrium states, *Nadia Larsen*
- Rigidity of Roe algebras, *Kang Li*
- Cartan subalgebras in classifiable C*-algebras, *Xin Li*
- Boundary theory and amenability: from Furstenberg's Poisson formula to boundaries of Drinfeld doubles of quantum groups, *Sergey Neshveyev*
- Property (T) for automorphism groups of free groups, *Piotr Nowak*
- Higher Kazhdan projections and the Baum-Connes conjecture, *Sanaz Pooya*
- Irreducible inclusions of simple C*-algebras, *Mikael Rordam*
- Quantum symmetry vs nonlocal symmetry, *Simon Schmidt*
- Constructions in minimal dynamics and applications to the classification of C*-algebras, *Karen Strung*
- The stable uniqueness theorem for equivariant Kasparov theory, *Gabor Szabo*
- Order isomorphisms of operator intervals, *Peter Šemrl*
- Higher structures in mathematics: buildings, C*-algebras and Drinfeld-Manin solutions of Yang-Baxter equations, *Alina Vdovina*
- Random quantum graphs are asymmetric, *Mateusz Wasilewski*
- Stable rank one and Cuntz semigroup regularity, *Wilhelm Winter*

SU(2)-symmetries, subproduct systems, and exact sequences of C*-algebras

Francesca Arici, f.arici@math.leidenuniv.nl
Leiden University, Netherlands

Motivated by the study of symmetries of C*-algebras and by multivariate operator theory, we will introduce the notion of an $SU(2)$-equivariant subproduct system of Hilbert spaces. We will describe the associated Toeplitz and Cuntz–Pimsner algebras and provide results about their topological invariants through K(K)-theory.

In particular, we will show how to compute the (K)K-theory groups of the Cuntz–Pimsner algebra of an (irreducible) $SU(2)$-subproduct system using an exact sequence that involves an operator theoretic analogue of the Euler class.

Based on joint work with Jens Kaad (SDU Odense).

A complete characterisation of algebraic number fields using Cartan pairs

Chris Bruce, Chris.Bruce@glasgow.ac.uk
Queen Mary University of London and *University of Glasgow, United Kingdom*
Coauthor: Xin Li

Given any two rings of algebraic integers, Li and Lück proved that the associated ring C*-algebras are always isomorphic. I will present a result showing that—in stark contrast with the result by Li and Lück—given any two such rings, there is a Cartan-preserving isomorphism between the ring C*-algebras if and only if the rings are isomorphic. As a consequence of this result, the semigroup C*-algebra of the (full) ax+b-semigroup over a ring of algebraic integers together with its canonical Cartan subalgebra completely characterises the ring. This is joint work with Xin Li (University of Glasgow).

Uniform property Gamma

Jorge Castillejos, jcastillejoslopez@impan.pl
Institute of Mathematics of the Polish Academy of Sciences, Poland

Property Gamma was introduced by Murray and von Neumann as a way to show the existence of non-hyperfinite factors. This property also played a key role in Connes' classification of injective factors and, outside the injective setting, it remains a key topic for von Neumann algebras. I will discuss a new C*-version of this property that has played a relevant role in the classification of simple nuclear C*-algebras.

KMS spectra for group actions on compact spaces

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Coauthor: Stefaan Vaes

The crossed product construction associates a C*-algebra to a countable group acting by homeomorphisms on a compact space, in such a way that the C*-algebra encodes information about the group action. This construction has stimulated a mutually beneficial interplay between dynamical systems and operator algebras.

In this talk I will uncover a surprising relation between geometric group theoretic properties of a group G and the so called *KMS spectra* for certain diagonal 1-parameter groups on the crossed product C^*-algebras of actions of G. The KMS spectrum for a 1-parameter group is the set of inverse temperatures for which there exists a *KMS state*, a concept originally studied in relation to quantum statistical mechanics, and that now plays a prominent role in the theory of C*-algebras.

I will present results which illustrates that the possible KMS spectra depend heavily on the acting group G: when G has subexponential growth, only the subsets $\{0\}$, $[0, +\infty)$, $(-\infty, 0]$ and $\mathbb{R}$ arise as KMS spectrum; for general amenable groups all closed subsets of $\mathbb{R}$ containing zero can arise and are concretely realized for certain wreath product groups; while for arbitrary countable groups, any closed subset of $\mathbb{R}$ may appear and is concretely realized for the free group with infinitely many generators.

The results I will present in this talk are joint work with Stefaan Vaes.

Nuclearity and generalized inductive limits

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Coauthor: Wilhelm Winter

Part of Alain Connes' groundbreaking work in von Neumann algebra theory was to show that any von Neumann algebra which can be well-approximated by matrix algebras can actually be built from matrix algebras via an inductive limit construction, i.e., semi-discrete von Neumann algebras are hyperfinite. In the setting of C*-algebras, such a tidy result is too much to ask. The C*-analogue of the semi-discrete von Neumann algebras are nuclear (or amenable) C*-algebras, and many of these, such as the Cuntz algebras or irrational rotation algebras, are not inductive limits of finite dimensional C*-algebras. Building on work of Blackadar and Kirchberg, we give a generalization of inductive systems for C*-algebras, which allows us to characterize separable nuclear C*-algebras as (generalized) inductive limits of finite dimensional C*-algebras.

Toeplitz quotient C^*-algebras and ratio limits for random walks

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We study quotients of the Toeplitz C*-algebra of a random walk, similar to those studied by the speaker and Markiewicz for finite stochastic matrices. We introduce a new Cuntz-type quotient C*-algebra for random walks that have convergent ratios of transition probabilities. These C*-algebras give rise to new notions of ratio limit space and boundary for such random walks, spurring further research in probability. Our results are leveraged to identify a unique symmetry-equivariant quotient C*-algebra for any symmetric random walk on a hyperbolic group, shedding light on a question of Viselter on C*-algebras of subproduct systems.

Refined moves for structure-preserving isomorphisms between graph C^*-algebras

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In work with Restorff, Ruiz and Sørensen, we recently provided a complete description of the equivalence class on the set of graphs with finitely many vertices and at most countably many edges induced by stable isomorphism of their associated graph C^*-algebras. This description was a list of “moves” that generate the equivalence relation in the sense that two such graphs are stably isomorphic if and only if one may transform one into another by a finite number of such moves.

A concurrent program pioneered by Kengo Matsumoto has established strikingly strong rigidity results for Cuntz-Krieger algebras when one considers them not just as C^*-algebras, but also take finer structure (a certain Abelian subalgebra and a certain circle action) into account, showing in a multitude of ways that operator algebraic objects completely remember the dynamics underlying their definition.

Since the Cuntz-Krieger algebras lie in the class of graph C^*-algebras, it is a natural (one might even say: pressing) question if a similar combinatorial description may be obtained for the finer equivalence relations induced by structure-preserving isomorphism of the graph C^*-algebras, and we present a list of moves which we conjecture solves this problem in the very satisfactory sense that the subset of moves preserving extra structure in fact also generates the relevant equivalence relation.

We can prove the conjecture for all graphs defining simple Cuntz-Krieger algebras, and in a large number of special cases beyond that. Apart from the core results obtained with Ruiz, I will also present recent work obtained with Becky Armstrong, Kevin Brix, Toke Meier Carlsen, Aidan Sims, and Gábor Szabó.

Bases, unitary group representations, and comparison of projections

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Many examples of bases and frames in applied harmonic analysis (e.g. wavelets and Gabor frames) share the property that they lie in the orbit of a single vector under the action of an irreducible, unitary group representation. Several problems related to the existence of such bases and frames remain open. In this talk I will show that when sampling from lattices, such existence problems can be formulated in terms of comparison of projections in (matrix algebras over) the group von Neumann algebra and the group C*-algebra of the lattice. Thus, new results in applied harmonic analysis can be deduced from structural properties of these operator algebras.

Nuclear Dimension of Simple C*-Algebras and Extensions

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The nuclear dimension of a C*-algebra, introduced by Winter and Zacharias, is a non-commutative generalisation of the covering dimension of a topological space.

Whilst any non-negative integer or infinity can be realised as the nuclear dimension of some commutative C*-algebra, the nuclear dimension of a simple C*-algebra must be either 0,1 or infinity. This trichotomy is just one application of my joint work on the Toms–Winter Conjecture with Castillejos, Tikuisis, White, and Winter. In this talk, I will outline the results, their application to classification theory, and the new ideas at the heart of our work.

I will then discuss the recent developments on the nuclear dimension of extensions, including the work on the Cuntz–Toeplitz algebras undertaken during the Glasgow Summer Project 2019.

Quantum groups in the heat

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Coauthors: Lucas Teyssier, Simeng Wang

In this talk, I will consider the diffusion of the heat semi-group on free orthogonal quantum groups. In the case of classical orthogonal groups, this is the Markov semi-group associated to the Brownian motion, and it is known to spread very abruptly in the sense that it exhibits a *cut-off phenomenon* (this is a result of P.-L. Meliot).

I will explain what this means and show that this phenomenon also occurs in the quantum setting. I will further detail how one can get a more precise description of the behaviour of the semi-group around the mixing time by computing the so-called *limiting profile*. This is based on a joint work with L. Teyssier and S. Wang.

Nuclear dimension of crossed products attached to partial homeomorphisms

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The concept of a C*-algebraic partial automorphism, namely an isomorphism between two ideals of a C*-algebra, was introduced by Exel in the 1990s. Many important C*-algebras that cannot be written as a crossed product by a (global) automorphism, have a description as a crossed product by a partial automorphism. In connection with the classification program, I show that in some cases crossed products attached to partial homeomorphisms on finite-dimensional spaces, have finite nuclear dimension.

Noncommutative ergodic theory of higher-rank lattices

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I will survey recent results in the study of dynamical properties of the space of positive definite functions and characters of higher-rank lattices. I will explain that for a large class of higher-rank lattices, all their Uniformly Recurrent Subgroups (URS) are finite and all their weakly mixing unitary representations weakly contain the left regular representation. These results strengthen celebrated results by Margulis (1978), Stuck-Zimmer (1992) and Nevo-Zimmer (2000). The key novelty in our work is a structure theorem for equivariant normal unital completely positive maps between von Neumann algebras and function spaces associated with Poisson boundaries.

Based on joint works with R. Boutonnet and U. Bader, R. Boutonnet, J. Peterson.

Boundary quotient C*-algebras for product systems

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A boundary quotient is a terminal object for a class of representations. In most of the cases boundary quotients are manifestations of corona sets and they are indispensable for our understanding beyond the "classical" universe of finite dimensional algebras and their norm-limits.

C*-terminal objects are central elements of the theory, while they provide key C*-constructs in the study of geometric and topological objects. This interplay goes as back as the classification of factors by Murray and von Neumann in the 1930's, and has been a continuous source of inspiration for further developments. Applications include for example detecting phase transitions of C*-invariants.

On the other hand the Shilov and the Choquet boundaries of (nonselfadjoint) function algebras have been the subject of intense research since the 1950's providing fruitful interactions with convexity and approximation theory. Their noncommutative analogues have been a groundbreaking foresight of Arveson's seminal work in the 1960's, and beyond. Applications include interactions with group theory, noncommutative geometry, and noncommutative convexity, to mention only but a few.

In this talk we will show how a combination of the selfadjoint and the nonselfadjoint viewpoints gives the existence of the boundary quotient C*-algebra for product systems. This class has been under consideration for the past 30 years and models a great number of C*-constructs, including graphs (of rank 1 or higher), C*-dynamics of several flavors (reversible and irreversible), semigroup C*-algebras, and Nica covariant representations of compactly aligned product systems.

The talk is based on joint works with Dor-On, Katsoulis, Laca and Li.

Elementary amenability and almost finiteness

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We show that every free continuous action of a countably infinite elementary amenable group on a finite-dimensional compact metrizable space is almost finite. As a consequence, the crossed products of minimal such actions are $\mathcal{Z}$-stable and classified by their Elliott invariant. This is joint work with Petr Naryshkin.

A duality theorem for non-unital operator systems

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Coauthors: Matthew Kennedy, Nicholas Manor

The recent work on nc convex sets of Davidson, Kennedy, and Shamovich show that there is a rich interplay between the category of operator systems and the category of compact nc convex sets, leading to new insights even in the case of C*-algebras. The category of nc convex sets are a generalization of the usual notion of a compact convex set that provides meaningful connections between convex theoretic notions and notions in operator system theory. In this talk, we present a duality theorem for norm closed self-adjoint subspaces of $B(H)$ that do not necessarily contain the unit. We will present some insights this duality presents to various notions in C*-algebras and operator systems such as simplicity. As well, we present a non-commutative dynamical characterization of locally compact property (T) groups. This is joint work with Matthew Kennedy and Nicholas Manor.

Positive trace polynomials

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Trace polynomials are polynomials in noncommuting variables and traces of their products. They can be naturally evaluated in finite von Neumann algebras. While originating in invariant theory as equivariant maps between tuples of matrices, trace polynomials more recently received attention in operator algebra, free probability and quantum information theory. This talk addresses positivity of their evaluations and presents new Positivstellensätze (=algebraic certificates for positivity) in terms of sums of squares and traces of sums of squares.

C*-algebras of right LCM monoids and their equilibrium states

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A right LCM monoid is characterised by the structural feature that the intersection of two principal right ideals is either empty or equal to another principal right ideal. The associated universal

C*-algebra of the monoid is generated by a family of isometries subject to an algebraic condition capturing the right LCM property. A choice of a monoid homomorphism into the positive real numbers leads to a time evolution on the C*-algebra and a zeta function that records the growth of the monoid elements. There is a generally rich structure of equilibrium states reflecting a fine interplay between critical values and values in an interval of convergence for this series. Many authors have contributed to this development in recent years. In the talk I will outline some key ideas and present the concrete case of Artin monoids, including the two large classes of right-angled and finite-type monoids.

Rigidity of Roe algebras

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(Uniform) Roe algebras are C^*-algebras associated to metric spaces, which reflect coarse properties of the underlying metric spaces. It is well-known that if X and Y are coarsely equivalent metric spaces with bounded geometry, then their (uniform) Roe algebras are (stably) $*$-isomorphic. The rigidity problem refers to the converse implication. The first result in this direction was provided by Ján Špakula and Rufus Willett, who showed that the rigidity problem has a positive answer if the underlying metric spaces have Yu's property A. I will in this talk review all previously existing results in literature, and then report on the latest development in the rigidity problem. This is joint work with Ján Špakula and Jiawen Zhang.

Cartan subalgebras in classifiable C*-algebras

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This talk is about Cartan subalgebras in classifiable C*-algebras. We will give an overview of some recent results, including a general construction of Cartan subalgebras in all stably finite classifiable C*-algebras and a detailed analysis of these Cartan subalgebras in example classes.

Boundary theory and amenability: from Furstenberg's Poisson formula to boundaries of Drinfeld doubles of quantum groups

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In his work on the Poisson formula for semisimple Lie groups Furstenberg attached two boundaries to every locally compact group G, which are now called the Poisson and Furstenberg boundaries of G. As has been observed over the years, both constructions can be approached from an operator algebraic point of view and extended to the noncommutative setting, leading to the theories of noncommutative Poisson boundaries by Izumi and of injective envelopes by Hamana. The noncommutative Poisson boundaries have been computed in a number of cases. An important aspect of the computations, both in the classical and noncommutative settings, is that the boundaries can be interpreted as universal objects measuring nonamenability of a (quantum) group G. I'll explain how such universal properties can be used to identify noncommutative Poisson and Furstenberg-Hamana boundaries in some cases, leading to quantum analogues of classical results of Furstenberg and Moore. (Joint work with Erik Habbestad and Lucas Hataishi.)

Property (T) for automorphism groups of free groups

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Kazhdan's property (T) is a powerful rigidity property of locally compact groups. It has many applications, including constructions of expander graphs and of counterexamples to certain versions of the Baum-Connes conjecture. I will describe a new approach to proving property (T) that uses optimization methods in the form of semidefinite programming. I will present the recent proof of property (T) for $\mathrm{Aut}(F_n)$, the automorphism group of the free group F_n on n generators, for $n \geq 5$. These results were obtained jointly with Marek Kaluba, Dawid Kielak and Narutaka Ozawa.

Higher Kazhdan projections and the Baum-Connes conjecture

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Coauthors: Piotr Nowak, Kang Li

The Baum-Connes conjecture, if it holds for a certain group, provides topological tools to compute the K-theory of its reduced group C*-algebra. This conjecture has been confirmed for large classes of groups, such as amenable groups, but also for some Kazhdan's property (T) groups. Property (T) and its strengthening are driving forces in the search for potential counterexamples to the conjecture. Having property (T) for a group is characterised by the existence of a certain projection in the universal group C*-algebra of the group, known as the Kazhdan projection. It is this projection and its analogues in other completions of the group ring, which obstruct known methods of proof for the Baum-Connes conjecture. In this talk, I will introduce a generalisation of Kazhdan projections. Employing these projections we provide a link between surjectivity of the Baum-Connes map and the ℓ^2-Betti numbers of the group. A similar relation can be obtained in the context of the coarse Baum-Connes conjecture. This is based on joint work with Kang Li and Piotr Nowak.

Irreducible inclusions of simple C*-algebras

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The literature contains a number interesting examples of inclusions of simple C*-algebras, typically arising from dynamical systems, with the property that all intermediate C*-algebras are also simple. One can argue that this property of an inclusion of C*-algebras is the natural

C*-analog of an irreducible inclusion of von Neumann algebras (i.e., one with trivial relative commutant). I will present an intrinsic description of when an inclusion of C*-algebras is C*-irreducible, and relate this to the parallel situation of von Neumann algebras. I will further show how C*-irreducible inclusions can arise from groups, dynamical systems, inductive limits (and AF-algebras), and tensor products.

Quantum symmetry vs nonlocal symmetry

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We will introduce the notion of nonlocal symmetry of a graph G, defined as winning quantum correlation for the G-automorphism game that cannot be produced classically. We investigate the differences and similarities between this and the quantum symmetry of the graph G, defined as non-commutativity of the algebra of functions on the quantum automorphism group of G. We show that quantum symmetry is a necessary but not sufficient condition for nonlocal symmetry. In particular, we show that the complete graph on four points does not exhibit nonlocal symmetry. We will also see that the complete graph on five or more points does have nonlocal symmetry. This talk is based on joint work with David Roberson.

Constructions in minimal dynamics and applications to the classification of C^*-algebras

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Coauthors: Robin Deeley, Ian Putnam

What abelian groups can arise as the K-theory of C^*-algebras arising from minimal dynamical systems? In joint work with Robin Deeley and Ian Putnam, we completely characterize the K-theory of the crossed product of a space X with finitely generated K-theory by an action of the integers and show that crossed products by a minimal homeomorphisms exhaust the range of these possible K-theories. We also investigate the K-theory and the Elliott invariants of orbit-breaking algebras. We show that given arbitrary countable abelian groups G_0 and G_1 and any Choquet simplex Δ with finitely many extreme points, we can find a minimal orbit-breaking relation such that the associated C^*-algebra has K-theory given by this pair of groups and tracial state space affinely homeomorphic to Δ. These results have important applications to the Elliott classification program for C^*-algebras. In particular, we make a step towards determining the range of the Elliott invariant of the C^*-algebras associated to étale equivalence relations.

The stable uniqueness theorem for equivariant Kasparov theory

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In this talk I will present a new approach to the classification of C*-dynamics up to cocycle conjugacy. After a few categorical preliminaries and a brief introduction to equivariant Elliott intertwinings, the talk will focus on how to exploit equivariant Kasparov theory for the purpose of classification. This leads to a strengthening of the Cuntz-Thomsen picture of equivariant KK-theory in the spirit of Lin and Dadarlat-Eilers, which is the main result of the talk. If time permits, I will speculate on some far-reaching consequences of this machinery. This is joint work with James Gabe.

Order isomorphisms of operator intervals

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A general theory of order isomorphisms of operator intervals will be presented. It unifies and extends several known results, among others Ludwig's description of ortho-order automorphisms of effect algebras and Molnar's characterization of bijective order preserving maps on bounded observables. The problem of the existence and uniqueness of extensions of order isomorphisms of operator intervals to operator domains will be discussed.

Higher structures in mathematics: buildings, C*-algebras and Drinfeld-Manin solutions of Yang-Baxter equations

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The most interesting mathematical structures are usually sufficiently rich and appear in several fields of mathematics, physics and computer science. We give a brief introduction to one such topic, namely buildings. We will present geometric, algebraic and arithmetic aspects of buildings. In particular, we present explicit constructions of infinite families of quaternionic cube complexes, covered by buildings.

We will use these cube complexes to describe new infinite families of Drinfeld-Manin solutions of Yang-Baxter equations. Another application of our constructions are new infinite families of higher-rank graph C*-algebras and von Neumann algebras.

Random quantum graphs are asymmetric

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The study of quantum graphs emerged from quantum information theory. One way to define them is to replace the space of functions on a vertex set of a classical graph with a noncommutative algebra and find a satisfactory counterpart of an adjacency matrix in this context. Another approach is to view undirected graphs as symmetric, reflexive relations and "quantize" the notion of a relation on a set. In this case quantum graphs are operator systems and the definitions are equivalent. Doing this has some consequences already for classical graphs; viewing them as operator systems of a special type has already led to the introduction of a few new "quantum" invariants.

Motivated by developing the general theory of quantum graphs, I will take a look at random quantum graphs, having in mind that the study of random classical graphs is very fruitful. I will show how having multiple perspectives on the notion of a quantum graph is useful in determining the symmetries of these objects – as expected, a generic quantum graph is asymmetric.

Stable rank one and Cuntz semigroup regularity

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A unital C*-algebra has stable rank one if its group of invertible elements is norm dense. While this plays an important role for many structural results especially for simple C*-algebras, its interplay with other regularity properties is subtle and somewhat mysterious.

Thiel has shown that if a simple unital C*-algebra has stable rank one and strict comparison of positive elements, then its Cuntz semigroup is almost divisible.

In this talk I will describe a partial converse: strict comparison and almost divisibility together imply stable rank one.

This is joint work in progress with Shirly Geffen.

Minisymposium

OPERATOR SEMIGROUPS AND EVOLUTION EQUATIONS (MS-29)

Organized by Christian Budde, *North-West University, South Africa*
Coorganized by David Seifert, *Newcastle University, United Kingdom*

- Spectral aspects of eventually positive C_0-semigroups, *Sahiba Arora*
- Bounded functional calculi for unbounded operators, *Charles Batty*
- Observability for Non-Autonomous Systems, *Fabian Gabel*
- Chernoff approximation of operator semigroups and applications, *Yana Kinderknecht (Butko)*
- Fractional semidiscrete evolution equations in Lebesgue sequence spaces, *Pedro J. Miana*
- Evolution equations on graph and networks: diffusion and beyond, *Delio Mugnolo*
- Weighted composition operators via hyperbolic C_0-groups on $\mathbb{D}$, *Jesús Oliva Maza*
- Embeddability of matrices into real and positive semigroups, *Agnes Radl*
- Subclasses of Partial Contraction Mapping and generating evolution system on Semigroup of Linear Operators, *Kamilu Rauf*
- Null-Controllability for Parabolic Equations, *Christian Seifert*
- Spectral Multiplier Theorems in L^p For Abstract Differential Operators, *Himani Sharma*
- Strong and Polynomial Stability for Delay Semigroups, *Sachi Srivastava*
- On decay rates of the solutions of parabolic Cauchy problems, *Jari Taskinen*

Spectral aspects of eventually positive C_0-semigroups

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The theory of positive one-parameter semigroups is rich and has applications in various mathematical fields. A closely related notion that appeared recently is that of *eventually positive* semigroups, i.e., semigroups that become (and stay) positive for sufficiently large times. A systematic study of this concept revealed that such semigroups exhibit spectral properties similar to those already known for positive semigroups.

We start with a few examples of eventually positive semigroups and then highlight several spectral theoretic behaviours of this notion. We follow this up with criteria for *strong convergence* of an eventually positive semigroup. Further, we look at a characterization of convergence of eventually positive semigroups in the operator norm which generalizes a result that was previously only known for positive semigroups.

Towards the end, we look at some spectral and convergence implications of *locally eventual positive* semigroups. In a loose sense, this means that the solution of the corresponding Cauchy problem becomes positive in a part of the domain for large times. While examples of this concept have been known for quite some time, a systematic study of it has only been recently initiated.

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Bounded functional calculi for unbounded operators

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Several new functional calculi for unbounded operators have been discovered recently. In particular, many semigroup generators have a bounded functional calculus for a Banach algebra of so-called analytic Besov functions. I will describe properties and significance of this calculus, and then briefly mention further extensions of that calculus for generators of bounded semigroups on Hilbert spaces and for generators of bounded holomorphic semigroups on Banach spaces.

This work has been in collaboration with Alexander Gomilko and Yuri Tomilov.

Observability for Non-Autonomous Systems

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Coauthors: Clemens Bombach, Christian Seifert, Martin Tautenhahn

We study non-autonomous abstract Cauchy problems

$$\dot{x}(t) = A(t)x(t)\,,\ y(t) = C(t)x(t)\,,\ t > 0\,, \quad x(0) = x_0 \in X\,,$$

where $A(t) : D(A) \to X$ is a strongly measurable family of operators on a Banach space X and $C(t) \in \mathcal{L}(X, Y)$ is a family of bounded observation operators from X to a Banach space Y.

For measurable subsets $E \subseteq (0, T)$, $T > 0$, we provide sufficient conditions such that the Cauchy problem satisfies a *final state observability estimate*

$$\|x(T)\|_X \lesssim \left(\int_E \|y(t)\|_Y^r \, \mathrm{d}t \right)^{1/r}, \quad r \in [1, \infty)\,,$$

where an analogous estimate holds for the case $r = \infty$.

An application of the above result to families of strongly elliptic differential operators $A(t)$ and observation operators

$$C(t)u := \mathbf{1}_{\Omega(t)}u\,, \quad \Omega(t) \subseteq \mathbb{R}^d\,, u \in \mathrm{L}^p(\mathbb{R}^d)\,,$$

is presented. In this setting, we give sufficient and necessary geometric conditions on the family of sets $(\Omega(t))$ such that the corresponding Cauchy problem satisfies a final state observability estimate.

Chernoff approximation of operator semigroups and applications

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We present a method to approximate operator semigroups with the help of the Chernoff theorem. We discuss different approaches to construct Chernoff approximations for semigroups, generated by Markov processes, and for Schrödinger groups. This method provides simultaneously some numerical schemes for PDEs and pseudo-differential equations (in particular, the operator splitting method), Euler–Maruyama schemes for the corresponding SDEs and other Markov chain approximations to the corresponding Markov processes, can be understood as a numerical path integration method. In some cases, Chernoff approximations have the form of limits of n iterated integrals of elementary functions as $n \to \infty$ (in this case, they are called *Feynman formulae*) and can be used for direct computations and simulations of stochastic processes. The limits in Feynman formulae sometimes coincide with (or give rise to) path integrals with respect to probability measures (such path integrals are usually called *Feynman-Kac formulae*) or with respect to Feynman type pseudomeasures. Therefore, Feynman formulae can be used to approximate the corresponding path integrals and to establish relations between different path integrals.

In this talk, we discuss Chernoff approximations for semigroups generated by Feller processes in $\mathbb{R}^d$. We are also interested in constructing Chernoff approximations for semigroups, generated by Markov processes which are obtained by different operations from some original Markov processes. In this talk, we discuss Chernoff approximations for such operations as: a random time change via an additive functional of a process, a subordination (i.e., a random time change via an independent a.s. nondecreasing 1-dim. Lévy process), killing of a process upon leaving a given domain, reflecting of a process. These results allow, in particular, to obtain Chernoff approximations for subordinate diffusions on strar graphs and compact Riemannian manifolds. Moreover, the constructed Chernoff approximations for evolution semigroups can be used further to approximate solutions of some time-fractional evolution equations describing anomalous diffusion (solutions of such equations do not posess the semigroup property and are related to some non-Markov processes).

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Fractional semidiscrete evolution equations in Lebesgue sequence spaces

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In this talk, we give representations for solutions of time-fractional differential equations that involve operators on Lebesgue spaces of sequences defined by discrete convolutions involving kernels through the discrete Fourier transform. We consider finite difference operators of first and second orders, which are generators of uniformly continuous semigroups and cosine functions. We present the linear and algebraic structures (in particular, factorization properties) and their norms and spectra in the Lebesgue space of summable sequences. We identify fractional powers of these generators and apply to them the subordination principle. We also give some applications and consequences of our results. These results have been published in a joint paper with Carlos Lizama and Jorge González-Camus from the Universidad de Santiago de Chile.

Evolution equations on graph and networks: diffusion and beyond

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I will present several models of hyperbolic and parabolic systems supported on metric graphs. Well-posedness and long-time behavior can be studied by an interplay of variational, functional analytical, and spectral theoretical methods. If time allows, I will also present a new class of functionals and use it to describe the diffusive efficiency of a given graph topology.

Weighted composition operators via hyperbolic C_0-groups on $\mathbb{D}$

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Coauthors: Luciano Abadías, José E. Galé, Pedro J. Miana

In this talk, we present our recent work regarding a family of C_0-groups of weighted composition operators on Hardy and Bergman spaces on the disk, i.e. $f \mapsto \frac{k\circ\varphi_t}{k} f \circ \varphi_t$, where $(\varphi_t)_{t\in\mathbb{R}}$ is a group of hyperbolic automorphisms of the disk $\mathbb{D}$ onto itself, and $k : \mathbb{D} \to \mathbb{C}$ is a holomorphic function with polynomial limits of arbitrary order $\alpha, \beta \in \mathbb{R}$ at the fixed points of $(\varphi_t)_{t\in\mathbb{R}}$.

In the first part, we are able to characterize in detail the spectra of the infinitesimal generator of the C_0-group, which only depends on the values of α and β. More precisely, we obtain its point spectra, approximate point spectra and essential spectra. This is done by using a mixture of tools of C_0-semigroups theory, and the theory of weighted composition operators on these spaces of holomorphic functions.

After that, an application of the spectral mapping theorem is shown with some subordinated operators, some of which resemble the Hilbert or Cesàro operators. In a final remark, we show that one cannot take these C_0-groups to $\ell^p(\mathbb{N}_0)$ spaces since they do not define bounded operators there for $p \neq 2$.

Embeddability of matrices into real and positive semigroups

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Coauthor: Tanja Eisner

It is a well-known problem whether a Markov matrix is embeddable into a Markov semigroup, see the recent survey [1]. We consider a similar problem: Given a (finite or infinite) real/positive matrix T, is it embeddable into a real/positive C_0-semigroup, i. e., is there a real/positive semigroup $(T(t))_{t\geq 0}$ such that $T(1) = T$? We will give necessary and sufficient conditions for embeddability.

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Subclasses of Partial Contraction Mapping and generating evolution system on Semigroup of Linear Operators

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Coauthor: Akinola Yussuff Akinyele

Some properties of C_0-Semigroup are investigated and used to derive properties of ω-Order Preserving and Reversing Partial Contraction Mapping where homogeneous, inhomogeneous and regularity of mild solution for analytic semigroups are engaged. Furthermore, the subclasses performed like semigroup of linear operators. Moreover, semigroup of linear operator generated by ω-order reversing partial contraction mapping (ω-$ORCP_n$) as the infinitesimal generator of a C_0-semigroup is discussed. It is an attempt to obtain results on evolution systems and stable families of generators considering the homogeneous and inhomogeneous initial value problem.

Null-Controllability for Parabolic Equations

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In this talk we study various notions of null-controllability of systems in Banach spaces. In an abstract Banach space setting we show that an uncertainty relation together with a dissipation estimate implies a so-called final state observability estimate with explicit dependence on the model parameters. This estimate applied to the dual system in turn is in general equivalent to an approximate notion of null-controllability, and in special cases also to null-controllability of the original system. Our approach unifies and generalizes the respective advantages from earlier results obtained in the context of Hilbert spaces. As an application we consider parabolic equations induced by strongly elliptic operators on L_p spaces for $1 \leq p < \infty$.

The talk is based on joint work with Clemens Bombach, Dennis Gallaun and Martin Tautenhahn.

Spectral Multiplier Theorems in L^p For Abstract Differential Operators

Himani Sharma, Himani.sharma@anu.edu.au
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For an operator generating a group on L^p spaces transference results give bounds on the Phillips functional calculus also known as spectral multiplier estimates. In this talk, we will consider specific group generators which are abstraction of first order differential operators and show similar spectral multiplier estimates assuming only that the group is bounded on L^2 rather than L^p. We will also show some R-bounded Hörmander calculus results. Firstly for the square of a perturbed Hodge-Dirac operator, by assuming an abstract Sobolev embedding property. Secondly for an abstract Harmonic oscillator obtained using Weyl pairs.

Strong and Polynomial Stability for Delay Semigroups

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In this talk we will discuss strong and polynomial stability for semigroups associated with delay differential equations. In particular we will study some conditions on the delay operator Φ and the generator B of the underlying semigroup that ensure strong and polynomial stability of the delay semigroup associated with the abstract delay differential equation

$$\begin{cases} u'(t) = Bu(t) + \Phi u_t, \ t > 0, \\ u(0) = x, \\ u_0 = f, \end{cases}$$

where X is a Banach space, $u_t(\sigma) = u(t+\sigma), -1 \leq \sigma \leq 0, x \in X, f$ lies in an appropriate space, $(B, D(B))$ generates a C_0-semigroup on X and Φ is the delay operator.

On decay rates of the solutions of parabolic Cauchy problems

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Coauthors: José Bonet, Wolfgang Lusky

We consider the Cauchy problem in the Euclidean space $\mathbb{R}^N \ni x$ for the parabolic equation $\partial_t u(x,t) = Au(x,t)$, where the operator A (e.g. the Laplacian) is assumed, among other things, to be a generator of a C_0 semigroup in a weighted L^p-space $L^p_w(\mathbb{R}^N)$ with $1 \leq p < \infty$ and a fast growing weight w. We show that there is a Schauder basis $(e_n)_{n=1}^{\infty}$ in $L^p_w(\mathbb{R}^N)$ with the following property: given an arbitrary positive integer m there exists $n_m > 0$ such that, if the initial data f belongs to the closed linear span of e_n with $n \geq n_m$, then the decay rate of the solution of the problem is at least t^{-m} for large times t. In other words, the Banach space of the initial data can be split into two components, where the data in the infinite-dimensional component leads to decay with any pre-determined speed t^{-m}, and the exceptional component is finite dimensional.

We discuss in detail the needed assumptions of the integral kernel of the semigroup e^{tA}.

We present variants of the result having different methods of proofs and also consider finite polynomial decay rates instead of unlimited m.

The results are contained in the following of papers published together with José Bonet (Valencia) and Wolfgang Lusky (Paderborn).

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Minisymposium

Orthogonal Polynomials and Special Functions (MS-10)

Organized by Juan J. Moreno-Balcázar, *University of Almería, Department of Mathematics, Spain*

Coorganized by

Galina Filipuk, *University of Warsaw, Poland*

Francisco Marcellán, *University Carlos III of Madrid, Spain*

- On some positive quadrature rules on the unit circle, *Cleonice F. Bracciali*
- Multiple Orthogonal Polynomials and Random Walks, *Ana Foulquié*
- d-orthogonal analogs of classical orthogonal polynomials, *Emil Horozov*
- Matrix valued multivariable orthogonal polynomials with BC_2-symmetry, *Erik Koelink*
- Periodic random tiling models and non-Hermitian orthogonality, *Arno Kuijlaars*
- Bivariate Koornwinder-Sobolev orthogonal polynomials, *Misael Enrique Marriaga Castillo*
- Poncelet's Theorem and Orthogonal Polynomials, *Andrei Martinez Finkelshtein*
- Local asymptotics for some q-hypergeometric polynomials, *Juan F. Mañas Mañas*
- A proof of a conjecture of Elbert and Laforgia on the zeros of cylinder functions, *Gergő Nemes*
- Multivariate hybrid orthogonal functions, *Teresa E. Perez*
- Comparative asymptotics of rational modified orthogonal polynomials, *Hector Pijeira Cabrera*
- Converting divergent asymptotic series into convergent series: factorial series for Laplace-type integrals, *Ester Pérez Sinusía*
- Khrushchev formulas for orthogonal polynomials, *Luis Velázquez*
- Dual bases and orthogonal polynomials, *Paweł Woźny*

On some positive quadrature rules on the unit circle

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We consider quadrature rules on the real line associated with a sequence of polynomials generated by a special R_{II} recurrence relation. With a simple transformation, these quadrature rules on the real line also lead to certain positive quadrature rules of highest algebraic degree of precision on the unit circle. We also show new approaches to evaluate the nodes and weights of these specific quadrature rules on the unit circle.
Joint work with Junior A. Pereira and A. Sri Ranga

Multiple Orthogonal Polynomials and Random Walks

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Coauthors: Amílcar Branquinho, Manuel Mañas, Carlos Álvarez-Fernández, Juan Fernández-Díaz

Given a non-negative Jacobi matrix describing higher order recurrence relations for multiple orthogonal polynomials of type II and corresponding linear forms of type I, a general strategy for constructing a pair of stochastic matrices, dual to each other, is provided. The corresponding Markov chains (or 1D random walks) allow, in one transition, to reach for the N-th previous states, to remain in the state or reach for the immediately next state. The dual Markov chains allow, in one transition, to reach for the N-th next states, to remain in the state or reach for immediately previous state. The connection between both dual Markov chains is discussed at the light of the Poincaré's theorem on ratio asymptotics for homogeneous linear recurrence relations and the Christoffel–Darboux formula within the sequence of multiple orthogonal polynomials and linear forms of type I.

The Karlin–McGregor representation formula is extended to both dual random walks, and applied to the discussion of the corresponding generating functions and first-passage distributions. Recurrent or transient character of the Markov chain is discussed. Steady state and some conjectures on its existence and the relation with mass points are also given.

The Jacobi–Piñeiro multiple orthogonal polynomials are taken as a case study of the described results.

d-orthogonal analogs of classical orthogonal polynomials

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Classical orthogonal polynomial systems of Jacobi, Hermite, Laguerre and Bessel have the property that the polynomials of each system are eigenfunctions of second order ordinary differential operator. According to a classical theorem by Bochner they are the only systems with this property. The othogonality property is equivalent to the 3-term recurrence relation according to the famous Favard-Shohat theorem.

Motivated by Bochner's theorem we are looking for d-orthogonal polynomials (systems that

satisfy d+2-term recurrence) that are also eigenfunctions of a differential operator. We call these simultaneous conditions Bochner's property.

Using purely algebraic methods and ideas from the bispectral problem, based on automorphisms of non-commutative algebras we construct polynomial systems with Bochner property.

Many properties of the constructed polynomial systems are obtained quite directly from their construction. In particular they have a number of similarities with the classical orthogonal polynomials, which makes them their natural analog - they have hypergeometric representations, ladder operators, generating functions, they can be presented via Rodrigues formulas, there are Pearson's equations for the weights of their measures, they possess the Hahn's property, i.e. the polynomial system of their derivatives are again orthogonal polynomials, etc. We conjecture that the proposed construction exhausts all systems with Bochner's property. Connections to integrable systems like KP and Toda hierarchies and matrix models will be discussed.

Matrix valued multivariable orthogonal polynomials with BC_2-symmetry

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Coauthor: Jie Liu

The relation between multivariable special functions and orthogonal polynomials has been influenced greatly by generalising spherical functions on Riemannian symmetric spaces by Heckman and Opdam. These functions and polynomials turn out to be important in mathematical physics. In recent years several approaches to vector and matrix valued analogues in the rank one case have been studied from the perspective of matrix valued spherical functions, and several extensions have been studied. We discuss a rank two case with BC_2-symmetry in detail, and we derive explicit results for the corresponding matrix valued orthogonal polynomials. In particular, the matrix weight function is given explicitly in terms of Krawtchouk polynomials, and we prsent the matrix valued linear partial differential operator for which the polynomials are eigenfunctions.

Periodic random tiling models and non-Hermitian orthogonality

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Certain random tiling models show characteristic features that are similar to the eigenvalues of large random matrices. They can be recast as determinantal point process from which limiting laws as the Tracy-Widom distribution at the edge and the Gaussian free field in the bulk can be deduced.

I will discuss a new technique, developed in collaboration with Maurice Duits, to study tiling models with periodic weights. The technique relies on a formulation of a correlation kernel as a double contour integral containing non-Hermitian orthogonal polynomials. In the case of periodic weightings, the orthogonal polynomials are matrix valued.

We use the Riemann-Hilbert problem for matrix valued orthogonal polynomials to obtain asymptotics for the two-periodic Aztec diamond. This model is remarkable since it gives rise to a gaseous phase, in addition to the more familiar solid and liquid phases.

Bivariate Koornwinder-Sobolev orthogonal polynomials

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Coauthors: Miguel Piñar, Teresa E. Pérez

The purpose of this talk is to introduce the so-called Koornwinder bivariate orthogonal polynomials. These polynomials are generated by means of a non-trivial procedure involving two families of univariate orthogonal polynomials and a function $\rho(t)$ such that $\rho(t)^2$ is a polynomial of degree less than or equal to 2. We also discuss how to extend the Koornwinder method to the case when one of the univariate families is orthogonal with respect to a Sobolev inner product. Therefore, we study the new Sobolev bivariate families obtaining relations between the classical original Koornwinder polynomials and the Sobolev one, deducing recursive methods in order to compute the coefficients. The case when one of the univariate families is associated to a classical inner product is analysed. Finally, some useful examples are given.

Poncelet's Theorem and Orthogonal Polynomials

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Poncelet's Theorem is one of the most beautiful and well known results from projective geometry. In the last few decades, the relationship between Poncelet's Theorem and other mathematical object, such as Blaschke products or numerical range of completely non-unitary contractions, has been the focus of extensive research. Recently, another connection, now with the theory of orthogonal polynomials on the unit circle has been revealed. These interconnections allow us to prove several new results, to interpret the existing theory in a new context, and also to understand further connections with other areas of geometry and analysis.

This is a joint work with M. Hunziker, T. Poe, and B. Simanek.

Local asymptotics for some q-hypergeometric polynomials

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The basic q-hypergeometric function ${}_r\phi_s$ is defined by the series

$$ {}_r\phi_s\left(\begin{array}{c} a_1,\ldots,a_r \\ b_1,\ldots,b_s \end{array};q,z\right) = \sum_{k=0}^{\infty} \frac{(a_1;q)_k\cdots(a_r;q)_k}{(b_1;q)_k\cdots(b_s;q)_k}\left((-1)^k q^{\binom{k}{2}}\right)^{1+s-r}\frac{z^k}{(q;q)_k}, \qquad (1)$$

where $0<q<1$ and $(a_j;q)_k$ and $(b_j;q)_k$ denote the q-analogues of the Pochhammer symbol.

When one of the parameters a_j in (1) is equal to q^{-n} the basic q-hypergeo-metric function is a polynomial of degree at most n in the variable z. Our objective is to obtain a type of local asymptotics, known as Mehler–Heine asymptotics, for q-hypergeometric polynomials when $r=s$.

Concretely, by scaling adequately these polynomials we intend to get a limit relation between them and a q-analogue of the Bessel function of the first kind. Originally, this type of local asymptotics was introduced for Legendre orthogonal polynomials (OP) by the German mathematicians H. E. Heine and G. F. Mehler in the 19th century. Later, it was extended to the families of classical OP (Jacobi, Laguerre, Hermite), and more recently, these formulae were obtained for other families as discrete OP, generalized Freud OP, multiple OP or Sobolev OP, among others.

These formulae have a nice consequence about the scaled zeros of the polynomials, i.e. using the well–known Hurwitz's theorem we can establish a limit relation between these scaled zeros and the ones of a Bessel function of the first kind. In this way, we are looking for a similar result in the context of the q-analysis and we will illustrate the results with numerical examples.

A proof of a conjecture of Elbert and Laforgia on the zeros of cylinder functions

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We prove the enveloping property of the known divergent asymptotic expansion of the large real zeros of the cylinder functions, and thereby answering in the affirmative a conjecture posed by Elbert and Laforgia in 2001 (*J. Comput. Appl. Math.* **133** (2001), no. 1–2, p. 683). The essence of the proof is the construction of an analytic function that returns the zeros when evaluated along certain discrete sets of real numbers. By manipulating contour integrals of this function, we derive the asymptotic expansion of the large zeros truncated after a finite number of terms plus a remainder that can be estimated efficiently. The conjecture is then deduced as a corollary of this estimate.

Multivariate hybrid orthogonal functions

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Coauthor: Cleonice F. Bracciali

We consider multivariate orthogonal functions satisfying hybrid orthogonality conditions with respect to a moment functional. This kind of orthogonality means that the product of functions of different parity order is computed by means of the moment functional, and the product of elements of the same parity order is computed by a modification of the original moment functional. Results about existence conditions, three term relations with matrix coefficients, a Favard type theorem for this kind of hybrid orthogonal functions are proved. In addition, a method to construct bivariate hybrid orthogonal functions from univariate orthogonal polynomials and univariate orthogonal functions is presented. Finally, we give a complete description of a sequence of hybrid orthogonal functions on the unit disk on $\mathbb{R}^2$, that includes, as particular case, the classical orthogonal polynomials on the disk.

Comparative asymptotics of rational modified orthogonal polynomials

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Coauthor: Javier Quintero-Roba

The talk deals with the relative asymptotic behavior between a sequence of orthogonal polynomials with respect to a measure with unbounded support and the orthogonal polynomials with respect to a rational modification of this measure. These results are applied to the study of the asymptotic behavior of Sobolev type orthogonal polynomials.

Converting divergent asymptotic series into convergent series: factorial series for Laplace-type integrals

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Asymptotic techniques for Laplace-type integrals are a useful tool to derive asymptotic approximations of special functions. But in most of the important examples of special functions, the asymptotic expansion is not convergent. In this paper we investigate a modification of those asymptotic techniques that transforms the unbounded integration region of the Laplace-type integral into a bounded region. Then, an elementary asymptotic analysis of the new integral shows that the asymptotically relevant integration point is attained at a point of the boundary of the integration region and an expansion of the integrand at that point gives an asymptotic expansion of the integral. But moreover, an analysis of the remainder of this new expansion shows that it is convergent under a mild condition over the integrand. We illustrate this modification with several examples of special functions, providing convergent and asymptotic expansions of these functions.

Khrushchev formulas for orthogonal polynomials

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Coauthors: Christopher Cedzich, F. Alberto Grünbaum, Albert H. Werner, Reinhard F. Werner

Since their origin in the early 20th century, the theory of orthogonal polynomials on the unit circle (OPUC) has proved to be intimately connected to harmonic analysis via the so called Schur functions. The beginning of this century has witnessed a renewed interest in this connection due to a revolutionary approach to OPUC by the hand of Sergei Khrushchev, which emphasizes the role of continued fractions and Schur functions. The name “Khrushchev theory”, coined by Barry Simon, refers to a body of methods and results on OPUC originated by this new approach, whose cornerstone is the so called “Khrushchev formula”. The interest of Schur functions and Khrushchev formulas has been fueled even more by a recently uncovered link between OPUC theory and the study of quantum walks, the quantum version of random walks, where Schur functions are central to develop the quantum version of Pólya’s renewal theory. This has led to a very general understanding of Khrushchev formula, susceptible to be applied to a wide range

of situations covering not only OPUC, but also orthogonal polynomials on the real line (OPRL), matrix valued measures built out of scalar OPUC and OPRL, as well as matrix valued OPUC and OPRL. This talk will give an overview of these different versions of Khrushchev formula, pointing also to their implications in other areas such as harmonic analysis or quantum renewal theory.

Dual bases and orthogonal polynomials

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Let $b_0, b_1, \ldots, b_n$ be linearly independent functions. Let us consider the linear space $\mathcal{B}_n$ generated by these functions with an inner product $\langle \cdot, \cdot \rangle : \mathcal{B}_n \times \mathcal{B}_n \to \mathbb{C}$. We say that the functions $D_n := \{d_0^{(n)}, d_1^{(n)}, \ldots, d_n^{(n)}\}$ form *a dual basis* of the space $\mathcal{B}_n$ with respect to the inner product $\langle \cdot, \cdot \rangle$, if the following conditions hold:

$$\begin{cases} \text{span}\left\{d_0^{(n)}, d_1^{(n)}, \ldots, d_n^{(n)}\right\} = \mathcal{B}_n, \\ \left\langle b_i, d_j^{(n)} \right\rangle = \delta_{ij} \qquad (0 \le i, j \le n), \end{cases}$$

where $\delta_{ii} = 1$, and $\delta_{ij} = 0$ for $i \neq j$.

In general, the dual basis D_n can be found with $O(n^2)$ computational complexity. However, if the dual basis D_n is known, it is possible to construct the dual basis D_{n+1} faster, i.e., with $O(n)$ computational complexity. Dual bases have many applications in numerical analysis, approximation theory or in computer aided geometric design. For example, skillful use of these bases often results in less costly algorithms which solve some computational problems.

It is also important that dual bases are very closely related to orthogonal bases. In the first part of the talk, we present general results on dual bases. Next, we focus on some important families of polynomial dual bases and their connections with classical, discrete and q-orthogonal polynomials. For example, the so-called dual Bernstein polynomials are related to orthogonal Hahn, dual Hahn and Jacobi polynomials. Using some of these connections, one can find differential-recurrence formulas, differential equation or recurrence relation satisfied by dual Bernstein polynomials. There also exists a first-order non-homogeneous recurrence relation linking dual Bernstein and orthogonal Jacobi polynomials. When used properly, it allows to propose fast and numerically efficient algorithms for evaluating all $n+1$ dual Bernstein polynomials of degree n with $O(n)$ computational complexity.

Minisymposium

RECENT DEVELOPMENTS ON PRESERVERS (MS-38)

Organized by
Lajos Molnár, *University of Szeged, Hungary*
Bojan Kuzma, *University of Primorska, Slovenia*

- On maps preserving products equal to fixed elements, *Louisa Catalano*
- Total variation distance isometries of probability measures, *Gregor Dolinar*
- The minimax principle in the Jordan setting, *Francisco J. Fernandez Polo*
- Recent generalisations of classical theorems on quantum mechanical symmetry transformations, *György Pál Gehér*
- Tingley's problem on uniform algebras, *Osamu Hatori*
- Wigner's theorem in normed spaces, *Dijana Ilišević*
- Meet preservers between lattices of real-valued continuous functions, *Kristopher Lee*
- On the linearity of order-isomorphisms, *Bas Lemmens*
- Unitarily Invariant Norms on Operators and Distance Preserving Maps, *Chi Kwong Li*
- Schur null preservers, *Ying-Fen Lin*
- Linear functions preserving Green's relations over fields, *Artem Maksaev*
- The Gleason–Kahane–Żelazko theorem and automatic countinuity, *Javad Mashreghi*
- Maps on positive definite cones of C^*-algebras preserving the Wasserstein mean, *Lajos Molnar*
- Loewner's theorem for maps on operator domains, *Michiya Mori*
- Surjective isometries on Banach algebras of Lipschitz maps taking values in a unital C^*-algebra, *Shiho Oi*
- Free functions preserving certain partial orders of operators, *Miklós Pálfia*
- Linear maps which are (triple) derivable or anti-derivable at a point, *Antonio M. Peralta*
- Singularity preserving maps on matrix algebras, *Valentin Promyslov*
- Maps preserving absolute continuity of positive operators, *Tamás Titkos*
- Isometries of Wasserstein spaces, *Dániel Virosztek*
- Positivity preservers forbidden to operate on diagonal blocks, *Prateek Kumar Vishwakarma*

On maps preserving products equal to fixed elements

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Coauthor: Hayden Julius

In this talk, we will characterize the form of bijective linear maps $f : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ satisfying $f(A)f(B) = M$ whenever $AB = N$, where M and N are fixed $n \times n$ complex matrices.

Total variation distance isometries of probability measures

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Coauthors: Bojan Kuzma, Djordje Mitrović

Some recent results about surjective isometries of Borel probability measures with respect to the total variation distance will be presented. It turns out that the characterization of such isometries under some additional assumption, e.g. w*-continuity or absolute continuity with respect to some fixed probability measure is quite different than in the general case of surjective isometries on all probability measures.

The minimax principle in the Jordan setting

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The celebrated minimax principle (also known as Courant-Fisher Minimax theorem) provides a characterization of the eigenvalues of a symmetric compact operator on a Hilbert space without any reference to eigenvectors or the characteristic polynomial.

In the Jordan setting it is remarkable the contribution of U. Hirzebruch, who obtained in 1970 a minimax principle for finite dimensional formally real Jordan algebras (finite dimensional JB-algebras).

We present a generalized minimax principle for weakly compact JB*-triples (and hence also for weakly compact JB*-algebras). This result provides a geometric characterization of the singular values associated to an element. As consequences of this principle a Weyl inequality and a Cauchy-Poincaré interlacing Theorem can also be obtained.

Recent generalisations of classical theorems on quantum mechanical symmetry transformations

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Symmetries of geometric and operator structures are important in the foundations of quantum mechanics. One famous example is Wigner's theorem on quantum mechanical symmetry transformations, which describes all maps on the space of quantum pure states that leave the transition probability invariant. In particular, Wigner's theorem is an important step in obtaining the general time-dependent Schrodinger equation from purely mathematical assumptions.

In my talk I'll review some of these classical results, and will present some of their recently obtained generalizations.

Part of my talk will be based on joint works with Peter Šemrl (University of Ljubljana, Slovenia) and Michiya Mori (University of Tokyo, Japan).

Tingley's problem on uniform algebras

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The Tingley's problem asks if a surjective isometry between the unit spheres of two Banach spaces is extended to a real-linear surjective isometry between whole of the Banach spaces. Several progresses have been done by many authors, although I do not show a complete list of the name of authors. Tanaka and Peralta initiated to study the case of algebras of matrices and operators. Recently Mori and Ozawa gave a positive partial solution by proving that C^*-algebras satisfy the Mazur-Ulam property. Several progresses are going on in this direction. Very recently, Becerra-Guerrero, Cueto-Avellaneda, ,Ferenández-Polo and Peralta published the results about JBW^*-triples. On the other hand, the case of Banach spaces of analytic functions, except Hiblert spaces, are still missing. I will give a talk in this case including uniform algebras.

This is a joint work with Shiho Oi and Rumi Shindo Togashi.

Wigner's theorem in normed spaces

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Let $(H, (\cdot,\cdot))$ and $(K, (\cdot,\cdot))$ be inner product spaces over $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$ and suppose that $f\colon H \to K$ is a mapping satisfying

$$|(f(x), f(y))| = |(x, y)|, \quad x, y \in H. \tag{1}$$

Then the famous Wigner's unitary–antiunitary theorem says that f is a solution of (1) if and only if it is phase equivalent to a linear or an anti-linear isometry, say U, that is,

$$f(x) = \sigma(x)Ux, \quad x \in H,$$

where $\sigma\colon H \to \mathbb{F}$, $|\sigma(x)| = 1$, $x \in H$, is a so called phase function. In this talk several generalizations of this theorem to the setting of normed spaces will be presented.

Meet preservers between lattices of real-valued continuous functions

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It is well-know that the set $C(X)$ of real-valued continuous functions defined on a compact Hausdorff space X becomes a lattice when equipped with the usual point-wise ordering; in particular, the join and meet of $f, g \in C(X)$ are given by

$$(f \vee g)(x) = \max\{f(x), g(x)\} \quad \text{and} \quad (f \wedge g)(x) = \min\{f(x), g(x)\},$$

respectively. We will demonstrate that any surjective $T\colon C(X) \to C(Y)$ satisfying

$$\mathrm{Ran}_\pi(f \wedge g) = \mathrm{Ran}_\pi(T(f) \wedge T(g))$$

for all $f, g \in C(X)$, where $\mathrm{Ran}_\pi(\cdot)$ denotes the set of range values of maximum absolute value, induces a homeomorphism $\psi\colon Y \to X$ such that

$$T(f) = f \circ \psi$$

holds for all $f \in C(X)$ with $0 \leq f$.

On the linearity of order-isomorphisms

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A basic problem in the theory of partially ordered vector spaces is to understand when order-isomorphisms are affine. This depends in a subtle way on the geometry of the cones involved. In this talk I will discuss some recent progress on this problem, mainly based on joint with van Gaans and van Imhoff. We will introduce a new condition on the extreme rays of the cone, which ensures that all order-isomorphisms are affine. The condition is milder than existing ones and is satisfied by, for example, the cone of positive operators in the space of bounded self-adjoint operators on a Hilbert space.

Unitarily Invariant Norms on Operators and Distance Preserving Maps

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We discuss some inequalities related to some classes of unitarily invariant norms on the set of bounded linear operators between two Hilbert spaces, and the characterization of the distance preserving maps for such norms.

Schur null preservers

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Schur multiplicative and Schur null preserving maps have been initially studied on matrix spaces, these are preservers with respect to the Schur product defined on matrices. We extend those results on matrices over some function spaces. In this talk, I will first introduce two different notions of Schur null preservers, then give some characterisation results on those operators.

Linear functions preserving Green's relations over fields

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The talk is based on the joined work with A. Guterman, M. Johnson, and M. Kambites [5].

Green's relations are a number of equivalence relations and pre-orders which are defined upon any semigroup. Introduced by J. Green [1] in 1951, they encapsulate the ideal structure of the semigroup, and play a central role in almost every aspect of semigroup theory. The following are the definitions of Green's relations $\mathcal{R}, \mathcal{L}, \mathcal{J}, \mathcal{H}, \mathcal{D}$.

Definition. Let $\mathcal{M}$ be a monoid. For $a, b \in \mathcal{M}$, we say that:

(i) $a\,\mathcal{R}\,b$ if $a\mathcal{M} = b\mathcal{M}$;

(ii) $a\,\mathcal{L}\,b$ if $\mathcal{M}a = \mathcal{M}b$;

(iii) $a\,\mathcal{J}\,b$ if $\mathcal{M}a\mathcal{M} = \mathcal{M}b\mathcal{M}$;

(iv) $a\,\mathcal{H}\,b$ if $a\,\mathcal{R}\,b$ and $a\,\mathcal{L}\,b$;

(v) $a\,\mathcal{D}\,b$ if there exists $c \in \mathcal{M}\colon\, a\,\mathcal{R}\,c$ and $c\,\mathcal{L}\,b$.

In particular, these are natural relations to define upon the set of $n \times n$ matrices over any semiring, when viewed as a semigroup under matrix multiplication.

The investigation of linear transformations preserving natural functions, invariants and relations on matrices has a long history, dating back to a result of Frobenius [1] describing maps which preserve the determinant. During the past century a lot of effort has been devoted to the development of this theory, particularly, for matrices over semirings.

In 2018, motivated by recent interest in the structure of the tropical semifield, A. Guterman, M. Johnson, and M. Kambites [3] characterized bijective linear maps which preserve (or strongly preserve) each of Green's relations on the space of $n \times n$ matrices over an anti-negative semifield. The results of [3] are rather unusual, since they provided a classification for all semifields except fields. Thus, the question arises, what the corresponding maps over fields are. The aim of this talk is to answer it.

Let $\mathbb{F}$ be a field and $M_n(\mathbb{F})$ be the monoid of $n \times n$ matrices over $\mathbb{F}$. Based on a convenient description of Green's relations for $M_n(\mathbb{F})$, we present a complete classification of linear maps

$T\colon M_n(\mathbb{F}) \to M_n(\mathbb{F})$ preserving each of these relations. However, some additional assumptions are needed, namely:

- for all relations ($\mathcal{R}, \mathcal{L}, \mathcal{J}, \mathcal{H}, \mathcal{D}$), the classification was carried out under the assumption that T is bijective;
- for the relations $\mathcal{R}, \mathcal{L}, \mathcal{H}$, it was done under the assumption that the field $\mathbb{F}$ contains roots of all polynomials from $\mathbb{F}[x]$ of degree n (particularly, for algebraically closed field);
- for the relations $\mathcal{J}, \mathcal{D}$, no additional assumptions are needed.

Also, for the relation $\mathcal{H}$, it holds that non-zero $\mathcal{H}$-preservers coincide exactly with invertibility preservers, which classification is a well-known preserver problem. Over an arbitrary field, linear maps preserving invertibility have been fully described by de Seguins Pazzis [4].

Furthermore, we show some examples of non-bijective linear $\mathcal{L}$-, $\mathcal{R}$-, and $\mathcal{H}$-preservers that are not of the form mentioned in the classification. This shows the importance of the restrictions on the field or map stated above.

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The Gleason–Kahane–Żelazko theorem and automatic countinuity

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The study of outer preserving linear maps on hardy spaces H^p led us to a generalized version of Gleason–Kahane–Żelazko theorem for modules. In particular, linear functionals $T : H^p \to \mathbb{C}$ (no continuity assumption) whose kernels do not include any outer function are not frequent and should be of a very specific form which entails to automatic continuity. In this new work, we take one step further and study such results in the general framework of reproducing kernel Hilbert spaces. In a sense, this is the most general setting which include numerous special function cases as a special case, e.g., Bergman, Dirichlet, Besov, the little Bloch, and VMOA.

Maps on positive definite cones of C^*-algebras preserving the Wasserstein mean

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In the past years, we have obtained some structural results for bijective maps between positive definite cones of operator algebras which preserve some specific Kubo-Ando means (power means, geometric mean).

Recently, Bhatia, Jain, and Lim have introduced a new mean for positive definite matrices called Wasserstein mean. Its importance lies in its close connection to the Bures-Wasserstein metric.

In this talk, we give the complete description of all (continuous) isomorphisms between positive definite cones of C^*-algebras with respect to the operation of the Wasserstein mean and present a result concerning the nonexistence of nonconstant such morphisms into the positive reals. Comments on the algebraic properties of the Wasserstein mean relating associativity are also made.

Loewner's theorem for maps on operator domains

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The classical Loewner's theorem states that operator monotone functions on real intervals are described by holomorphic functions on the upper half-plane. We characterize local order isomorphisms on operator domains by biholomorphic automorphisms of the generalized upper half-plane, which is the collection of all operators with positive invertible imaginary part. We describe such maps in an explicit manner, and examine properties of local order isomorphisms. Moreover, in the finite-dimensional case, we explain that every order embedding of a matrix domain is a homeomorphic order isomorphism onto another matrix domain. This is joint work with Peter Šemrl (University of Ljubljana).

Surjective isometries on Banach algebras of Lipschitz maps taking values in a unital C^*-algebra

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Let $\mathcal{A}$ be a unital C^*-algebra. If its center is trivial, i.e. $\mathcal{A} \cap \mathcal{A}' = \mathbb{C}1$, we call it a unital factor C^*-algebra. In this talk, we consider unital surjective complex linear isometries on $\mathrm{Lip}(X, \mathcal{A})$ with $\|\cdot\|_L = \|\cdot\|_\infty + L(\cdot)$. The celebrated Kadison theorem yields that unital surjective linear isometries between unital C^*-algebras are Jordan $*$-isomorphisms. We consider surjective linear isometries on Banach algebras of continuous maps taking values in a C^*-algebra and derive extensions of the Kadison theorem.

In [O. Hatori, K. Kawamura and S. Oi, *Hermitian operators and isometries on injective tensor products of uniform algebras and C^*-algebras,* JMAA, 2019], we proved that if $\mathcal{A}_i$ is a unital factor C^*-algebra for $i = 1, 2$, every surjective linear isometry U from $C(K_1, \mathcal{A}_1)$ onto

$C(K_2, \mathcal{A}_2)$ is a weighted composition operator of the form $UF(y) = uV_y(F(\varphi(y)))$, where $\varphi : K_2 \to K_1$ is a homeomorphism, $\{V_y\}_{y \in K_2}$ is a strongly continuous family of Jordan $*$-isomorphisms from $\mathcal{A}_1$ onto $\mathcal{A}_2$, and $u \in C(K_2, \mathcal{A}_2)$ is a unitary element. A main result in this talk is a version of the Banach algebras of all Lipschitz maps of the theorem. Recently in [S. Oi, *Hermitian operators and isometries on algebras of matrix-valued Lipschitz maps,* Linear Multilinear Algebra, 2020], we gave a complete description of surjective linear isometries on $\mathrm{Lip}(X, M_n(\mathbb{C}))$, where $M_n(\mathbb{C})$ is the Banach algebra of complex matrices of order n. Hence the main result is the generalization of it.

In the course of the proof, we characterize hermitian operators on $\mathrm{Lip}(X, E)$ with $\|\cdot\|_L$ for any Banach space E. Note that similar results characterizing hermitian operators on $\mathrm{Lip}(X, E)$ with $\|\cdot\|_M = \max\{\|\cdot\|_\infty, L(\cdot)\}$ have been already obtained in [F. Botelho, J. Jamison, A. Jiménez-Vargas and M. Villegas-Vallecillos, *Hermitian operators on Lipschitz function spaces,* Studia Math., 2013].
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Free functions preserving certain partial orders of operators

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Recently free analysis has been a very active topic of study in operator and function theory. In particular free functions that preserve partial orders of operators have been studied by a number of authors, in connection with Loewner's theorem. Also operator concave and convex free functions naturally get into the picture as we study the positive definite order preserving free functions. We will go through recent results and open problems in this field, and we will cover some recent joint work with M. Gaál on real operator monotone functions.

Linear maps which are (triple) derivable or anti-derivable at a point

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A typical challenge in the setting of preservers asks whether a linear map T from a C*-algebra A into a Banach A-bimodule X behaving like a derivation (i.e. $D(ab) = D(a)b + aD(b)$) or like an anti-derivation ($D(ab) = D(b)a + bD(a)$) only on those pairs of elements (a, b) in a proper subset $\mathfrak{D} \subset A^2$ is in fact a derivation or an anti-derivation. A protagonist role is played by sets of the form $\mathfrak{D}_z := \{(a, b) \in A^2 : ab = z\}$, where z is a fixed point in A. A linear map $T : A \to X$ is said to be a *derivation* or an *anti-derivation at a point* $z \in A$ if it behaves like a derivation or like an anti-derivation on pairs $(a, b) \in \mathfrak{D}_z$. These maps are usually called *derivable or anti-derivable at* z. Let us simply observe that applying a similar method to define linear maps which are homomorphisms at zero, we find a natural link with the fruitful line of results on zero products preservers.

A recent study developed by B. Fadaee and H. Ghahramani in [3] characterizes continuous linear maps from a C*-algebra A into its bidual which are derivable at zero. A similar problem was considered by H. Ghahramani and Z. Pan for linear maps on a complex Banach algebra which is zero product determined [4]. These authors also find necessary conditions to guarantee that a continuous linear map $T : A \to A^{**}$ is anti-derivable at zero, where A is a C*-algebra, and also for linear maps on a zero product determined unital *-algebra to be anti-derivable at zero.

We have been involved in the study of those linear maps on C*-algebras which are derivations or triple derivations at zero or at the unit [2]. We shall revisit some of the main conclusions on these kind of maps from the perspective of preservers. We have further explored in [1] whether a full characterization of those (continuous) linear maps on a C*-algebra which are (*-)anti-derivable at zero can be given in pure algebraic terms. In this talk we shall present the latest advances in [1], which provide a complete solution to this problem.

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Singularity preserving maps on matrix algebras

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The talk is based on the joined work with Alexander Guterman and Artem Maksaev.

The first result on linear preservers was obtained by Ferdinand Georg Frobenius, who characterized linear maps on complex matrix algebra preserving the determinant.

Let $M_n(\mathbb{F})$ be the $n \times n$ matrix algebra over a field $\mathbb{F}$ and $\mathcal{Y}$ be a subset of $M_n(\mathbb{F})$. We say that a transformation $T\colon \mathcal{Y} \to M_n(\mathbb{F})$ is of a *standard form* if there exist non-singular matrices P, Q such that

$$T(A) = PAQ \quad \text{or} \quad T(A) = PA^TQ \quad \text{for all } A \in \mathcal{Y}. \tag{1}$$

Frobenius [1] proved that if $T\colon M_n(\mathbb{C}) \to M_n(\mathbb{C})$ is linear and preserves the determinant, i. e., $\det(T(A)) = \det(A)$ for all $A \in M_n(\mathbb{C})$, then T is of the standard form (1) with $\det(PQ) = 1$. In 1949 Jean Dieudonné [2] generalized this result for an arbitrary field $\mathbb{F}$. He replaced the determinant preserving condition by the singularity preserving condition and proved the corresponding result for a bijective map T.

In 2002 Gregor Dolinar and Peter Šemrl [3] modified the classical result of Frobenius by removing the linearity and replacing the determinant preserving condition by

$$\det(A + \lambda B) = \det(T(A) + \lambda T(B)) \quad \text{for all } A, B \in M_n(\mathbb{F}) \text{ and all } \lambda \in \mathbb{F} \tag{2}$$

for $\mathbb{F} = \mathbb{C}$. They proved that if $T\colon M_n(\mathbb{C}) \to M_n(\mathbb{C})$ is surjective and satisfies (2), then T is linear and hence is of the standard form (1) with $\det(PQ) = 1$.

Soon after that, Victor Tan and Fei Wang [4] generalized this proof for a field $\mathbb{F}$ with $|\mathbb{F}| > n$ and showed that under the condition (2) the map T is linear even without the surjectivity condition. Moreover, they revealed that if T is surjective, then only two different values of λ are required in (2). To be more precise, if $|\mathbb{F}| > n$ and $T\colon M_n(\mathbb{F}) \to M_n(\mathbb{F})$ is a surjective map satisfying

$$\det(A + \lambda_i B) = \det(T(A) + \lambda_i T(B)) \quad \text{for all } A, B \in M_n(\mathbb{F}) \text{ and } i = 1, 2,$$

where $\lambda_i \neq 0$ and $(\lambda_1/\lambda_2)^k \neq 1$ for $1 \leqslant k \leqslant n-2$, then T is of the standard form (1).

Nevertheless, this result was also further generalized by Constantin Costara [5]. Suppose $|\mathbb{F}| > n^2$ and $\lambda_0 \in \mathbb{F}$. Let $T\colon M_n(\mathbb{F}) \to M_n(\mathbb{F})$ be a surjective map satisfying (2) only for one fixed value of $\lambda = \lambda_0$: $\det(A + \lambda_0 B) = \det(T(A) + \lambda_0 T(B))$ for all $A, B \in M_n(\mathbb{F})$.

Costara obtained that if $\lambda_0 \neq -1$, then such T is of the standard form (1) with $\det(PQ) = 1$.

For $\lambda_0 = -1$, he showed that there exist $P, Q \in GL_n(\mathbb{F})$, $\det(PQ) = 1$, and $A_0 \in M_n(\mathbb{F})$ such that

$$T(A) = P(A + A_0)Q \quad \text{or} \quad T(A) = P(A + A_0)^T Q \quad \text{for all } A \in \mathcal{Y}.$$

The aim of this work is to relax the condition (2) for T. It has been revealed that if $\mathbb{F}$ is an algebraically closed field, then the conditions on determinant in the above results of Dieudonné or Tan and Wang can be replaced by less restrictive. The following result has been obtained.

Theorem. Suppose $\mathcal{Y} = GL_n(\mathbb{F})$ or $\mathcal{Y} = M_n(\mathbb{F})$, $T\colon \mathcal{Y} \to M_n(\mathbb{F})$ is a map satisfying the following conditions:

- for all $A, B \in \mathcal{Y}$ and $\lambda \in \mathbb{F}$, the singularity of $A + \lambda B$ implies the singularity of $T(A) + \lambda T(B)$;
- the image of T contains at least one non-singular matrix.

Then T is of the standard form (1).

(Note that in the theorem above $\det(PQ)$ possibly differs from 1.)

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Maps preserving absolute continuity of positive operators

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Motivated by their measure theoretic analogues, Ando introduced in [1] the notions of absolute continuity and singularity of positive operators, and proved a Lebesgue-type decomposition theorem. Since then, similar results have been proved in more general contexts [3, 4, 5, 7, 8], just to mention a few. Molnár in [6] described the structure of bijective maps on the cone of positive operators that preserve the Lebesgue decomposition in both directions. It turned out that the cone is quite rigid in the sense that these maps can be always written in the form $A \mapsto SAS^*$ with a bounded, invertible, linear- or conjugate linear operator S. A natural question arises: is it possible to weaken the preserver property, and to characterize those bijections that preserve absolute continuity only? The aim of this talk to answer this question in the affirmative.

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Isometries of Wasserstein spaces

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I will report on our systematic study of isometries of classical Wasserstein spaces with various underlying spaces. The starting point was the case of the discrete underlying space, where we found a rich family of non-surjective isometries. However, we proved isometric rigidity on the contrary, which means that every surjective isometry is governed by a permutation of the underlying space [G.-T.-V., J. Math. Anal. Appl. 480 (2019), 123435].

The next step was the description of the isometries of $\mathcal{W}_p(\mathbb{R})$ for $p \neq 2$. Here, we proved isometric rigidity and classified the non-surjective isometries for $p > 1$, as well. The study of $\mathcal{W}_p([0,1])$ led to the discovery of a mass-splitting isometry for $p = 1$, which turned out to be also a key step in giving an affirmative answer to Kloeckner's questions from 2010 concerning the existence of exotic and mass-splitting isometries on quadratic Wasserstein spaces [G.-T.-V., Trans. Amer. Math. Soc. 373 (2020), 5855-5883].

The most recent work of ours concerns Wasserstein-Hilbert spaces. We extended Kloeckner's result on the quadratic case to the infinite-dimensional setting and proved isometric rigidity for the non-quadratic cases. The main tool we introduced to study the non-quadratic cases is the Wasserstein potential of measures. As a byproduct of our results, we showed that $\mathcal{W}_p(X)$ is isometrically rigid for every Polish space X and parameter $0 < p < 1$ [G.-T.-V., arXiv:2102.02037 (2021)].

Positivity preservers forbidden to operate on diagonal blocks

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The question of which functions acting entrywise preserve positive semidefiniteness has a long history, beginning with the Schur product theorem [*Crelle* 1911], which implies that absolutely monotonic functions (i.e., power series with nonnegative coefficients) preserve positivity on matrices of all dimensions. A famous result of Schoenberg and of Rudin [*Duke Math. J.* 1942, 1959] shows the converse: there are no other such functions.

Motivated by modern applications, Guillot and Rajaratnam [*Trans. Amer. Math. Soc.* 2015] classified the entrywise positivity preservers in all dimensions, which act only on the off-diagonal entries. These two results are at "opposite ends", and in both cases the preservers have to be absolutely monotonic.

We complete the classification of positivity preservers that act entrywise except on specified "diagonal/principal blocks", in every case other than the two above. (In fact we achieve this in a more general framework.) This yields the first examples of dimension-free entrywise positivity preservers – with certain forbidden principal blocks – that are not absolutely monotonic.

Minisymposium

Topological Methods in Differential Equations (MS-13)

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- Existence results of fourth order equations with perturbed two-point boundary conditions, *Alberto Cabada*
- About coupled gradient-type quasilinear elliptic systems with supercritical growth, *Anna Maria Candela*
- On the stability of the oscillations for some singular models, *Jose Angel Cid*
- Multiple oscillating BV-solutions for a mean-curvature Neumann problem, *Francesca Colasuonno*
- Traveling waves for advection-reaction-diffusion equations with negative diffusivity, *Andrea Corli*
- A Topological Approach to Nonlocal Differential Equations with Convolution Coefficients, *Christopher Goodrich*
- Quasilinear conservation laws with discontinuous flux as singular limit of semilinear parabolic equations, *Graziano Guerra*
- Multiple solutions for the fractional p-Laplacian via degree theory, *Antonio Iannizzotto*
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- Flow invariance of closed sets, *Wojciech Kryszewski*
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- Periodic solutions to a forced Kepler problem in the plane, *Duccio Papini*
- Existence of radial bounded solutions for some quasilinear elliptic equation in $\mathbb{R}^N$, *Addolorata Salvatore*
- Solutions to problem driven by A-Laplacian operator, *Calogero Vetro*
- Complex dynamics in periodically perturbed Duffing equations with singularities, *Fabio Zanolin*
- An averaging method for a semilinear equation, *Mirosława Zima*

Existence results of fourth order equations with perturbed two-point boundary conditions

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In this talk we establish the existence and multiplicity of positive solutions for a fourth-order boundary value problem. Integral perturbations of some kind of two-point boundary conditions are considered. After the construction of a Green's function and the study of its constant sign, it is defined a positive cone, where to apply the Krasnoselskii compression/expansion and Leggett-Williams fixed point theorems in cones. A generalization for a higher order case is also considered. Some particular examples are given.

About coupled gradient-type quasilinear elliptic systems with supercritical growth

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The aim of this talk is pointing out some recent results on the coupled gradient–type quasilinear elliptic system

$$(P)\quad \begin{cases} -\mathrm{div}(A(x,u)|\nabla u|^{p_1-2}\nabla u) + \frac{1}{p_1}A_u(x,u)|\nabla u|^{p_1} = G_u(x,u,v) & \text{in } \Omega, \\ -\mathrm{div}(B(x,v)|\nabla v|^{p_2-2}\nabla v) + \frac{1}{p_2}B_v(x,v)|\nabla v|^{p_2} = G_v(x,u,v) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded domain, $p_1, p_2 > 1$ and $A(x,u)$, $B(x,v)$ are $\mathcal{C}^1$–Carathéodory functions on $\Omega \times \mathbb{R}$ with partial derivatives $A_u(x,u)$, respectively $B_v(x,v)$. Here, $(G_u(x,u,v), G_v(x,u,v)) = \nabla G(x,u,v)$ where $G(x,u,v)$ is a given function on $\Omega \times \mathbb{R}^2$.

Even if the coefficients $A(x,u)$ and $B(x,v)$ make the variational approach more difficult, suitable hypotheses allow us to prove that the weak bounded solutions of problem (P) are critical points of the functional

$$\mathcal{J}(u,v) = \frac{1}{p_1}\int_\Omega A(x,u)|\nabla u|^{p_1}dx + \frac{1}{p_2}\int_\Omega B(x,v)|\nabla v|^{p_2}dx - \int_\Omega G(x,u,v)dx$$

in the Banach space $X = X_1 \times X_2$, where $X_i = W_0^{1,p_i}(\Omega) \cap L^\infty(\Omega)$ for $i = 1, 2$.

Unluckily, classical variational theorems cannot apply to $\mathcal{J}$ in X but, following an approach which exploits the interaction between $\|\cdot\|_X$ and the standard norm on $W_0^{1,p_1}(\Omega) \times W_0^{1,p_2}(\Omega)$, the existence of critical points of $\mathcal{J}$ can be proved by means of a generalized Mountain Pass Theorem.

In particular, if the coefficients $A(x,u)$ and $B(x,v)$ grow in the "right" way then $G(x,u,v)$ can have a suitable supercritical growth and if $\mathcal{J}$ is even then (P) has infinitely many weak bounded solutions.

These results are part of joint works with Caterina Sportelli and Addolorata Salvatore.

On the stability of the oscillations for some singular models

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We present some sufficient conditions for the existence and Lyapunov stability of periodic solutions for a kind of equations allowing singularities. Our approach relies on the third order approximation and the averaging method. We will provide applications to some singular models like the Brillouin beam focusing equation.

The talk is based on the paper *Wang, F., Cid, J. Á., Li, S. and Zima, M., Lyapunov stability of periodic solutions of Brillouin type equations, Appl. Math. Lett. 101 (2020).*

Multiple oscillating BV-solutions for a mean-curvature Neumann problem

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Coauthors: Alberto Boscaggin, Colette De Coster

In this talk, I will focus on a one-dimensional mean-curvature problem under Neumann boundary conditions. I will describe how we can find multiple positive oscillating BV-solutions, using an approximation of the mean curvature operator and the shooting method. This is joint work with Alberto Boscaggin and Colette De Coster.

Traveling waves for advection-reaction-diffusion equations with negative diffusivity

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Coauthors: Diego Berti, Luisa Malaguti

In the talk I shall present some recent results, motivated by the modeling of collective movements, about traveling-wave solutions for advection-reaction-diffusion equations

$$u_t + f(u)_x = (D(u)u_x)_x + g(u),$$

with $g(0) = g(1) = 0$ and $u \in [0, 1]$. The main issue is that the diffusivity D, that may vanish at 0 or 1, can be *negative*. More precisely, we first deal with the case when $g > 0$ in $(0, 1)$ and D changes sign once, either from the positive to the negative or conversely. These results are extended to a finite number of sign changes of D. Then, we also admit the source term g to change sign.

In every case, the presence of the convective term f leads to new behaviors of the profiles with respect to the pure reaction-diffusion case.

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A Topological Approach to Nonlocal Differential Equations with Convolution Coefficients

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I will consider the nonlocal problem

$$-A\Big(\big(b*(g\circ u)\big)(1)\Big)u''(t)=\lambda f\big(t,u(t)\big),\, t\in(0,1),$$

where $*$ denotes a finite convolution and b and g are given functions. By means of a nonstandard cone, together with a specially tailored open set, I will demonstrate the existence of at least one positive solution to this class of problem under given boundary conditions. It will be shown that this approach improves results which rely on a more standard cone.

Quasilinear conservation laws with discontinuous flux as singular limit of semilinear parabolic equations

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Coauthors: Alberto Bressan, Wen Shen

Scalar conservation laws with a discontinuous flux with respect to the space variable arise in many applications where the conservation laws describe physical models in rough non homogeneous media. For example, traffic flows with rough road conditions and polymer flooding in porous media. We are interested in solutions to this type of equations obtained by approximating them with solutions to semilinear parabolic equations. We show that the Crandall Liggett theory [3] of nonlinear semigroups provides a very elegant framework for proving existence and uniqueness of solutions to the Cauchy problem for the semilinear parabolic equation

$$u_t+f\left(x,u\right)_x=\varepsilon u_{xx} \tag{1}$$

where the function f is only $\mathbf{L}^\infty$ regular with respect to x. Then, when the flux has a single discontinuity at $x=0$, we show that the Brezis & Pazy convergence theorem [2] can be used to show existence and uniqueness of the singular limit, as $\varepsilon\to 0$, of solutions to (1) [4]. The limits obtained in this way are the unique vanishing viscosity solutions to the quasilinear conservation law (1) with $\varepsilon=0$.

The convergence result can be generalised to fluxes less regular than BV [1] but counterexamples show that it is not true for the simple $\mathbf{L}^\infty$ regularity.

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Multiple solutions for the fractional p-Laplacian via degree theory

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Coauthor: Silvia Frassu

We study a Dirichlet type problem for a nonlinear, nonlocal equation driven by the degenerate fractional p-Laplacian with a jumping reaction, crossing the first eigenvalue. By means of Browder's degree for $(S)_+$-mappings and fractional spectral theory (in particular, monotonicity of weighted eigenvalues with respect to the weight), we prove existence of two nontrivial solutions.

Compactness properties of operator of translation along trajectories in evolution equations

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Nicolaus Copernicus University in Toruń, Poland

Our aim is to present some results about compactness properties of operator of translation along trajectories (which is also known as Poincaré operator) which is associated with some evolution equation. Fixed points of this operator are periodic solutions of connected evolution equation. In order to apply some kind of topological degree we study just compactness properties of operator of translation. We consider two types of evolution equations: first is linked with parabolic problems and second with hyperbolic problems. In case of parabolic equations we discuss results which come from A. Ćwiszewski and R. Łukasiak ([1], [2]). Next we present approach (but not yet with specific results) to hyperbolic problems which is a part of collaboration with A. Ćwiszewski. Our method uses so-called „tail estimates" (which were firstly introduced by B. Wang in [4]) and is based on work of D. Fall and Y. You (see [3], but note that they looked for global attractors).

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Flow invariance of closed sets

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When studying the existence of solutions to systems of parabolic equations of the form $u_t + Lu = f(x, u)$, $u \in \mathbb{R}^M$, $x \in \Omega \subset \mathbb{R}^N$, subject to boundary conditions, where L is an elliptic (vector valued) differential operator and $f : \Omega \times \mathbb{R}^M \to \mathbb{R}^M$ is a continuous map, such that $u(x) \in C$ for a.a. $x \in \Omega$, where $C \subset \mathbb{R}^M$ is a given closed set of state constraints, an important hypotheses concern the so-called resolvent invariance of K, i.e. $(I + \lambda\mathbf{A})^{-1}(\mathbf{K}) \subset \mathbf{K}$ for sufficiently small $\lambda > 0$, where $\mathbf{A}$ is the sectorial operator corresponding to L, $\mathbf{K} := \{u \in L^2(\Omega, \mathbb{R}^M) \mid u(x) \in C, \text{for a.a. } x \in \Omega\}$ (being equivalent to the invariance of $\mathbf{K}$ with respect to the semigroup generated by $-\mathbf{A}$) and the so-called tangency of the nonlinear perturbation f. We will discuss the sufficient and necessary conditions for the invariance stated in terms of the coefficients of the operator L as well as in the language of Dirichlet bilinear form associated to L. This topic is strictly related to the study of the viability and invariance questions for partial differential equations.

This is a joint work with Jakub Siemianowski, Grzegorz Gabor and Aleksander Ćwiszewski.

Positive solutions for systems of Riemann-Liouville fractional differential equations with coupled nonlocal boundary conditions

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Coauthor: Alexandru Tudorache

We investigate the existence and nonexistence of positive solutions for a system of Riemann-Liouville fractional differential equations with p-Laplacian operators, nonnegative nonlinearities and positive parameters, subject to coupled nonlocal boundary conditions which contain various fractional derivatives and Riemann-Stieltjes integrals. In the proof of the main existence results we use the Guo-Krasnosel'skii fixed point theorem.

Lower and upper solutions for even order boundary value problems

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We will prove the existence of solutions of nonlinear boundary value problems of arbitrary even order using the lower and upper solutions method. In particular, we will show that the existence of a pair of lower and upper solutions of a considered problem could imply the existence of solution of another one with different boundary conditions.

L^p-exact controllability of partial differential equations with nonlocal terms

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Coauthors: Stefania Perrotta, Valentina Taddei

This talk deals with the exact controllability of some classes of partial differential equations by means of linear controls. The discussion takes place in infinite dimensional state spaces since these equations are considered in their abstract formulation as semilinear equations. The linear parts are densely defined and generate strongly continuous semigroups. The nonlinear terms may also include a nonlocal part. The solutions satisfy suitable nonlocal constraints, which are possibly nonlinear. Several contributions already appeared on this topic and the novelty of our investigation lies in the introduction of an approximation solvability method which involves a sequence of controllability problems in finite-dimensional spaces. This approach is made possible by means of a Schauder basis in the Banach state space. The results exploit topological methods. The exact controllability of nonlocal solutions can then be proved, with controls in arbitrary L^p spaces, $1 < p < \infty$. The results apply, in particular, to the transport equation in arbitrary Euclidean spaces and to the nonlinear wave equation with possibly localized controls.

Boundary value problems associated with singular strongly nonlinear equations with functional terms

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Coauthors: Stefano Biagi, Alessandro Calamai, Francesca Papalini

The talk concerns boundary value problems associated with singular, strongly nonlinear differential equations with functional terms of the type

$$\begin{cases} (\Phi(k(t)x'(t)))' + f(t, G_x(t))h(t, x'(t)) = 0 \ , \ t \in [a, b] \\ x(a) = H_a[x], \ x(b) = H_b[x]. \end{cases}$$

The nonlinear differential operator Φ is a general strictly increasing homeomorphism; the coefficient k is non-negative and it may vanish on a set of null measure. Moreover, the differential equation depends on a general functional term G_x. By means of a fixed point argument, we provide sufficient conditions for the existence of solutions, in a suitable weak sense, satisfying general boundary conditions expressed by means of a functional term H.

Second order necessary conditions for PDEs optimal control problems with state constraints

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This talk is devoted to second order necessary optimality conditions for optimal control problems under pure state constraints together with end point constraints, involving PDEs. Using tools of second order variational analysis, we derive necessary optimality conditions in the form of a maximum principle and a second order variational inequality. We further propose sufficient conditions guaranteeing normality of the maximum principle. Applications to heat and wave models will be proposed.

The results have been obtained jointly with Hélène Frankowska and Marco Mazzola

A fixed point approach for decaying solutions of difference equations

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A boundary value problem associated to the difference equation with advanced argument

$$\Delta\big(a_n\Phi(\Delta x_n)\big) + b_n\Phi(x_{n+p}) = 0, \quad n \geq 1 \tag{$*$}$$

is presented, where $\Phi(u) = |u|^\alpha \text{sgn}\, u$, $\alpha > 0, p$ is a positive integer and the sequences a, b, are positive. We deal with a particular type of decaying solutions of ($*$), the so-called intermediate solutions, that is solutions x of ($*$) such that $x_n > 0, \Delta x_n < 0$ for large n and

$$\lim_n x_n = 0, \;\; \lim_n x_n^{[1]} = a_n\Phi(\Delta x_n) = -\infty,$$

where $x^{[1]}$ is called the quasidifference of x. In particular, we prove the existence of these type of solutions for ($*$) by reducing it to a suitable boundary value problem associated to a difference equation without deviating argument. Our approach is based on a fixed point result for difference equations, which originates from existing ones stated in the continuous case, but take into account some peculiarities of the discrete case.

A Neumann p-Laplacian problem on metric spaces

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We use a variational approach to study existence and regularity of solutions for a Neumann p-Laplacian problem with a reaction term on metric spaces equipped with a doubling measure and supporting a Poincare' inequality. Trace theorems for functions with bounded variation are applied in the definition of the variational functional and minimizers are shown to satisfy De Giorgi type conditions.

Multiplicity of finite energy solutions for singular elliptic equations

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Our main goal is to investigate the following class of nonlinear elliptic equations with subquadratic growth with respect to the gradient

$$div(a(||x||)\nabla u(x)) + h(x, u(x), \nabla u(x)) = 0 \text{ in } \Omega_R$$

where $\Omega_R = \{x \in \mathbb{R}^n, ||x|| > R\}$, $n > 2$ and $h(x, u, z) = f(x, u) - b(x, u)||z||^\beta + g(x)x \cdot z$ with $\beta \in (0, 2\rangle$. We consider the case when b may be singular at zero and describe conditions guaranteeing the existence of a large number of positive solutions such that for a certain $A > 0$, $u(x) \le A||x||^{2-n}$ in Ω_{R_1}, where R_1 is sufficiently large. The rate of decay of ∇u is also discussed. We present the approach based on the subsolution and supersolution method for bounded subdomains and a certain convergence procedure. These results cover both sublinear and superlinear f.

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Periodic solutions to a forced Kepler problem in the plane

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Coauthors: Alberto Boscaggin, Walter Dambrosio

We investigate the following forced Kepler problem in the plane:

$$\ddot{x} = -\frac{x}{|x|^3} + \nabla_x U(t, x), \qquad x \in \mathbb{R}^2 \setminus \{(0, 0)\},$$

where $U(t, x)$ is T-periodic in the first variable and satisfies $U(t, x) = \mathcal{O}(|x|^\alpha)$ for some $\alpha \in (0, 2)$ as $|x| \to \infty$. We look for a T-periodic solution which minimimizes the corresponding action functional on a space of loops which are not null-homotopic in the punctured plane.

On one hand, we do not impose further symmetry conditions on the perturbation's potential U. On the other, the solution we find is generalised, according to the definition given in the paper [Boscaggin, Ortega, Zhao, *Periodic solutions and regularization of a Kepler problem with time-dependent perturbation*, Trans. Amer. Math. Soc. **372** (2018), 677–703]. In particular, such solution may have a finite number of collisions with the origin in each period, while its energy and bouncing directions behave in a regular way at each collision time.

Existence of radial bounded solutions for some quasilinear elliptic equation in $\mathbb{R}^N$

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We study the quasilinear equation

$$(P) \quad -\operatorname{div}(A(x,u)|\nabla u|^{p-2}\nabla u) + \frac{1}{p}\, A_t(x,u)|\nabla u|^p + |u|^{p-2}u \;=\; g(x,u) \quad \text{in } \mathbb{R}^N,$$

with $N \geq 3$, $p > 1$, where $A(x,t)$, $A_t(x,t) = \frac{\partial A}{\partial t}(x,t)$ and $g(x,t)$ are Carathéodory functions on $\mathbb{R}^N \times \mathbb{R}$.

Under suitable assumptions on $A(x,t)$ and $g(x,t)$ the problem has a good variational structure, i.e. the weak bounded solutions of problem (P) are critical points of the C^1 functional

$$\mathcal{J}(u) \;=\; \frac{1}{p}\int_{\mathbb{R}^N} A(x,u)|\nabla u|^p dx + \frac{1}{p}\int_{\mathbb{R}^N} |u|^p dx - \int_{\mathbb{R}^N} G(x,u)dx,$$

on the Banach space $X = W^{1,p}(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, with $G(x,t) = \int_0^t g(x,s)ds$.

In order to overcome the lack of compactness, we assume that the problem has radial symmetry, then we look for critical points of $\mathcal{J}$ restricted to X_r, subspace of the radial functions in X.

Following an approach which exploits the interaction between $\|\cdot\|_X$ and the norm on $W^{1,p}(\mathbb{R}^N)$, we prove the existence of at least one weak bounded radial solution of (P) by applying a generalized version of the Ambrosetti–Rabinowitz Mountain Pass Theorem.

The result is contained in a joint work with Anna Maria Candela.

Solutions to problem driven by A-Laplacian operator

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We give sufficient conditions for existence of weak solutions to quasilinear elliptic Dirichlet problem driven by A-Laplace operator in a bounded domain Ω. The techniques, based on a variant of the symmetric mountain pass theorem, exploit variational methods. This study extends and complements various qualitative results for some standard cases of Laplace-type operators. We also provide information about the asymptotic behavior of the solutions as a suitable parameter goes to 0^+.

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Complex dynamics in periodically perturbed Duffing equations with singularities

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We present some examples of periodically perturbed Duffing equations $x'' + g(x) = p(t)$, where $g : (0, +\infty) \to \mathbb{R}$ has a singularity at the origin: $g(0^+) = -\infty$. We prove the existence of infinitely many subharmonic solutions, as well as the presence of chaotic-like dynamics, as a consequence of a topological horseshoe type geometry.

An averaging method for a semilinear equation

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We discuss an abstract averaging method for a semilinear equation

$$Lx = \varepsilon N(x, \varepsilon),$$

where L is a Fredholm mapping of index zero, and N is a nonlinear operator. The applicability of the method for periodic solutions to n-th order differential equation and nonlocal nonlinear boundary problems is discussed. The talk is based on a joint paper
J.A. Cid, J. Mawhin, M. Zima, An abstract averaging method with applications to differential equations, J. Differential Equations 274 (2021), 231–250.

Minisymposium

VARIATIONAL AND EVOLUTIONARY MODELS INVOLVING LOCAL/NONLOCAL INTERACTIONS (MS-58)

Organized by Sara Daneri, *Gran Sasso Science Institute, Italy*
Coorganized by Matteo Novaga, *University of Pisa, Italy*

- Minimal planar N-partitions for large N, *Giovanni Alberti*
- Phase Separation in Nonlocal Multispecies Models, *Martin Burger*
- One dimensional multi-agent optimal control and Mean Field limits with density constraints, *Annalisa Cesaroni*
- The antiferromagnetic XY model, *Marco Cicalese*
- Deterministic particle approximation of aggregation-diffusion equations on unbounded domains, *Sara Daneri*
- Deterministic many-particle limit for a system of interaction equations driven by Newtonian potentials, *Marco Di Francesco*
- Minimisers of a fractional seminorm and nonlocal minimal surfaces, *Serena Dipierro*
- Striped patterns for generalized antiferromagnetic functionals with power law kernels of exponent smaller than $d+2$, *Alicja Kerschbaum*
- Gamma–limit for zigzag walls, *Hans Knüpfer*
- Stability of nonlocal geometric evolutions, *Matteo Novaga*
- Torus-like solutions for the Landau-de Gennes model, *Adriano Pisante*
- Optimal design spectral problems with repulsion, *Berardo Ruffini*
- Pattern formation in local/non-local models interaction functionals, *Eris Runa*
- Convexity properties of the isoperimetric profile, *Giorgio Saracco*
- Stripe formation in Ising models with competing interactions, *Robert Seiringer*
- Domain walls in thin ferromagnetic strips, *Valeriy Slastikov*
- Long time behaviour of discrete volume preserving mean curvature flows, *Emanuele Spadaro*

Minimal planar N-partitions for large N

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By minimal N-partition of a given planar domain E we mean a partition which consists of N sets (cells) with equal area, that minimize the total perimeter (that is, the length of the union of the boundaries of all cells).
T. C. Hales proved in 2001 that if E is a flat 2-dimensional torus, then a regular hexagonal N-partition (if there is any) is minimal. It is then interesting to understand what happens for a planar domain E that does not admit a regular hexagonal N-partition; in particular the following questions naturally arise: Are the cells asymptotically hexagonal as N tends to infinity, and to which extent the partition looks locally hexagonal? Is the partition rigid, in the sense that the orientation of the cells is (essentially) the same through the domain?
In this talk I will describe some results obtained in these directions together with Marco Caroccia (Politecnico di Milano) and Giacomo Del Nin (University of Warwick).

Phase Separation in Nonlocal Multispecies Models

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In this talk we discuss some nonlocal variational problems arising as macroscopic steady states of some many particle systems with different species. We will demonstrate the the emergence of phase separation effects, explained from relations to Cahn-Hilliard Systems, and network formation, whose theoretical understanding is still rather open. We will also comment on corresponding evolution equations formulated in terms gradient flows, e.g. in Wasserstein spaces.

One dimensional multi-agent optimal control and Mean Field limits with density constraints

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In this talk I will consider a deterministic system (in one dimension) of many evolving interacting agents with constraints on the reciprocal distance between agents, in which each agent chooses its speed in order to minimize an energy depending on the position of the other agents through an aggregative potential (given in term of an interaction kernel and a coercive function). I will focus on periodic (in time) patterns of this model, discussing their qualitative properties, and its macroscopic mean-field limit as the number of agents tends to infinity. The talk is based on joint works with Marco Cirant (Padova).

The antiferromagnetic XY model

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Technical University of Munich, Germany

We introduce the antiferromagnetic XY system on the triangular lattice, a spin model driven by an energy functional that favours anti-alignment on each pair of interacting spins. We start recalling the main results concerning the variational discrete-to-continuum analysis at the surface scaling at which chirality transitions take place. Then, we focus on the vortex scaling: we introduce a notion of discrete vorticity and explain how to gain compactness for this order parameter and how to prove a Gamma-limit result of the energy functionals as the lattice spacing goes to zero.

Joint work with A. Bach, L. Kreutz and G. Orlando.

Deterministic particle approximation of aggregation-diffusion equations on unbounded domains

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Coauthors: Eris Runa, Emanuela Radici

We consider a one-dimensional aggregation-diffusion equation, which is the gradient flow in the Wasserstein space of a functional with competing attractive-repulsive interactions. We prove that the fully deterministic particle approximations with piecewise constant densities introduced by Di Francesco and Rosini starting from general bounded initial densities converge strongly in L^1 to bounded weak solutions of the PDE. In particular, the result is achieved in unbounded domains and for arbitrary nonnegative bounded initial densities, thus extending the results by Gosse-Toscani and Matthes-Osberger (in which a no-vacuum condition is required) and giving an alternative approach to the one proposed by Carrillo-Craig-Patacchini in the one-dimensional case, including also subquadratic and superquadratic diffusions.

Deterministic many-particle limit for a system of interaction equations driven by Newtonian potentials

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We consider a discrete particle system of two species coupled through nonlocal interactions driven by the one-dimensional Newtonian potential, with repulsive self-interaction and attractive cross-interaction. After providing a suitable existence theory in a finite-dimensional framework, we explore the behaviour of the particle system in case of collisions and analyse the behaviour of the solutions with initial data featuring particle clusters. Subsequently, we prove that the empirical measure associated to the particle system converges to the unique 2-Wasserstein gradient flow solution of a system of two partial differential equations (PDEs) with nonlocal interaction terms in a proper measure sense. The latter result uses uniform estimates of the Lm-norms of a piecewise constant reconstruction of the density using the particle trajectories. The results are a joint work with A. Esposito and M. Schmidtchen.

Minimisers of a fractional seminorm and nonlocal minimal surfaces

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The recent literature has intensively studied two classes of nonlocal variational problems, namely the ones related to the minimisation of energy functionals that act on functions in suitable Sobolev-Gagliardo spaces, and the ones related to the minimisation of fractional perimeters that act on measurable sets of the Euclidean space. In this talk, we relate these two types of variational problems. In particular, we investigate the connection between the nonlocal minimal surfaces and the minimisers of a Gagliardo seminorm, showing that a function is a minimiser for the fractional seminorm if and only if its level sets are minimisers for the fractional perimeter, and that the characteristic function of a nonlocal minimal surface is a minimiser for the fractional seminorm. We also discuss an existence result for minimisers of the fractional seminorm, an explicit non-uniqueness example for nonlocal minimal surfaces, and a Yin-Yang result describing the full and void patterns of nonlocal minimal surfaces. This is a joint work with Claudia Bucur, Luca Lombardini and Enrico Valdinoci.

Striped patterns for generalized antiferromagnetic functionals with power law kernels of exponent smaller than $d + 2$

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In this talk I will consider a class of continuous sharp interface generalized antiferromagnetic models previously studied by Daneri, Goldman and Runa. The functional consists of a perimeter term (retaining discrete symmetry) and a repulsive nonlocal term with a power law kernel. In a suitable regime the two terms enter in competition and symmetry breaking with formation of periodic striped patterns is expected to occur. We will show that the results obtained by Daneri and Runa showing striped pattern formation for power law kernels with exponents $p \geq d + 2$ can be extended to power law kernels within a range of exponents strictly smaller than $d + 2$ and strictly larger than $d + 1$, being d the dimension of the underlying space. Notice that the exponent $p = d + 1$ corresponds to an anisotropic version (retaining discrete symmetry) of the model for pattern formation in thin magnetic films.

Gamma–limit for zigzag walls

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Ferromagnets typically exhibit the formation of magnetic domains with uniform magnetization separated by thin transition layers. The Zigazag wall is one type of such transition layers which particularly appears in thin ferromagnetic films. In order to investigate this transition layer, we consider a sample in the form a thin strip and enforce a transition layer by suitable boundary

conditions on m. The associated thin–film ferromagnetic energy is

$$E_\varepsilon[m] \ = \ \frac{\varepsilon}{2}\|\nabla m\|^2_{L^2} + \frac{1}{2\varepsilon}\|m\cdot e_2\|^2_{L^2} + \frac{\pi\lambda}{2|\ln\varepsilon|}\|\nabla\cdot(m-M)\|^2_{\dot H^{-\frac{1}{2}}},$$

where M is an arbitrary fixed background field to ensure global neutrality of magnetic charges. In the macroscopic limit $\varepsilon \to 0$ we show that the energy Γ–converges to a limit energy where jump discontinuities of the magnetization are penalized anisotropically. In particular, in the subcritical regime $\lambda \le 1$ one–dimensional charged domain walls are favorable, in the supercritical regime $\lambda > 1$ the limit model allows for zigzaging two–dimensional domain walls.

Stability of nonlocal geometric evolutions

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We introduce a notion of uniform convergence for local and nonlocal curvatures, and we propose an abstract method to prove the convergence of the corresponding geometric flows, within the level set formulation. We apply such a general method to characterize the limits of fractional mean curvature flows and Riesz curvature flows.

Torus-like solutions for the Landau-de Gennes model

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We study global minimizers of a continuum Landau-De Gennes energy functional for nematic liquid crystals in three-dimensional domains, possibly in a restricted class of axisymmetric configurations. Assuming smooth and uniaxial (e.g. homeotropic) boundary conditions and a corresponding physically relevant norm constraint (Lyuksyutov constraint) in the interior, we discuss partial regularity of the minimizers away from a possible finite set of interior singularities lying on the symmetry axis or even full regularity when no symmetry is imposed. As a consequence, we discuss boundary data which yield as minimizers smooth configuration with maximally biaxial set carrying nontrivial topology. In the axially symmetric case we show how singular (split) solutions or smooth (torus) solutions (or even both) for the Euler-Lagrange equations do appear for boundary data and/or domains which are smooth deformation of the radial hedgehog in a nematic droplet.

Optimal design spectral problems with repulsion

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In the short talk I will quickly review the state of the art of some variational problems based on physical models (the Rayleigh liquid drop model, the Gamow liquid drop and the reduced Hartree equation). Then I will focus on recent results about an optimal design problem related to the latter of these models. The talk will take into consideration collaborations with M. Goldman, C. B. Muratov, M. Novaga and D. Mazzoleni.

Pattern formation in local/non-local models interaction functionals

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In this talk I will review some recent results on the one-dimensionality of the minimizers of a family of continuous local/nonlocal interaction func- tionals in general dimension. Such functionals have a local term, typically a perimeter term or its Modica-Mortola approximation, which penalizes inter- faces, and a nonlocal term favouring oscillations which are high in frequency and in amplitude. The competition between the two terms is expected by experiments and simulations to give rise to periodic patterns at equilibrium. Functionals of this type are used to model pattern formation, either in mate- rial science or in biology. One of the main difficulties in proving the emergence of such regular structures, together with nonlocality, is due to the fact that the functionals retain more symmetries (in this case symmetry with respect to permutation of coordinates) than the minimizers. We will present new techniques and results showing that for two classes of functionals (used to model generalized anti-ferromagnetic systems, respectively colloidal suspensions), both in sharp interface and in diffuse interface models, minimizers are (in general dimension) one-dimensional and periodic.

Convexity properties of the isoperimetric profile

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Coauthor: Gian Paolo Leonardi

Given an open, bounded set Ω we consider the isoperimetric profile $\mathcal{J}$ that to each volume $V \in [0, |\Omega|]$ associates the least perimeter $P(E)$ among Borel subsets E of Ω needed to enclose the given volume. We shall prove that for a wide class of planar sets, which encompasses convex sets, there exists a threshold $\bar{V}$ such that $\mathcal{J}$ is concave below it and convex above it. Moreover, $\mathcal{J}^2$ is globally convex. In order to prove these properties, a full characterization of the isoperimetric sets will be provided. Some comments on the n-dimensional case will be given.

Stripe formation in Ising models with competing interactions

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We consider Ising models in two and three dimensions, with short range ferromagnetic and long range, power-law decaying, antiferromagnetic interactions. The competition between these two kinds of interactions induces the system to form domains of minus spins in a background of plus spins, or vice versa. If the decay exponent of the long range interaction is large enough, this happens if the ratio J between the strength of the ferromagnetic and antiferromagnetic interactions is smaller than a critical value J_c, beyond which the ground state is homogeneous. We give a characterization of the infinite volume ground states of the system, for J in a left neighborhood of J_c. In particular, we prove that the quasi-one-dimensional states consisting of infinite stripes ($d = 2$) or slabs ($d = 3$), all of the same optimal width and orientation, and alternating magnetization, are infinite volume ground states. Our proof is based on localization bounds combined with reflection positivity. (Joint work with Alessandro Giuliani.)

Domain walls in thin ferromagnetic strips

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Coauthors: Matteo Novaga, Cyrill Muratov, Massimiliano Morini

We present a characterization of the domain wall solutions arising as minimizers of an energy functional obtained in a suitable asymptotic regime of micromagnetics for infinitely long thin film ferromagnetic strips in which the magnetization is forced to lie in the film plane. For the considered energy, we provide existence, uniqueness, monotonicity, and symmetry of the magnetization profiles in the form of 180 and 360 degree walls. We also demonstrate how this energy arises as a Gamma-limit of the reduced two-dimensional thin film micromagnetic energy that captures the non-local effects associated with the stray field.

Long time behaviour of discrete volume preserving mean curvature flows

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Coauthors: Massimiliano Morini, Marcello Ponsiglione

Volume preserving mean-curvature flow is a model for coarsening phenomena in physical systems exhibiting a combination of local effects driven by curvature and nonlocal ones driven by the volume constraint. In this talk I will analyse the Euler implicit scheme for the volume preserving mean curvature flow, following the scheme introduced by Almgren-Taylor-Wang and Luckhaus-Sturzenhecker for the flat flows, and I will show the exponential convergence of the scheme to a finite union of disjoint balls with equal volume for any bounded initial set with finite perimeter.

Minisymposia in

Combinatorics and Discrete Mathematics

- Algorithmic Graph Theory (MS-54)
- Applied Combinatorial and Geometric Topology (MS-34)
- Combinatorial Designs (MS-16)
- Configurations (MS-81)
- Extremal and Probabilistic Combinatorics (MS-20)
- Graph polynomials (MS-62)
- Graphs and Groups, Geometries and GAP - G2G2 (MS-7)
- Graphs, Polynomials, Surfaces, and Knots (MS-49)
- Groups, Graphs and Networks (MS-75)
- Spectral Graph Theory (MS-46)
- Symmetry of Graphs, Maps and Polytopes (MS-9)

Minisymposium

ALGORITHMIC GRAPH THEORY (MS-54)

Organized by Nicolas Trotignon, *CNRS, France*
Coorganized by
Martin Milanič, *Univerza na Primorskem, Slovenija*
Daniel Paulusma, *Durham University, United Kingdom*
Sandi Klavžar, *University of Ljubljana, Slovenija*

- On Efficient Domination for H-free bipartite graphs, *Andreas Brandstaedt*
- Domination number of graphs with fixed minimum degree, *Csilla Bujtás*
- Extending Thomason's Algorithm, *Kathie Cameron*
- Clique-Width: Harnessing the Power of Atoms, *Konrad K. Dabrowski*
- Colourful components in k-caterpillars and planar graphs, *Clément Dallard*
- Firebreaking and Firefighting, *Jess Enright*
- Graphs with two moplexes are more than perfect, *Meike Hatzel*
- Computing Weighted Subset Transversals in H-free Graph, *Matthew Johnson*
- Interference-free Walks in Time: Temporally Disjoint Paths, *Nina Klobas*
- Shifting any path to an avoidable one, *Matjaž Krnc*
- Model-checking in multilayer graphs, *Kitty Meeks*
- Beyond treewidth: the tree-independence number, *Martin Milanič*
- Width parameters and graph classes: the case of mim-width, *Andrea Munaro*
- Coloring $(4K_1, C_4, C_6)$-free graphs, *Irena Penev*
- Circularly compatible ones, D-circularity, and proper circular-arc bigraphs, *Martín Safe*
- On the treewidth of even-hole-free graphs, *Ni Luh Dewi Sintiari*
- On 12-regular nut graphs, *Riste Škrekovski*
- Treewidth versus clique number: a complete dichotomy for one forbidden structure, *Kenny Štorgel*
- The Slow-Coloring Game on a Graph, *Douglas West*
- Approximate and Randomized algorithms for Computing a Second Hamiltonian Cycle, *Viktor Zamaraev*

On Efficient Domination for H-free bipartite graphs

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A vertex set D in a finite undirected graph G is an *efficient dominating set* (*e.d.s.* for short) of G if every vertex of G is dominated by exactly one vertex of D. The *Efficient Domination* (*ED*) problem, asks for the existence of an e.d.s. in G; it is the Exact Cover problem for the closed neighborhood hypergraph of G. ED is known to be NP-complete even for very restricted H-free graph classes such as for $2P_3$-free chordal graphs (and thus, for P_7-free graphs) while it is solvable in polynomial time for P_6-free graphs. For H-free graphs, ED is either NP-complete or polynomial, i.e., a dichotomy. However, for H-free bipartite graphs, there is no such dichotomy.

Lu and Tang showed that ED is NP-complete for chordal bipartite graphs and for planar bipartite graphs; actually, ED is NP-complete even for planar bipartite graphs with vertex degree at most 3 and girth at least g for every fixed g. Thus, ED is NP-complete for $K_{1,4}$-free bipartite graphs and for C_4-free bipartite graphs. For classes of bounded clique-width, ED is solvable in polynomial time. Dabrowski and Paulusma published a dichotomy for clique-width of H-free bipartite graphs. For instance, clique-width of $S_{1,2,3}$-free bipartite graphs is bounded.

In Discrete Applied Math. 270 (2019), we published a manuscript "On efficient domination for some classes of H-free bipartite graphs". We showed that (weighted) ED can be solved in polynomial time for H-free bipartite graphs when H is P_7 or ℓP_4 for fixed ℓ, and similarly for P_9-free bipartite graphs with vertex degree at most 3, and when H is $S_{2,2,4}$. In this talk, we also mention a polynomial time solution for P_8-free bipartite graphs (which was an open problem in our publication).

Domination number of graphs with fixed minimum degree

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We discuss upper bounds on the domination number of graphs with a fixed minimum degree. The main result states that for every graph G on n vertices and with minimum degree 5, the domination number $\gamma(G)$ cannot exceed $n/3$. The proof is obtained via an algorithmic approach where a weighting method is combined with a discharging process.

Extending Thomason's Algorithm

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Wilfrid Laurer University, Canada

Carsten Thomassen and I proved that in any graph G, the number of cycles containing a specified edge as well as all the odd-degree vertices is odd if and only if G is eulerian. Where all vertices have even degree this is a theorem of Shunichi Toida and where all vertices have odd degree it is Andrew Thomason's extension of Smith's Theorem. Andrew Thomason proved his theorem by constructing a graph X(G) in which the odd-degree vertices correspond precisely to the things he wants to show there are an even number of, namely the hamiltonian cycles containing the specified edge. This provides an algorithm for given one of the objects, finding another. I have extended Thomason's algorithm to one which, in a non-eulerian graph, finds a second cycle containing a specified edge and all the odd-degree vertices. I will discuss some other parity theorems about paths, cycles, and trees in graphs; in particular, attempts to find proofs of them by showing that the objects of interest are the odd-degree vertices of an associated (generally large) graph.

Clique-Width: Harnessing the Power of Atoms

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Coauthors: Tomáš Masařík, Jana Novotná, Daniël Paulusma, Paweł Rzazewski

Many NP-complete graph problems are polynomial-time solvable on graph classes of bounded clique-width. Several of these problems are polynomial-time solvable on a hereditary graph class $\mathcal{G}$ if they are so on the atoms (graphs with no clique cut-set) of $\mathcal{G}$. Hence, we initiate a systematic study into boundedness of clique-width of atoms of hereditary graph classes. A graph G is H*-free* if H is not an induced subgraph of G, and it is (H_1, H_2)*-free* if it is both H_1-free and H_2-free. A class of H-free graphs has bounded clique-width if and only if its atoms have this property. This is no longer true for (H_1, H_2)-free graphs, as evidenced by one known example. We prove the existence of another such pair (H_1, H_2) and classify the boundedness of clique-width on (H_1, H_2)-free atoms for all but 18 cases.

References

[1] Konrad K. Dabrowski, Tomáš Masařík, Jana Novotná, Daniël Paulusma, and Paweł Rzazewski. Clique-width: Harnessing the power of atoms. *Proc. WG 2020, LNCS*, 12301:119–133, 2020. Full version: arXiv:2006.03578.

Colourful components in k-caterpillars and planar graphs

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Coauthor: Janka Chlebíková

A connected component of a vertex-coloured graph is said to be *colourful* if all its vertices have different colours. By extension, a graph is colourful if all its connected components are colourful. Given a vertex-coloured graph and an integer p, the COLOURFUL COMPONENTS problem asks whether there exist at most p edges whose removal makes the graph colourful. Bulteau, Dabrowski, Fertin, Johnson, Paulusma, and Vialette (2019) proved that COLOURFUL COMPONENTS is NP-complete on trees with maximum degree at most 6 and asked whether the same would hold if the maximum degree is at most 5. We show that the problem remains NP-complete on binary 4-caterpillars, ternary 3-caterpillars and quaternary 2-caterpillars, where a k*-caterpillar* is a tree containing a path P such that every vertex is at distance at most k from P. On the other hand, we provide a linear-time algorithm for 1-caterpillars (without restriction on the maximum degree), and thus almost settle the complexity dichotomy on k-caterpillars with respect to the maximum degree. COLOURFUL COMPONENTS has also been studied in vertex-coloured graphs with a bounded number of colours. Bruckner, Hüffner, Komusiewicz, Niedermeier, Thiel, and Uhlmann (2012) showed that the problem is NP-complete on 3-coloured graphs with maximum degree 6, and Bulteau et al. asked whether the problem would remain NP-complete on graphs with bounded number of colors and maximum degree 3. We answer their question in the affirmative by showing that the problem is NP-complete on 5-coloured planar graphs with maximum degree 4 and on 12-coloured planar graphs with maximum degree 3.

Firebreaking and Firefighting

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In the classic firefighter game, a fire burns and firefighters defend in turns. Motivated by a practical epidemiological question, we defined a one-shot version of this game, and plan to discuss both some new progress on firefighting and this one-shot version: the FIREBREAK problem.

Suppose we have a network that is represented by a graph G. Potentially a fire (or other type of contagion) might erupt at some vertex of G. We are able to respond to this outbreak by establishing a firebreak at k other vertices of G, so that the fire cannot pass through these. fortified vertices. The question that now arises is which k vertices will result in the greatest number of vertices being saved from the fire, assuming that the fire will spread to every vertex that is not fully behind the k vertices of the firebreak. This is the essence of the FIREBREAK decision problem, which is the focus of this talk. We establish that the problem is intractable on the class of split graphs as well as on the class of bipartite graphs, but can be solved in linear time when restricted to graphs having constant-bounded treewidth, or in polynomial time when restricted to convenient classes of intersection graphs.

Graphs with two moplexes are more than perfect

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A well-known result by Dirac (1961) states that every chordal graph contains a simplicial vertex. This theorem proved to be very useful for structural and algorithmic applications. Moplexes are a generalisation of simplicial vertices in chordal graphs to the setting of general graphs, as Berry and Bordat (1998) proved that every non-complete graph contains at least two moplexes.

There are results on the structure of chordal graphs with a bounded number of simplicial modules, for example the chordal graphs having at most two simplicial modules are interval. This motivates the research of graphs with a bounded number of moplexes. As only complete graphs have exactly one moplex, we consider the smallest interesting case: the class of graphs with at most two moplexes. Berry and Bordat (2001) proved that this class of graphs contains all connected proper interval graphs and is contained in the class of AT-free graphs. We strengthen the latter inclusion in two ways. First, we generalise it by proving that the asteroidal number yields a lower bound on the number of moplexes. Second, as our main structural result, we show that graphs with at most two moplexes are cocomparability.

So, as the class of connected graphs with at most two moplexes is sandwiched between the connected proper interval graphs and cocomparability graphs, this leads to the natural question of whether the presence of at most two moplexes guarantees a sufficient amount of structure to efficiently solve problems that are known to be intractable on cocomparability graphs, but not on proper interval graphs. For two such problems, namely GRAPH ISOMORPHISM and MAX-CUT, we show that they stay hard on the graphs with two moplexes. On the other hand, we prove that every connected graph with two moplexes contains a Hamiltonian path.

Computing Weighted Subset Transversals in H-free Graph

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In graph transversal problems, one seeks, given a graph, to find a small subset of the vertex set — a *transversal* — that intersects every subgraph of a specified kind. Examples include vertex cover, feedback vertex set and odd cycle transversal. In *subset* versions of the problem, a subset of the vertex set is also given and the transversal that is found must also intersect this subset. In *weighted* versions, a vertex weighting is given and the aim is to find a transversal such that the sum of the weights of the transversal vertices is small.

These problems are NP-hard in general. We discuss recent work on graph transversal problems for hereditary graph classes. We will focus in particular on the problems WEIGHTED SUBSET ODD CYCLE TRANSVERSAL and WEIGHTED SUBSET FEEDBACK VERTEX SET for graph classes that are H-free. We present new algorithms that lead us to classify the complexity of the problems except in the cases where H is $2P_1+P_3$, P_1+P_4 or $2P_1+P_4$. We note that in the latter two cases the complexity remains open even for the unweighted non-subset versions of the problem. One interesting aspect of our classification is the discovery that the complexities of weighted and unweighted versions of SUBSET ODD CYCLE TRANSVERSAL do not coincide for H-free graphs.

Interference-free Walks in Time: Temporally Disjoint Paths

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We investigate the computational complexity of finding *temporally disjoint* paths or walks in temporal graphs. There, the edge set changes over discrete time steps and a temporal path (resp. walk) uses edges that appear at monotonically increasing time steps. Two paths (or walks) are temporally disjoint if they never use the same vertex or edge at the same time; otherwise, they interfere.

We show that on general graphs the problem is computationally hard. The "walk version" is W[1]-hard when parameterized by the number of routes. However, it is polynomial-time solvable for any constant number of walks. The "path version" remains NP-hard even if we want to find only two temporally disjoint paths. On the other extreme, restricting the underlying graph to be a path, we find a polynomial-time algorithm for a relevant special case while the problem remains NP-hard in general (for both paths and walks).

Shifting any path to an avoidable one

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A vertex v in a graph G is avoidable if every induced path on three vertices with middle vertex v is contained in an induced cycle. Dirac's classical result from 1961 on the existence of simplicial vertices in chordal graphs is equivalent to the statement that every chordal graph has an avoidable vertex. Beisegel, Chudovsky, Gurvich, Milanič, and Servatius (2019) generalized the notion of avoidable vertices to *avoidable paths*, and conjectured that every graph that contains an induced k-vertex path also contains an avoidable induced k-vertex path; they proved the result for $k = 2$. The case $k = 1$ was known much earlier, due to a work of Ohtsuki, Cheung, and Fujisawa in 1976. The conjecture was proved for all k in 2020 by Bonamy, Defrain, Hatzel, and Thiebaut. This result generalizes the mentioned results regarding avoidable vertices, as well as a result by Chvátal, Rusu, and Sritharan (2002) suggested by West on the existence of simplicial k-vertex paths in graphs excluding induced cycles of length at least $k + 3$.

In this talk we discuss an adaptation of the approach by Bonamy et al. from a reconfiguration point of view. We say that two induced k-vertex paths are *shifts* of each other if their union is an induced path with $k + 1$ vertices and show that in every graph, every induced path can be shifted to an avoidable one. We also present an analogous result about paths that are not necessarily induced, where an efficient reconfiguration sequence relies on the properties of depth-first search trees.

Model-checking in multilayer graphs

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Real-world networks often involve qualitatively different types of relationships between different objects: for example, in order to understand a social network, we can consider both online and face-to-face contact. Many computational questions we might want to answer about such systems are intractable unless we have some additional information about the structure. The different layers typically have different structural properties (for example, face-to-face contact will be much more influenced by geography than online contact) and this talk will address the following question: when can we make use of algorithmically useful structure in the individual layers to solve problems efficiently on the whole system?

Beyond treewidth: the tree-independence number

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We introduce a new graph invariant called *tree-independence number*. This is a common generalization of treewidth and independence number, in the sense that bounded treewidth or bounded independence number implies bounded tree-independence number. The tree-independence number of a graph G is defined as the smallest positive integer k such that G has a tree decomposition whose bags induce subgraphs of G with independence number at most k. While for $k = 1$ we obtain the well-known class of chordal graphs, we show that the problem of computing the tree-independence number of a graph is NP-hard in general.

We consider six graph containment relations (the subgraph, topological minor, and minor relations, as well as their induced variants) and for each of them completely characterize graph classes of bounded tree-independence number defined by a single forbidden graph with respect to the relation. In each of these bounded cases, a tree decomposition with small independence number can be computed efficiently, which implies polynomial-time solvability of the Maximum Weight Independent Set (MWIS) problem. This in particular applies to an infinite family of generalizations of the class of chordal graphs, for which a polynomial-time algorithm for the MWIS problem was given by Frank in 1976.

Width parameters and graph classes: the case of mim-width

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Coauthors: Flavia Bonomo-Braberman, Nick Brettell, Daniël Paulusma, Jake Horsfield, Giacomo Paesani

A large number of NP-hard graph problems become polynomial-time solvable on graph classes where the mim-width is bounded and quickly computable. Hence, when solving such problems on special graph classes, it is helpful to know whether the graph class under consideration has bounded mim-width. We extend the toolkit for proving (un)boundedness of mim-width of graph classes and initiate a systematic study into bounding mim-width from the perspective of hereditary graph classes. We present summary theorems of the current state of the art for the boundedness of mim-width for (H_1, H_2)-free graphs and observe several interesting consequences. We also study the mim-width of generalized convex graphs. This allows us to re-prove and strengthen a large number of known results.

Coloring $(4K_1, C_4, C_6)$-free graphs

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A graph is *even-hole-free* if it contains no induced cycles of even length. Even-hole-free graphs can be recognized in polynomial time, and furthermore, the MAXIMUM CLIQUE problem can be solved in polynomial time for such graphs. However, the time complexity of the VERTEX

COLORING problem is open for this class. The time complexity of VERTEX COLORING is also open for graphs of stability number at most three. In this talk, we consider the intersection of these two classes: even-hole-free graphs of stability number at most three, or equivalently, $(4K_1, C_4, C_6)$-free graphs. We show that such graphs can be recognized and colored in cubic time.

Circularly compatible ones, D-circularity, and proper circular-arc bigraphs

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In 1969, Alan Tucker characterized proper circular-arc graphs as those graphs whose augmented adjacency matrices have the circularly compatible ones property. Moreover, he also found a polynomial-time algorithm for deciding whether any given augmented adjacency matrix has the circularly compatible ones property. These results allowed him to devise the first polynomial-time recognition algorithm for proper circular-arc graphs. However, as Tucker himself remarks, he did not solve the problems of finding a structure theorem and an efficient recognition algorithm for the circularly compatible ones property in arbitrary matrices (i.e., not restricted to augmented adjacency matrices only). In this work, we solve these problems. More precisely, we give a minimal forbidden submatrix characterization for the circularly compatible ones property in arbitrary matrices and a linear-time recognition algorithm for the same property. We derive these results from analogous ones for the related D-circular property. Interestingly, these results lead to a minimal forbidden induced subgraph characterization and a linear-time recognition algorithm for proper circular-arc bigraphs, solving a problem first posed by Basu, Das, Ghosh, and Sen [J. Graph Theory, 73(4):361–376, 2013]. Our findings generalize some known results about D-interval hypergraphs and proper interval bigraphs.

On the treewidth of even-hole-free graphs

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Even-hole-free graphs attracted attention lately (see [1]). However, many questions about them remain unanswered: polytime algorithm to color them, or to find a maximum stable set, despite the existence of several decomposition theorems. In general, the class of all even-hole-free graphs has unbounded tree-width, as it contains all complete graphs. Nonetheless, bounding the size of the maximum clique does not turn the class into having bounded treewidth, because there exists a family of even-hole-free graphs with no 4-vertex clique which has unbounded tree-width [2]. We observe that the graph constructed in [2] has unbounded degree *and* contains arbitrarily large clique-minors. We ask whether this is necessary.

We prove that for every graph G, if G excludes a fixed graph H as a minor, then G either has small tree-width, or G contains a large wall or the line graph of a large wall as *induced* subgraph. This can be seen as a strengthening of Robertson and Seymour's excluded grid theorem for the case of minor-free graphs. Our theorem implies that every class of even-hole-free graphs excluding a fixed graph as a minor has bounded tree-width. In fact, our theorem applies to a more general class: (theta, prism)-free graphs. Furthermore, we conjecture that even-hole-free graphs of bounded degree have bounded tree-width, and we prove it for subcubic graphs and give a partial result when the maximum degree is four. This conjecture is now proved in [3].

Based on a joint work with Pierre Aboulker, Isolde Adler, Eun Jung Kim, and Nicolas Trotignon.

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[3] T. Abrishami, M. Chudnovsky, K. Vušković. Even-hole-free graphs with bounded degree have bounded treewidth. arXiv 2009.01297, 2020.

On 12-regular nut graphs

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Coauthors: Nino Bašić, Martin Knor

A nut graph is a simple graph whose adjacency matrix is singular with 1-dimensional kernel and corresponding eigenvector with non-zero elements. For each $d \in \{3, 4, \ldots, 11\}$ are known all values n for which there exists a d-regular nut graph of order n. In the talk, we consider all values n for which there exists a 12-regular nut graph of order n.

Treewidth versus clique number: a complete dichotomy for one forbidden structure

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Coauthors: Clément Dallard, Martin Milanič

Treewidth is an important graph invariant, relevant for both structural and algorithmic reasons. A necessary condition for a graph class to have bounded treewidth is the absence of large cliques. We study graph classes closed under taking induced subgraphs in which this condition is also sufficient, which we call (tw, ω)-bounded. For six graph containment relations (the subgraph, topological minor, and minor relations, as well as their induced variants) we give a complete characterization of graphs H for which the class of graphs excluding H is (tw, ω)-bounded.

Our results yield an infinite family of χ-bounded induced-minor-closed graph classes and imply that the class of 1-perfectly orientable graphs is (tw, ω)-bounded, leading to linear-time algorithms for k-coloring 1-perfectly orientable graphs for every fixed k. This answers a question of Brešar, Hartinger, Kos, and Milanič (2018) and one of Beisegel, Chudnovsky, Gurvich, Milanič, and Servatius (2019), respectively. We also reveal some further algorithmic implications of (tw, ω)-boundedness related to list k-coloring and clique problems.

The Slow-Coloring Game on a Graph

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The *slow-coloring game* is played by Lister and Painter on a graph G. Initially, all vertices of G are uncolored. In each round, Lister marks a non-empty set M of uncolored vertices, and Painter colors a subset of M that is independent in G. The game ends when all vertices are colored. The score of the game is the sum of the sizes of all sets marked by Lister. The goal of Painter is to minimize the score, while Lister tries to maximize it; the score under optimal play is the *cost* of the graph.

A greedy strategy for Painter keeps the cost of G to at most $\chi(G)n$ when G has n vertices, which is asyptotically sharp for Turán graphs. On various classes Painter can do better. For n-vertex trees the maximum cost is $\lfloor 3n/2 \rfloor$. There is a linear-time algorithm and inductive formula to compute the cost on trees, and we know all the extremal n-vertex trees. Also, Painter can keep the cost to at most $(1 + 3k/4)n$ when G is k-degenerate, $7n/3$ when G is outerplanar, $3.9857n$ when G is acyclically 5-colorable, and $3.4n$ when G is planar. These bounds are not believed to be sharp.

We will discuss strategies (algorithms) for Lister and Painter that establish various lower and upper bounds. The results appear in three papers with various subsets of Grzegorz Gutowski, Tomasz Krawczyk, Thomas Mahoney, Krzysztof Maziarz, Gregory J. Puleo, Michal Zajac, and Xuding Zhu.

Approximate and Randomized algorithms for Computing a Second Hamiltonian Cycle

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In 1946 Cedric Smith proved, using a non-constructive parity argument, that any cubic Hamiltonian graph contains at least two Hamiltonian cycles.

This motivated the following computational problem, which is still largely open: given a Hamiltonian cycle C in a cubic Hamiltonian graph G, can we efficiently compute a second Hamiltonian cycle?

In this talk, I will discuss various open questions surrounding this problem and present some efficient approximate and randomized algorithms for related problems.

Joint work with Argyrios Deligkas, George B. Mertzios, and Paul G. Spirakis

Minisymposium

Applied Combinatorial and Geometric Topology (MS-34)

Organized by
Luigi Grasselli, *University of Modena and Reggio Emilia, Italy*
Maria Rita Casali, *University of Modena and Reggio Emilia, Italy*
Coorganized by
Bruno Benedett, *University of Miami, United States*
Antonio Felix Costa, *UNED Madrid, Spain*

- Hamiltonian complexes, interval graphs, determinantal ideals, *Bruno Bendetti*
- A colored approach for the self-assembly of DNA structures, *Simona Bonvicini*
- Universality classes of triangulations in dimensions greater than 2, *Valentin Bonzom*
- Kirby diagrams, edge-colored graphs and trisections of PL 4-manifolds, *Maria Rita Casali*
- Complexity of graph manifolds, *Alessia Cattabriga*
- Periodic projections of alternating knots, *Antonio F. Costa*
- Classifying compact PL 4-manifolds according to generalized regular genus and G-degree, *Paola Cristofori*
- Partition extenders, skeleta of simplices, and Simon's conjecture, *Bennet Goeckner*
- On the Connectivity of Branch Loci of Spaces of Curves, *Milagros Izquierdo*
- Invariants for tame parametrised chain complexes, *Claudia Landi*
- Transitive factorizations of pairs of permutations and three-dimensional constellations, *Luca Lionni*
- Explicit computation of some families of Hurwitz numbers, *Carlo Petronio*
- Partitioning the projective plane and the dunce hat, *Andrés David Santamaría-Galvis*
- Shellings from relative shellings, *Russ Woodroofe*

Hamiltonian complexes, interval graphs, determinantal ideals

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Coauthors: Lisa Seccia, Matteo Varbaro

There are three mutually-related notions in graph theory, from three different centuries: Hamiltonian paths, (unit)-interval graphs, and binomial edge ideals. We study the d-dimensional generalization of these ideas.

This is joint work with Matteo Varbaro and Lisa Seccia (arXiv:2101.09243).

A colored approach for the self-assembly of DNA structures

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Coauthor: Margherita Maria Ferrari

We study a graph theory problem related to the self-assembly of DNA structures. The self-assembly can be obtained by several methods that are based on the Watson-Crick complementary properties of DNA strands. We consider the method based on branched junction molecules, that is, star-shaped molecules whose arms have cohesive ends that allow the molecules to join together in a prescribed way and form a larger molecule (DNA complex).

In the language of graphs, a branched junction molecule is called a tile and consists of a vertex with labeled half-edges; labels represent the cohesive ends and belong to a set $\{a, \hat{a} : a \in \Sigma\}$, where Σ is a finite set of symbols; a tile is denoted by the multiset consisting of the labels of the half-edges; and two tiles are of the same tile type if they are denoted by the same multiset.

We can create an edge between the vertices u, v if and only if u has a half-edge labeled by a and v has a half-edge labeled by $\hat{a}$; the edge thus obtained is said to be a bond-edge of type $a\hat{a}$; by connecting the vertices according to the labels, we can construct a graph G representing a DNA complex.

The following problem is considered: given a graph G, determine the minimum number of tile types and bond-edge types that are necessary to construct G. We describe the problem by edge-colored graphs and show some upper bounds for the number of bond-edge types that are necessary to construct an arbitrary graph.

Universality classes of triangulations in dimensions greater than 2

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Combinatorial maps are cellular topological models for surfaces, which include triangulations, quadrangulations, etc. of all genera. It is well-known that at fixed topology, all models lie in the same universality class. It means in particular that they have the same critical exponents in the enumeration formula and the same limit object at large scales. For instance, the universality class for planar maps is also known as the universality class of 2D pure quantum gravity and its scaling limit is known as the Brownian sphere. It is important to generalize this framework to greater dimensions, with motivations from combinatorics, topology, mathematical physics and quantum gravity. However, it has been a challenge due to the intricate nature of combinatorics and topology in dimensions 3 and greater. Here I will present results based on the model of colored triangulations for PL-manifolds. They admit a representation as edge-colored graphs which is amenable to combinatorial analysis. I will focus on a combinatorial generalization of Euler's relation for combinatorial maps, which is to bound the number of $(d-2)$-simplices linearly in the number of d-simplices, and identifying the triangulations which maximize the bound for different colored building blocks. In 3d, I prove that those triangulations, for any set of colored building blocks homeomorphic to the 3-ball, are in bijection with trees. In even dimensions, we have proved that several universality classes can be achieved. Both those results are in sharp contrast with the case of surfaces.

Kirby diagrams, edge-colored graphs and trisections of PL 4-manifolds

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Coauthor: Paola Cristofori

It is well-known that any *framed link* (L, c) uniquely represents the 3-manifold $M^3(L, c)$ obtained from $\mathbb{S}^3$ by Dehn surgery along (L, c), as well as the PL 4-manifold $M^4(L, c)$ obtained from $\mathbb{D}^4$ by adding 2-handles along (L, c). Moreover, if trivial dotted components are also allowed (i.e. in case of a *Kirby diagram* $(L^{(*)}, c)$), the associated PL 4-manifold $M^4(L^{(*)}, c)$ is obtained from $\mathbb{D}^4$ by adding 1-handles along the dotted components and 2-handles along the framed components.

In the present talk we present the relationship between framed links and/or Kirby diagrams and the so called *crystallization theory*, which represents compact PL manifolds of arbitrary dimension by regular edge-colored graphs: in particular, we describe how to construct a 5-colored graph representing $M^4(L^{(*)}, c)$, directly "drawn over" a planar diagram of $(L^{(*)}, c)$).

As a consequence, the combinatorial properties of Kirby diagrams yield upper bounds for both the graph-defined invariants *gem-complexity* and *generalized regular genus* of the associated 4-manifold.

Further, the described relationship turns out to be strictly related to the possibility of studying *trisections* of 4-manifolds via edge-colored graphs, also in the extended case of compact PL 4-manifolds with connected boundary.

Complexity of graph manifolds

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Coauthor: Michele Mulazzani

The notion of complexity for compact 3-dimensional manifolds, was introduced by Matveev in the nineties as a way to measure how complicated a manifold is; indeed, for closed orientable irreducible manifolds, the complexity coincides with the minimum number of tetrahedra needed to construct the manifold, with the only exceptions of the 3-sphere, the projective space and the lens space L(3, 1) all having complexity zero. There exists a census of manifolds according to increasing complexity that, for the orientable case, is Recognizer catalogue and includes all manifolds up to complexity 12 (see http://matlas.math.csu.ru/?page=search). In this talk, after recalling some general result about complexity, I will focus on an important class of manifolds: graph manifolds. These manifolds have been introduced and classified by Waldhausen in the sixties and are defined as compact 3-manifolds obtained by gluing Seifert fibre spaces along toric boundary components. I will present an upper bound for their complexity obtained in a joint work with Michele Mulazzani (University of Bologna) that is sharp for all the 14502 graph manifolds included in the Recognizer catalogue

Periodic projections of alternating knots

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Coauthor: Cam Van Quach-Hongler

Let K be an oriented prime alternating knot that is q-periodic with $q \geq 3$, i.e. K admits a symmetry that is a rotation of order q. We present a proof that K has an alternating q-periodic projection.

The main tool is the Menasco-Thistlethwaite's Flyping theorem.

We present also some results about the visibility on projections of q-freely periodicity of alternating knots.

Classifying compact PL 4-manifolds according to generalized regular genus and G-degree

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Coauthor: Maria Rita Casali

(d+1)-colored graphs, that is $(d+1)$-regular graphs endowed with a proper edge-coloration, are the objects of a long-studied representation theory for closed PL d-manifolds, which has been recently extended to the whole class of compact PL d-manifolds.

In this context, combinatorially defined PL invariants play a relevant role; in this talk we will focus on two of them: the *generalized regular genus* and the *G-degree*. The former extends to higher dimension the classical notion of Heegaard genus for 3-manifolds; the latter has arisen in connection with *Colored Tensor Models* (CTM), a particular kind of tensor models, that have been intensively studied in the last years, mainly as an approach to quantum gravity in dimension greater than two. CTMs established a link between colored graphs and tensor models, since the Feynman graphs of a d-dimensional CTM are precisely $(d+1)$-colored graphs. Furthermore, the G-degree of a colored graph is a crucial quantity driving the 1/N expansion of the free energy of a CTM.

This talk will mainly concern recent results achieved in dimension 4: in particular, the classification of all compact PL 4-manifolds with generalized regular genus at most one or with G-degree at most 18. Furthermore, we will discuss interesting classes of 5-colored graphs (*semi-simple and weak semi-simple crystallizations*), representing compact PL 4-manifolds with empty or connected boundary and minimizing the invariants. In the simply-connected case they also belong to a wider class of 5-colored graphs which are proved to induce handle-decompositions of the represented 4-manifold lacking in 1- and/or 3-handles; therefore their study is strictly related to the problem, posed by Kirby, of the existence of special handle-decompositions for any simply-connected closed PL 4-manifold.

Partition extenders, skeleta of simplices, and Simon's conjecture

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Coauthors: Joseph Doolittle, Alexander Lazar

If a pure simplicial complex is partitionable, then its h-vector has a combinatorial interpretation in terms of any partitioning of the complex. Such an interpretation does not exist for non-partitionable complexes. Given a non-partitionable complex, we will construct a relative complex—called a *partition extender*—that allows us to write the h-vector of a non-partitionable complex as the difference of two h-vectors of partitionable complexes in a natural way. We will show that all pure complexes have partition extenders.

A similar notion can be defined for Cohen–Macaulay and shellable complexes. We will show precisely which complexes have Cohen–Macaulay extenders, and we will discuss a connection to a conjecture of Simon on the extendable shellability of uniform matroids. This is joint work with Joseph Doolittle and Alexander Lazar.

On the Connectivity of Branch Loci of Spaces of Curves

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Coauthor: Antonio F. Costa

Since the 19th century the theory of Riemann surfaces has a central place in mathematics putting together complex analysis, algebraic and hyperbolic geometry, group theory and combinatorial methods.

Since Riemann, Klein and Poincar'e among others, we know that a compact Riemann surface is a complex curve, and also the quotient of the hyperbolic plane by a Fuchsian group.

In this talk we study the connectivity of the moduli spaces of Riemann surfaces (i.e in spaces of Fuchsian groups). Spaces of Fuchsian groups are orbifolds where the singular locus is formed by Riemann surfaces with automorphisms: *the branch loci*: With a few exceptions the branch loci is disconnected and consists of several connected components.

This talk is a survey of the different methods and topics playing together in the theory of Riemann surfaces.

Joint wotk with Antonio F. Costa.

Invariants for tame parametrised chain complexes

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Coauthors: Wojciech Chachólski, Barbara Giunti

Data analysis is often about simplifying, ignoring most of the information available and extracting what might be meaningful to a task at hand. This strategy of making sense by disregarding some information and focusing on aspects that might be relevant is very common across mathematics. Colocalization in homotopy theory is an example of such a process. In colocalization, simplification is achieved by approximating arbitrary objects by other objects that one considers simpler and more manageable. For instance, by approximating a given space by n-connected spaces, one obtains its n-connected cover. The aim of our presentation is to explain why extracting persistence invariants in Topological Data Analysis (TDA) is an example of the homotopical colocalization process. This allows us to extract computable invariants also in certain cases, such as commutative ladders, that have not been covered by more standard approaches. Furthermore, it allows for a comprehensive theory including several cases that standard persistence theory handles separately, such as persistence modules, zigzag modules, and commutative ladders.

Transitive factorizations of pairs of permutations and three-dimensional constellations

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Factorizations of pairs of permutations that generate the full symmetric group on n-symbols are shown to be in bijection with a certain family of three-dimensional 4-colored triangulations that generalize constellations. These spaces are ramified coverings of the three-dimensional sphere, branched over the n-unlink. We will present a generalization of the Riemann-Hurwitz formula, and identify the spaces that maximize the number of branching edges for a fixed number of sheets.

Explicit computation of some families of Hurwitz numbers

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I will describe the computation (based on dessins d'enfant) of the number of equivalence classes of surface branched covers matching a given branch datum belonging to a certain very specific family.

Partitioning the projective plane and the dunce hat

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The faces of a simplicial complex induce a partial order by inclusion in a natural way. We say that the complex is partitionable if its poset can be partitioned into boolean intervals, with a maximal face at the top of each.

In this work we show that all the triangulations of the real projective plane, the dunce hat, and the open Möbius strip are partitionable. To prove that, we introduce simple yet useful gluing tools that allow us to abridge the discussion about partitionability of a given complex in terms of smaller constituent relative subcomplexes. The gluing process generates partitioning schemes with a distinctive shelling-like flavor.

Shellings from relative shellings

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Coauthor: Andrés David Santamaría-Galvis

It is frequently helpful to build complicated mathematical objects by breaking down into sub-objects with simpler properties. In joint work with Andrés Santamaría-Galvis, we have shown how to usefully glue together shellings of relative simplicial complexes to construct a shelling of a large simplicial complex. Indeed, one of the main other ways of using the "divide and conquer" approach to build a shelling, vertex-decomposability, can be viewed as a consequence of our approach. One useful consequence is that our approach makes it relatively straightforward to find a shellable simplicial complex satisfying any of a variety of conditions on its facets that contain a free face. Another is an improved proof that the shellability decision problem is NP-complete.

Minisymposium

COMBINATORIAL DESIGNS (MS-16)

Organized by Marco Buratti, *Università di Perugia, Italy*

- Kirkman triple systems with many symmetries, *Marco Buratti*
- Mutually orthogonal cycle systems, *Andrea C. Burgess*
- Testing Arrays for Fault Localization, *Charles Colbourn*
- Factorizations of infinite graphs, *Simone Costa*
- On the mini-symposium problem, *Peter Danziger*
- A lower bound on permutation codes of distance $n-1$, *Peter Dukes*
- Characterizing isomorphism classes of Latin squares by fractal dimensions of image patterns, *Raúl M. Falcón*
- Novák's conjecture on cyclic Steiner triple systems and its generalization, *Tao Feng*
- An Evans-style result for block designs, *Daniel Horsley*
- Balancedly splittable orthogonal designs, *Hadi Kharaghani*
- Resolving sets and identifying codes in finite geometries, *György Kiss*
- 20 years of Legendre Pairs, *Ilias Kotsireas*
- Packings of Partial Difference Sets, *Shuxing Li*
- Balanced Equi-n-squares, *Trent Marbach*
- Equitably 2-colourable even cycle systems, *Francesca Merola*
- Weakly self-orthogonal designs and related linear codes, *Vedrana Mikulić Crnković*
- The classification of 2-(v, k, λ) designs, with $\lambda > 1$ and $(r, \lambda) = 1$, admitting a flag-transitive automorphism group, *Alessandro Montinaro*
- Testing isomorphism of circulant objects in polynomial time, *Mikhael Muzychuk*
- Strictly additive 2-designs, *Anamari Nakić*
- Sequences in $\mathbb{Z}_n$ with Distinct Partial Sums, *Matt Ollis*
- Universal sequences and Euler tours in hypergraphs, *Deryk Osthus*
- A reduction of the spectrum problem for sun systems, *Anita Pasotti*
- Regular 1-factorizations of complete graphs with orthogonal spanning trees, *Gloria Rinaldi*
- A few new triplanes, *Sanja Rukavina*
- Merging Combinatorial Design and Optimization: the Oberwolfach Problem, *Fabio Salassa*
- On the existence of large set of partitioned incomplete Latin squares, *Cong Shen*
- Bipartite 2-factorizations of complete multigraphs via layering, *Mateja Šajna*
- On some periodic Golay pairs and pairwise balanced designs, *Andrea Švob*
- Dual incidences and t-designs in elementary abelian groups, *Kristijan Tabak*
- On the Oberwolfach Problem for single-flip 2-factors via graceful labelings, *Tommaso Traetta*
- On the classification of unitals on 28 points of low rank, *Alfred Wassermann*
- Heffter Arrays and Biembbedings of Cycle Systems on Orientable Surfaces, *E. Şule Yazıcı*
- Block designs constructed from orbit matrices using a modified genetic algorithm, *Tin Zrinski*

Kirkman triple systems with many symmetries

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Coauthors: Simona Bonvicini, Martino Garonzi, Gloria Rinaldi, Tommaso Traetta

A *Steiner triple system* of order v, $\mathrm{STS}(v)$ for short, is a set V with v *points* together with a collection $\mathcal{B}$ of 3-subsets of V (*blocks*) such that that any pair of distinct points is contained in exactly one block. A *Kirkman triple system* of order v, $\mathrm{KTS}(v)$ for short, is a $\mathrm{STS}(v)$ whose blocks are partitioned in *parallel classes* each of which is a partition of the point-set V.

Steiner and Kirkman triple systems are among the most popular objects in combinatorics and their existence has been established a long time ago. Yet, very little is known about the automorphism groups of KTSs of an arbitrary order. In particular, from the very beginning of my research in design theory, I found surprising that there was no known pair (r, n) for which whenever $v \equiv r \pmod{n}$ one may claim that there exists a $\mathrm{KTS}(v)$ with a number of automorphisms at least close to v.

After pursuing the target of getting such a pair (r, n) for more than twenty years, we recently managed to find the following: $(39, 72)$ and $(4^e 48 + 3, 4^e 96)$ for any $e \geq 0$. Indeed, for $v \equiv r \pmod{n}$ with (r, n) as above, we are able to exhibit, concretely, a $\mathrm{KTS}(v)$ with an automorphism group of order at least equal to $v - 3$. The proof is very long and elaborated; so I will try to speak about the main ideas which led us to this result as for instance the invention of some variants of well known combinatorial objects which could be also used in the search of other combinatorial designs.

Mutually orthogonal cycle systems

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Coauthors: Nicholas Cavenagh, David A. Pike

An ℓ-cycle system of order n is a set of ℓ-cycles whose edges partition the edge set of K_n. We say that two cycle systems, say $\mathcal{C}$ and $\mathcal{C}'$, are *orthogonal* if any cycle of $\mathcal{C}$ and any cycle of $\mathcal{C}'$ share at most one edge. Orthogonal cycle systems arise naturally from simple Heffter arrays and biembeddings of cycle decompositions.

A collection of cycle systems is *mutually orthogonal* if any two of the systems are orthogonal. In this talk, we give bounds on the number of mutually orthogonal ℓ-cycle systems of order n, and provide constructions for sets of mutually orthogonal cyclic cycle systems.

Testing Arrays for Fault Localization

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A *separating hash family* $\mathsf{SHF}_\lambda(N; k, v, \{w_1, \ldots, w_s\})$ is an $N \times k$ array on v symbols, with the property that no matter how we choose disjoint sets $C_1, \ldots, C_s$ of columns with $|C_i| = w_i$, there are at least λ rows in which, for every $1 \leq i < j \leq s$, no entry in a column of C_i equals that in a column of C_j. (That is, there are λ rows in which sets $\{C_1, \ldots, C_s\}$ are *separated*.)

Separating hash families have numerous applications in combinatorial cryptography and in the construction of various combinatorial arrays; typically, one only considers whether two symbols are the same or different. We instead employ symbols that have algebraic significance.

We consider an $\mathsf{SHF}_\lambda(N; k, q^s, \{w_1, \ldots, w_s\})$ whose symbols are column vectors from $\mathbb{F}_q^s$. The entry in row r and column c of the SHF is denoted by $\mathbf{v}_{r,c}$. Suppose that $C_1, \ldots, C_s$ is a set of disjoint sets of columns. Row r is *covering* for $\{C_1, \ldots, C_s\}$ if, whenever we choose s columns $\{\gamma_i \in C_i : 1 \leq i \leq s\}$, the $s \times s$ matrix $[\mathbf{v}_{r,\gamma_1} \cdots \mathbf{v}_{r,\gamma_s}]$ is nonsingular over $\mathbb{F}_q$. Then the $\mathsf{SHF}_\lambda(N; k, q^s, \{w_1, \ldots, w_s\})$ is *covering* if, for every way to choose $\{C_1, \ldots, C_s\}$, there are at least λ covering rows.

We establish that covering separating hash families of type $1^t d^1$ give an effective construction for detecting arrays, which are useful in screening complex systems to find interactions among t or fewer factors without being masked by d or fewer other interactions. This connection easily accommodates outlier and missing responses in the screening. We explore asymptotic existence results and explicit constructions using finite geometries for covering separating hash families. We develop randomized and derandomized construction algorithms and discuss consequences for detecting arrays. This is joint work with Violet R. Syrotiuk (ASU).

Factorizations of infinite graphs

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Let $\mathcal{F} = \{F_\alpha : \alpha \in \mathcal{A}\}$ be a family of infinite graphs. The Factorization Problem $FP(\mathcal{F}, \Lambda)$ asks whether $\mathcal{F}$ can be realized as a factorization of a given infinite graph Λ, namely, whether there is a factorization $\mathcal{G} = \{\Gamma_\alpha : \alpha \in \mathcal{A}\}$ of Λ such that each Γ_α is a copy of F_α.

Inspired by the results on regular 1-factorizations of infinite complete graphs [1] and on the resolvability of infinite designs [4], we study this problem when Λ is either the Rado graph R or the complete graph $K_\aleph$ of infinite order $\aleph$. When $\mathcal{F}$ is a countable family, we show that $FP(\mathcal{F}, R)$ is solvable if and only if each graph in $\mathcal{F}$ has no finite dominating set. Generalizing the existence result of [2], we also prove that $FP(\mathcal{F}, K_\aleph)$ admits a solution whenever the cardinality $\mathcal{F}$ coincides with the order and the domination numbers of its graphs.

Finally, in the case of countable complete graphs, we show some non-existence results when the domination numbers of the graphs in $\mathcal{F}$ are finite.

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On the mini-symposium problem

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Joint work with E. Mendelsohn, B. Stevens, T. Traetta.

The Oberwolfach problem was originally stated as a seating problem:

> Given v attendees at a conference with t circular tables each of which seats a_i people $\left(\sum_{i=1}^{t} a_i = v\right)$. Find a seating arrangement so that every person sits next to each other person around a table exactly once over the r days of the conference.

The Oberwolfach problem thus asks for a decomposition of K_n ($K_n - I$ when n is even) into 2-factors consisting of cycles with lengths $a_1, \ldots, a_t$.

In this talk we introduce the related *mini-symposium problem,* which asks for solutions to the Oberwolfach problem on v points which contains a subsystem on m points. In the seating context above, the larger conference contains a mini-symposium of m participants, and we also require these m participants to be seated together for $\left\lfloor \frac{m-1}{2} \right\rfloor$ of the days.

We obtain a complete solution when the cycle sizes are as large as possible, i.e. m and $v-m$. In addition, we provide extensive results in the case where all cycle lengths are equal, of size k say, completely solving all cases when $m \mid v$, except possibly when k is odd and v is even. In particular, we completely solve the case when all cycles are of length m ($k = m$).

A lower bound on permutation codes of distance $n - 1$

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A construction for mutually orthogonal latin squares (MOLS) inspired by pairwise balanced designs is shown to hold more generally for a class of permutation codes of length n and minimum distance $n-1$. Using ingredients when n equals a prime or a prime plus one, and applying a number sieve, we obtain a general lower bound $M(n, n-1) \geq n^{1.0797}$ on the size of such codes for large n. This represents a small improvement on the guarantee given from MOLS.

Characterizing isomorphism classes of Latin squares by fractal dimensions of image patterns

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Based on the construction of pseudo-random sequences arisen from a given Latin square, Dimitrova and Markovski [1] described in 2007 a graphical representation of quasigroups by means of fractal image patterns. The recognition and analysis of such patterns have recently arisen [2,3] as an efficient new approach for classifying Latin squares into isomorphism classes. This talk delves into this topic by focusing on the use of the differential box-counting method for determining the mean fractal dimension of the homogenized standard sets associated to these fractal image patterns. It constitutes a new Latin square isomorphism invariant which is analyzed in this talk for characterizing isomorphism classes of non-idempotent Latin squares in an efficient computational way.

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Novák's conjecture on cyclic Steiner triple systems and its generalization

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Novák conjectured in 1974 that for any cyclic Steiner triple systems of order v with $v \equiv 1 \pmod 6$, it is always possible to choose one block from each block orbit so that the chosen blocks are pairwise disjoint.

In this talk, we shall consider the generalization of this conjecture to cyclic (v, k, λ)-designs with $1 \leq \lambda \leq k-1$. Superimposing multiple copies of a cyclic symmetric design shows that the generalization cannot hold for all v, but we conjecture that it holds whenever v is sufficiently large compared to k. We confirm that the generalization of the conjecture holds when v is prime and $\lambda = 1$ and also when $\lambda \leq (k-1)/2$ and v is sufficiently large compared to k. As a corollary, we show that for any $k \geq 3$, with the possible exception of finitely many composite orders v, every cyclic $(v, k, 1)$-design without short orbits is generated by a $(v, k, 1)$-disjoint difference family.

An Evans-style result for block designs

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A now-proven conjecture of Evans states that any partial latin square with at most $n-1$ filled cells can be completed to a latin square. This is sharp: there are uncompletable partial latin squares with n filled cells. This talk will discuss the analogous problem for block designs.

An $(n, k, 1)$-*design* is a collection of k-subsets (*blocks*) of a set of n *points* such that each pair of points occur together in exactly one block. If this restriction is relaxed to require only that each pair of points occur together in at most one block we instead have a *partial* $(n, k, 1)$-*design*. I will outline a proof that any partial $(n, k, 1)$-design with at most $\frac{n-1}{k-1} - k + 1$ blocks is completable to a $(n, k, 1)$-design provided that n is sufficiently large and obeys the obvious necessary conditions for an $(n, k, 1)$-design to exist. This result is sharp for all k. I will also mention some related results concerning edge decompositions of almost complete graphs into copies of K_k.

Balancedly splittable orthogonal designs

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As an extension of balancedly splittable Hadamard matrices, the concept of balancedly splittable orthogonal designs is introduced along with a recursive construction. Among the other results, a class of equiangular tight frames over the real, complex, and quaternions meeting the Delsarte-Goethals-Seidel upper bound is obtained.

Resolving sets and identifying codes in finite geometries

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Let $\Gamma = (V, E)$ be a finite, simple, undirected graph. A vertex $v \in V$ is resolved by $S = \{v_1, \ldots, v_n\} \subset V$ if the list of distances $(d(v, v_1), d(v, v_2), \ldots, d(v, v_n))$ is unique. S is a *resolving set* for Γ if it resolves all the elements of V.

A subset $D \subset V$ is a *dominating set* if each vertex is either in D or adjacent to a vertex in D. A vertex s *separates* u and v if exactly one of u and v is in $N[s]$. A subset $S \subset V$ is a *separating set* if it separates every pair of vertices of G. Finally, a subset $C \subset V$ is an *identifying code* for V if it is both a dominating and separating set.

In this talk resolving sets and identifying codes for graphs arising from finite geometries (e.g. Levi graphs of projective and affine planes and spaces, generalized quadrangles) are considered. We present several constructions and give estimates on the sizes of these objects.

20 years of Legendre Pairs

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Legendre pairs were introduced in 2001 by Seberry and her students, as a means to construct Hadamard matrices via a two-circulant core construction. A Legendre pair consists of two sequences of odd length ℓ, with elements from $\{-1, +1\}$, such that their respective autocorrelation coefficients sum to -2, or (equivalently) their respective power spectral density coefficients sum to $2\ell + 2$. Legendre pairs of every odd prime length ℓ exist, via a simple construction using the Legendre symbol. We will review known constructions for Legendre pairs. We will discuss various results on Legendre pairs during the past 20 years, including the concept of compression, introduced in a joint paper with Djokovic, as well as the computational state-of-the-art of the search for Legendre pairs. In particular, we recently contributed the only known Legendre

pair of length $\ell = 77$ in a joint paper with Turner/Bulutoglu/Geyer. In addition, we recently contributed in a joint paper with Koutschan, several Legendre pairs of new lenghts $\ell \equiv 0 \pmod 3$, as well as an algorithm that allows one to determine the full spectrum of values for the $\frac{\ell}{3}$-th power spectral density value. The importance of Legendre pairs lies in the fact that they constitute a promising avenue to the Hadamard conjectrure.

Packings of Partial Difference Sets

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As the underlying configuration behind many elegant finite structures, partial difference sets have been intensively studied in design theory, finite geometry, coding theory, and graph theory. Over the past three decades, there have been numerous constructions of partial difference sets in abelian groups with high exponent, accompanied by numerous very different and delicate techniques. Surprisingly, we manage to unify and extend a great many previous constructions in a common framework, using only elementary methods. The key insight is that, instead of focusing on one single partial difference set, we consider a packing of partial difference sets, namely, a collection of disjoint partial difference sets in a finite abelian group. Although the packing of partial difference sets has been implicitly studied in various contexts, we recognize that a particular subgroup reveals crucial structural information about the packing. Identifying this subgroup allows us to formulate a recursive lifting construction of packings in abelian groups of increasing exponent.

This is joint work with Jonathan Jedwab.

Balanced Equi-n-squares

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We define a d-balanced equi-n-square $L = (l_{ij})$, for some divisor d of n, as an $n \times n$ matrix containing symbols from $\mathbb{Z}_n$ in which any symbol that occurs in a row or column, occurs exactly d times in that row or column. We show how to construct a d-balanced equi-n-square from a partition of a Latin square of order n into $d \times (n/d)$ subrectangles. In design theory, L is equivalent to a decomposition of $K_{n,n}$ into d-regular spanning subgraphs of $K_{n/d,n/d}$. We also study when L is diagonally cyclic, defined as when $l_{(i+1)(j+1)} = l_{ij} + 1$ for all $i, j \in \mathbb{Z}_n$, which corresponds to cyclic such decompositions of $K_{n,n}$ (and thus α-labellings).

We identify necessary conditions for the existence of (a) d-balanced equi-n-squares, (b) diagonally cyclic d-balanced equi-n-squares, and (c) Latin squares of order n which partition into $d \times (n/d)$ subrectangles. We prove the necessary conditions are sufficient for arbitrary fixed $d \geq 1$ when n is sufficiently large, and we resolve the existence problem completely when $d \in \{1, 2, 3\}$.

Along the way, we identify a bijection between α-labellings of d-regular bipartite graphs and, what we call, d-starters: matrices with exactly one filled cell in each top-left-to-bottom-right unbroken diagonal, and either d or 0 filled cells in each row and column. We use d-starters to construct diagonally cyclic d-balanced equi-n-squares, but this also gives new constructions of α-labellings.

Equitably 2-colourable even cycle systems

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An ℓ-cycle decomposition of a graph G is said to be *equitably c-colourable* if there is a c-vertex-colouring of G such that each colour is represented (approximately) an equal number of times on each cycle: more precisely, we ask that in each cycle C of the decomposition, each colour appears on $\lfloor \ell/c \rfloor$ or $\lceil \ell/c \rceil$ of the vertices of C. In this talk, we consider the case $c = 2$ and present some new results on the existence of 2-colourable even ℓ-cycle systems of the cocktail party graph $K_v - I$. In particular, we determine a complete existence result for equitably 2-colourable ℓ-cycle decompositions of $K_v - I$, ℓ even, in the cases that $v \equiv 0, 2 \pmod{\ell}$, or ℓ is a power of 2, or $\ell \in \{2q, 4q\}$ for q an odd prime power, or $\ell \leq 30$. We will also discuss some work in progress on analogous problems for cycles of odd length.

Weakly self-orthogonal designs and related linear codes

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A 1-design is weakly self-orthogonal if all the block intersection numbers have the same parity. If both k and the block intersection numbers are even then 1-design is called self-orthogonal and its incidence matrix generates a self-orthogonal code. We analyze extensions, of the incidence matrix and an orbit matrix of a weakly self-orthogonal 1-design, that generates a self-orthogonal code over the finite field. Additionally, we develop methods for constructing LCD codes by extending the incidence matrix and an orbit matrix of a weakly self-orthogonal 1-design.

The classification of 2-(v, k, λ) designs, with $\lambda > 1$ and $(r, \lambda) = 1$, admitting a flag-transitive automorphism group

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A classical subject in Theory of Designs is the study of 2-designs admitting an automorphism group fulfilling prescribed properties. Within this research area, it is of great interest the study of 2-(v, k, λ) designs $\mathcal{D}$ admitting a flag-transitive automorphism group G. Since they have been classified for $\lambda = 1$ and $G \nleq A\Gamma L_1(q)$ by Buekenhout et al. (1990), a special attention is devoted to the general case $\lambda > 1$. In this setting, a first natural generalization of the case $\lambda = 1$ is represented by $\lambda > 1$ and $\gcd(r, \lambda) = 1$, where r is the replication number of $\mathcal{D}$. Then G acts point-primitively on $\mathcal{D}$ by a result of Dembowski (1968), and $Soc(G)$, the socle of G, is either an elementary abelian p-group for some prime p, or a non abelian simple group by a result of Zeischang (1988). Starting from these two results, such 2-designs have been recently classified for $G \nleq A\Gamma L_1(q)$ by Biliotti et al. and by Alavi, Zhou et al. according to whether $Soc(G)$ is an elementary abelian p-group or a non abelian simple group, respectively.

The aim of the talk is to survey the classification of 2-(v, k, λ) designs $\mathcal{D}$, with $\lambda > 1$

and $(r, \lambda) = 1$, admitting a flag-transitive automorphism group G, mostly focusing on the constructions of the various examples contained in it.

Testing isomorphism of circulant objects in polynomial time

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We show that isomorphism testing of two cyclic combinatorial objects may be done in a polynomial time of their sizes provided that both objects share the same regular cyclic group of automorphisms given in advance.

Strictly additive 2-designs

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This work draws inspiration from an interesting theory developed in [1]. A design $(V, \mathcal{B})$ is said to be *additive* if V is a subset of an abelian group G and the elements of any block $B \in \mathcal{B}$ sum up to zero. We propose to speak of a *strictly additive* design when V coincides with G.

Up to last year, apart from the obvious examples of the $2-(q^n, q, 1)$ designs associated with the affine geometry AG(n, q), all known strictly additive 2-designs had a quite “big" λ. Very recently, a strictly additive $2-(81, 6, 2)$ design has been found in [3]. This design, besides being *simple* (the only design with these parameters previously known [2] has sixteen pairs of repeated blocks), has the property that every block is union of two parallel lines of AG$(4, 3)$.

In the attempt of getting other strictly additive designs with this property we found some infinite series of 2-designs whose parameter-sets are probably new.

In this talk, besides presenting the above series, I will try to outline a proof that for every odd k, there are infinitely many values of v for which a strictly additive $2-(v, k, 1)$ design exists.

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Sequences in $\mathbb{Z}_n$ with Distinct Partial Sums

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Let $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ be the integers modulo n. For a sequence of elements $a_1, \ldots, a_k$ of $\mathbb{Z}_n$, define its *partial sums* $b_0, \ldots, b_k$ by $b_0 = 0$ and $b_i = a_1 + \cdots + a_i$ for $1 \leq i \leq k$. For which subsets $S \subseteq \mathbb{Z}_n \setminus \{0\}$ is it possible to order the elements of S so that the partial sums are distinct?

When the sum of the elements of S is 0, there can be no such ordering. Alspach conjectures that this is the only obstacle; that is, every subset S whose sum is nonzero has an ordering with distinct partial sums.

We show how to translate the problem into one of finding monomials with non-zero coefficients in particular polynomials over $\mathbb{Z}_p$, where p is a prime divisor of n, using Alon's Combinatorial Nullstellensatz. The approach offers hope for a full proof of the conjecture, and can be used in conjunction with a computational approach in cases where $n = pt$ with p prime and t and $|S|$ small.

Universal sequences and Euler tours in hypergraphs

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We show that a quasirandom k-uniform hypergraph G has a tight Euler tour subject to the necessary condition that k divides all vertex degrees. The case when G is complete confirms a $100 conjecture of Chung, Diaconis and Graham from 1989 on the existence of universal cycles for the k-subsets of an n-set.

Our proof is based on random walks and the proof of the existence of H-designs (by Glock, Kühn, Lo and Osthus), i.e. decompositions of a complete hypergraph into copies of an arbitrary hypergraph H (subject to divisibility conditions).

A reduction of the spectrum problem for sun systems

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A k-cycle with a pendant edge attached to each vertex is called a k-sun. When we approached the existence problem for k-sun systems of order v, complete solutions were known only for $k = 3, 4, 5, 6, 8, 10, 14$ and for $k = 2^t$. Here, we reduce this problem to the orders v in the range $2k < v < 6k$ satisfying the obvious necessary conditions. Thanks to this result, we provide a complete solution whenever k is an odd prime, and some partial results whenever k is twice a prime.

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Regular 1-factorizations of complete graphs with orthogonal spanning trees

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A 1-factorization $\mathcal{F}$ of a complete graph K_{2n} is said to be G-regular if G is an automorphism group of $\mathcal{F}$ acting sharply transitively on the vertex-set. The problem of determining which groups can realize such a situation dates back to a result by Hartman and Rosa (1985) which solved the problem when G is a cyclic group. It is also well known that this problem simplifies somewhat when n is odd: G must be the semi-direct product of Z_2 with its normal complement and G always realizes a $1-$factorization of K_{2n} upon which it acts sharply transitively on vertices. When n is even the problem is still open, even though several classes of groups were tested in the recent past. An attempt to obtain a fairly precise description of groups and 1-factorizations satisfying this symmetry constraint could be done by imposing further conditions. For example some non existence results were achieved by assuming the existence of a 1-factor fixed by the action of the group, further results were obtained when the number of fixed $1-$factors is as large as possible. In this talk we focus our attention on the possibility of constructing G-regular 1-factorizations of K_{2n} together with a complete set of isomorphic spanning trees orthogonal to the $1-$factorization. Here orthogonal tree means that the tree shares exactly one edge with each $1-$factor. We see how to realize such a situation when n is odd and examine some classes of groups in the case n even.

A few new triplanes

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An incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$, with point set $\mathcal{P}$, block set $\mathcal{B}$ and incidence I is a t-(v, k, λ) design, if $|\mathcal{P}| = v$, every block $B \in \mathcal{B}$ is incident with precisely k points, and every t distinct points are together incident with precisely λ blocks. We consider triplanes, i.e. symmetric block designs with $\lambda = 3$. Triplanes of order 12, i.e. symmetric (71,15,3) designs, have the greatest number of points among all known triplanes and it is not known if a triplane $(v, k, 3)$ exists for $v > 71$.

In this talk, in addition to reviewing previously known results, we give the first example of a triplane of order 12 that doesn't admit an automorphism of order 3.

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Merging Combinatorial Design and Optimization: the Oberwolfach Problem

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Combinatorial optimization is a subset of mathematical optimization that is related to operations research, algorithm theory, and computational complexity theory. It consists of finding an (optimal) object from a finite set of objects and in many such problems, exhaustive search is not tractable. Combinatorial design theory is the part of combinatorial mathematics that deals with the existence, construction and properties of systems of finite sets whose arrangements satisfy generalized concepts of balance and/or symmetry. Combinatorial Design and Combinatorial Optimization, though apparently different research fields, share common problems, such as for example sudokus, covering arrays, tournament design and more in general problems that can be represented on graphs. Aim of the talk is to present intersections and possible contributions to Combinatorial Design given by the application of Combinatorial Optimization techniques and solution methods. This is accomplished by the presentation of results on the Oberwolfach Problem (OP) where interaction of methods from both domains enabled us to solve large OP instances in limited computational time and at the same time to derive a theoretical result for general classes of instances.

On the existence of large set of partitioned incomplete Latin squares

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In this talk, we survey the existence of large sets of partitioned incomplete Latin squares (LSPILS). Algebraic and combinatorial methods are employed to construct the large sets of partitioned incomplete Latin squares of type $g^n u^1$. Furthermore, we prove that there exists a pair of orthogonal LSPILS$(1^p u^1)$s for any odd u and some even values of u, where p is a prime. Lastly, we propose some problems for further research.

Bipartite 2-factorizations of complete multigraphs via layering

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Layering is in principle a simple method that allows us to obtain a type-specific 2-factorization of a complete multigraph (or complete multigraph minus a 1-factor) from existing 2-factorizations of complete multigraphs and complete multigraphs minus a 1-factor. This technique is particularly effective when constructing bipartite 2-factorizations; that is, 2-factorizations with all cycles of even length.

In this talk, we shall give a thorough introduction to layering, and then describe new bipartite 2-factorizations of complete multigraphs obtained by layering. In particular, for complete multigraphs and bipartite 2-factors with no 2-cycles, we obtain a complete solution to the Oberwolfach Problem and an almost complete solution to the Hamilton-Waterloo Problem.

This is joint work with Amin Bahmanian.

On some periodic Golay pairs and pairwise balanced designs

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In this talk we will show how a relationship between certain pairwise balanced designs with v points and periodic Golay pairs of length v can be useful to construct periodic Golay pairs. The talk is based on recent work [1] where we construct pairwise balanced designs with v points under specific block conditions having an assumed cyclic automorphism group, and using isomorph rejection which is compatible with equivalence of corresponding periodic Golay pairs, we complete a classification of periodic Golay pairs of length less than 40, up to equivalence. Further, we will show how we use similar tools to construct new periodic Golay pairs of lengths greater than 40 where classifications remain incomplete, and demonstrate that under some extra conditions on its automorphism group, a periodic Golay pair of length 90 do not exist.

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Dual incidences and t-designs in elementary abelian groups

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Let q be a prime and E_{q^n} is an elementary abelian group of order q^n. Let $\mathcal{H}$ be a collection of some subgroups of E_{q^n} of order q^k. A pair $(E_{q^n}, \mathcal{H})$ is a $t-(n,k,\lambda)_q$ design if every subgroup of E_{q^n} of order q^t is contained in exactly λ groups from $\mathcal{H}$. This definition corresponds to the classical definition of a q-analog design.

We introduce two incidence structures denoted by $\mathcal{D}_{max}$ and $\mathcal{D}_{min}$ with $\mathcal{H}$ as set of points. The blocks of $\mathcal{D}_{max}$ are labeled by maximal subgroups of E_{q^n}, while the blocks of $\mathcal{D}_{min}$ are labeled by groups of order q.

We fully describe a duality between $\mathcal{D}_{max}$ and $\mathcal{D}_{min}$ by proving some identities over group rings. The proven results are used to provide a full description of incidence matrices of $\mathcal{D}_{max}$ and $\mathcal{D}_{min}$ and their mutual dependance.

On the Oberwolfach Problem for single-flip 2-factors via graceful labelings

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The Oberwolfach Problem, posed by Ringel in 1967 and still open, asks for each odd integer $v > 1$ and each 2-regular graph F of order v to determine whether there is a decomposition of the complete graph K_v into copies of F.

We construct solutions whenever F has a sufficiently large odd cycle meeting a specified lower bound and, in addition, F has a single-flip automorphism (i.e. an involutory automorphism acting as a reflection on exactly one cycle). For even orders v, we give analogous results for the maximum packing and minimum covering variants of the problem. We also show a similar result when the edges of K_v have multiplicity 2, but in this case we only require that F has a sufficiently large cycle.

Our methods build on the techniques used in [2] and involve a doubling construction defined in [1] which we apply to graceful labelings of 2-regular graphs with a vertex removed, allowing us to explicitly construct solutions to the Oberwolfach Problem with well-behaved automorphisms.

This is joint work with Andrea Burgess and Peter Danziger.

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On the classification of unitals on 28 points of low rank

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Unitals are combinatorial 2-$(q^3+1, q+1, 1)$ designs. In 1981, Brouwer constructed 138 non-isomorphic unitals for $q = 3$, i.e. 2-$(28, 4, 1)$ designs. He observed that the 2-rank of the constructed unitals is at least 19, where the p-rank of a design is defined as the rank of the incidence matrix between points and blocks of the design over the finite field GF(p). In 1998, McGuire, Tonchev and Ward proved that indeed the 2-rank of a unital on 28 points is between 19 and 27 and that there is a unique 2-$(28, 4, 1)$ design, the Ree unital $R(3)$, with 2-rank 19. In the same year, Jaffe and Tonchev showed that there is no unital on 28 points of 2-rank 20 and there are exactly 4 isomorphism classes of unitals of rank 21.

Here, we present the complete classification by computer of unitals of 2-rank 22, 23 and 24. There are 12 isomorphism classes of unitals of 2-rank 22, 78 isomorphism classes of unitals of 2-rank 23, and 298 isomorphism classes of unitals of 2-rank 24.

Heffter Arrays and Biembbedings of Cycle Systems on Orientable Surfaces

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Coauthors: Diane Donovan, Kevin Burrage, Nicholas Cavenagh

In this talk we will review the recent developments on square Heffter arrays, $H(n; k)$, and their applications on face 2-colourable embeddings of the complete graph K_{2nk+1} on an orientable surfaces.

Square Heffter arrays, $H(n; k)$, are $n\times n$ arrays such that each row and each column contains k filled cells, each row and column sum is divisible by $2nk + 1$ and either x or $-x$ appears in the array for each integer $1 \leq x \leq nk$.

Archdeacon noted that a Heffter array, satisfying two additional conditions, yields a face 2-colourable embedding of the complete graph K_{2nk+1} on an orientable surface, where for each colour, the faces give a k-cycle system. These necessary conditions pertain to cyclic orderings of the entries in each row and each column of the Heffter array and are: (1) for each row and each column the sequential partial sums determined by the cyclic ordering must be distinct modulo $2nk + 1$; (2) the composition of the cyclic orderings of the rows and columns is equivalent to a single cycle permutation on the entries in the array.

We construct Heffter arrays that satisfy condition (1) whenever (a) $k \equiv 0 \ mod\ 4$; or (b) $n \equiv 1 \ mod\ 4$ and $k \equiv 3 \ mod\ 4$; or (c) $n \equiv 0 \ mod\ 4$, $k \equiv 3 \ mod\ 4$ and $n \gg k$. As a corollary to the above we obtain pairs of orthogonal k-cycle decompositions of K_{2nk+1}.

Furthermore we study when these arrays satisfy condition (2). We show the existence of face 2-colourable embeddings of cycle decompositions of the complete graph when $n \equiv 1 \ mod\ 4$ and $k \equiv 3 \ mod\ 4$, $n \gg k \geq 7$ (provided that when $n \equiv 0 \ mod\ 3$ then $k \equiv 7 \ mod\ 12$).

Block designs constructed from orbit matrices using a modified genetic algorithm

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Genetic algorithms (GA) are search and optimization heuristic population-based methods which are inspired by the natural evolution process. In this talk, we will present a method of constructing incidence matrices of block designs combining the method of construction with orbit matrices and a modified genetic algorithm. With this method we managed to find some new non-isomorphic S(2,5,45) designs.

Minisymposium

CONFIGURATIONS (MS-81)

Organized by Gábor Gévay, *University of Szeged, Bolyai Institute, Hungary*

- Configurations from strong deficient difference sets, *Marién Abreu*
- Splittability of cubic bicirculants and their related configurations, *Nino Bašić*
- Connected (n_k) configurations exist for almost all n, *Leah Berman*
- Transitions between configurations, *Gábor Gévay*
- Combinatorial Configurations and Dessins d'Enfants, *Milagros Izquierdo*
- Highly symmetric configurations, *Jurij Kovič*
- Strongly regular configurations, *Vedran Krčadinac*
- Hirzebruch-type inequalities and extreme point-line configurations, *Piotr Pokora*
- 4-lateral matroids induced by n_3-configurations, *Michael Raney*
- Jordan schemes, *Sven Reichard*
- Taxonomy of Three-Qubit Doilies, *Metod Saniga*
- Barycentric configurations in real space, *Hendrik Van Maldeghem*

Configurations from strong deficient difference sets

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Coauthors: Martin Funk, Domenico Labbate, Vedran Krčadinac

In [1] we have studied combinatorial configurations with the associated point and line graphs being strongly regular, which we call *strongly regular configurations*. In the talk "Strongly regular configurations" of this minisymposium, Vedran Krčadinac will present existing known families of strongly regular configurations; constructions of several other families; necessary existence conditions and a table of feasible parameters with at most 200 points.

Let G be a group of order v. A subset $D \subset G$ of size k is a *deficient difference set* the left differences $d_1^{-1}d_2$ are all distinct. Considering the elements of G as points and the *development* $devD = \{gD|g \in G\}$ as lines a symmetric (v_k) configuration is obtained and it has G as its automorphism group acting regularly on the points and lines. Let $\Delta(D) = \{d_1^{-1}d_2|d_1, d_2 \in D, d_1 \neq d_2\}$ be the set of left differences of D. For a group element $x \in G \setminus \{1\}$, denote by $n(x) = |\Delta(D) \cap x\Delta(D)|$. If $n(x) = \lambda$ for every $x \in \Delta(D)$, and $n(x) = \mu$ for every $x \notin \Delta(D)$, D is said to be a *strong deficient difference set* $(SDDS)$ for $(v_k; \lambda, \mu)$.

Here, we present one of the new families of strongly regular configurations constructed in [1], with parameters different from semipartial geometries and arising from strong deficient difference sets, as well as two examples arising from Hall's plane and its dual. Moreover, from the exhaustive search performed in groups of order $v \leq 200$ further four examples corresponding to strong deficient difference sets, but not in the previous families, are obtained.

References

[1] M. Abreu, M. Funk, V. Krčadinac, D. Labbate, Strongly regular configurations, preprint, 2021. https://arxiv.org/abs/2104.04880

Splittability of cubic bicirculants and their related configurations

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Coauthors: Matjaž Krnc, Tomaž Pisanski

Recently, it was shown that there exist infinitely many splittable and also infinitely many unsplittable cyclic (n_3) configurations. This was achieved by studying splittability of trivalent cyclic Haar graphs. We extend this study to include cubic bicirculant and their related configurations.

Connected (n_k) configurations exist for almost all n

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Coauthors: Gábor Gévay, Tomaž Pisanski

A geometric (n_k) configuration is a collection of points and straight lines, typically in the Euclidean plane, so that each line passes through k of the points and each of the points lies on k of the lines. In a series of papers, Branko Grünbaum showed that geometric (n_4) configurations exist for all $n \geq 24$, using a series of geometric constructions later called the "Grünbaum Calculus". In this talk, we will show that for each $k > 4$, there exists an integer N_k so that for *all* $n \geq N_k$, there exists at least one (n_k) configuration, by generalizing the Grünbaum Calculus operations to produce more highly incident configurations. This is joint work with Gábor Gévay and Tomaž Pisanski.

Transitions between configurations

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In this talk we briefly review (without aiming at completeness) various procedures by means of which new configurations can be obtained from old configurations. Among them, there are binary operations, like e.g. the *Cartesian product* and the *incidence sum*. Several other operations are collectively called the *Grünbaum calculus*. The *incidence switch* operation can be defined on the level of incidence graphs (also called Levi graphs) of configurations, and in some cases it is in close connection with realization problems of configurations as well as with incidence theorems. Considering point-line, point-circle and point-conic configurations, there are interesting ad hoc constructions by which from a configuration of one of these geometric types another one can be derived, thus realizing *transitions* between configurations in a very general sense. We also present interesting examples and applications. The most recent results mentioned in this talk are based on joint work with Tomaž Pisanski and Leah Wrenn Berman.

Combinatorial Configurations and Dessins d'Enfants

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Linköping University, Sweden

In this talk we give an overview, with many examples, of how dessins d'enfants (maps and hypermaps) on Riemann surfaces produce point-circle realisations on higher genus of combinatorial configurations. The symmetry of the hypermap is transfered to the symmetry of the configuration.

Highly symmetric configurations

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Coauthor: Aleksander Simonič

In the talk some general methods and special techniques for the construction of highly symmetric configurations will be presented.

As an application of these theoretical principles, some examples of highly symmetric spatial geometric configurations of points and lines with the symmetry of Platonic solids will be given.

Strongly regular configurations

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Coauthors: Marien Abreu, Martin Funk, Domenico Labbate

We report on the recent study [1] of combinatorial configurations with the associated point and line graphs being strongly regular. A prominent family of such configurations are the partial geometries, introduced by R. C. Bose in 1963. We focus on such configurations that are not partial geometries nor known generalisations such as semipartial geometries and strongly regular (α, β)-geometries, neither are they elliptic semiplanes of P. Dembowski. Several families are constructed, necessary existence conditions are proved, and a table of feasible parameters with at most 200 points is presented.

References

[1] M. Abreu, M. Funk, V. Krčadinac, D. Labbate, *Strongly regular configurations*, preprint, 2021. https://arxiv.org/abs/2104.04880

Hirzebruch-type inequalities and extreme point-line configurations

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In my talk I would like to report on very recent developments devoted to extreme point-line configurations from an algebraic perspective of Hirzebruch-type inequalities. We recall some inequalities, especially the most powerful variant based on orbifolds, and then I will report on extreme combinatorial problems for which Hirzebruch-type inequalities played a decisive role. Time permitting, we will present a short proof of the Weak Dirac Conjecture.

4-lateral matroids induced by n_3-configurations

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A *4-lateral matroid* induced by an n_3-configuration is a rank-4 matroid whose ground set consists of the blocks (lines) of the configuration, and for which any 4 elements of the ground set are independent if and only if they do not determine a 4-lateral within the configuration. We characterize the n_3-configurations which induce 4-lateral matroids, and provide examples of small n_3-configurations which do so.

Jordan schemes

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Association schemes (also called homogeneous coherent configurations) first appeared in Statistics in relation to the design of experiments, due to the efforts of Bose and his collaborators.

These systems of binary relations defined on the same set, color graphs in other terms, give rise to certain matrix algebras, not necessarily commutative. In this way a bridge is formed between combinatorics and algebra, in particular with permutation groups.

In statistial applications we want the relations to be symmetric, while the product of symmetric matrices is not symmetric in general. Therefore Bailey and Cameron, following Shah (from the same school as Bose) suggested to replace the matrix product AB by the Jordan product $A*B = (AB+BA)/2$, which is commutative but not necessarily associative. The resulting structures are called Jordan schemes.

Given any association scheme, its symmetrization is a Jordan scheme. This led Peter Cameron to the following question: Do all Jordan schemes arise in this way, or do there exist "proper" Jordan schemes?

We gave a positive answer to this question by constructing a first proper Jordan scheme on 15 points using so-called Siamese color graphs, investigated earlier by our group. First elements of the theory of these structures were established; a few infinite classes of proper Jordan schemes were discovered. Moreover an efficient computational criterion for recognizing proper Jordan schemes is given, based on the classical Weisfeiler-Leman stabilization.

The current talk mainly focuses on the computer search for proper Jordan schemes, which is based on algorithmic ideas of Hanaki and Miyamoto, who enumerated all small association schemes. It turns out that the initial example is indeed the smallest. Besides it, up to isomorphism three more examples on less than 20 points were discovered. Each such small example of orders 15, 16 and 18 can be constructed from a suitable association scheme by a certain switching operation.

For more on the background of Jordan schemes see Cameron's blog:
https://cameroncounts.wordpress.com/2019/06/28/proper-jordan-schemes-exist/

This is part of a joint project with Misha Klin and Misha Muzychuk from Ben-Gurion University, Beer Sheva.

Taxonomy of Three-Qubit Doilies

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Coauthors: Henri de Boutray, Frédéric Holweck, Alain Giorgetti

We study doilies (i. e., $W(3,2)$'s) living in $W(5,2)$, when the points of the latter space are parametrized by canonical three-fold products of Pauli matrices and the associated identity matrix (i. e., by three-qubit observables). Key characteristics of such a doily are: the number of its negative lines, distribution of types of observables, character of the geometric hyperplane the doily shares with the distinguished (non-singular) quadric of $W(5,2)$ and the structure of its Veldkamp space. $W(5,2)$ is endowed with 90 negative lines of two types and its 1344 doilies fall into 13 types. 279 out of 480 doilies with three negative lines are composite, i. e. they all originate from the two-qubit doily by selecting in the latter a geometric hyperplane and formally adding to each two-qubit observable, at the same position, the identity matrix if an observable lies on the hyperplane and the same Pauli matrix for any other observable. Further, given a doily and any of its geometric hyperplanes, there are other three doilies possessing the same hyperplane. There is also a particular type of doilies a representative of which features a point each line through which is negative.

Barycentric configurations in real space

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Barycentric configurations are configurations with three points per line in real projective space such that (homogeneous) coordinates exist with the property that, for each line, the sum of the coordinate tuples of the three points on that line is the zero tuple. Such configurations turn up naturally and we develop some theory about them. In particular there exist universal barycentric embeddings and a general construction method if the geometry is self-polar. We apply these results to the Biggs-Smith geometry on 102 points, providing a (new) geometric construction of the Biggs-Smith graph making the full automorphism group apparent. We also mention a connection with ovoids.

Minisymposium

Extremal and Probabilistic Combinatorics (MS-20)

Organized by Deryk Osthus, *University of Birmingham, United Kingdom*

- Universal phenomena for random constrained permutations, *Jacopo Borga*
- Progress towards Nash-Williams' Conjecture on Triangle Decompositions, *Michelle Delcourt*
- New results for MaxCut in H-free graphs, *Stefan Glock*
- How does the chromatic number of a random graph vary?, *Annika Heckel*
- On graph norms, *Jan Hladky*
- Extremal density for sparse minors and subdivisions, *Jaehoon Kim*
- Progress on intersecting families, *Andrey Kupavskii*
- Geometric constructions for Ramsey-Turán theory, *Hong Liu*
- A solution to Erdős and Hajnal's odd cycle problem, *Richard Montgomery*
- Counting transversals in group multiplication tables, *Rudi Mrazović*
- Recent advances in Ramsey theory, *Dhruv Mubayi*
- On a problem of M. Talagrand, *Jinyoung Park*
- Ramsey properties of randomly perturbed sets of integers and the asymmetric random Rado theorem, *Yury Person*
- Packing D-degenerate graphs, *Diana Piguet*
- Erdos-Rademacher Problem, *Oleg Pikhurko*
- On Hadwiger's Conjecture, *Luke Postle*

Universal phenomena for random constrained permutations

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How do local/global constraints affect the limiting shape of random permutations? This is a classical question that has received considerable attention in the last 15 years. In this talk we give an overview of some recent results on this topic, mainly focusing on random pattern-avoiding permutations. We first introduce a notion of scaling limit for permutations, called permutons. Then we present some recent results that highlight certain universal phenomena for permuton limits of various families of pattern-avoiding permutations. These results will lead us to the definition of three remarkable new limiting random permutons: the “biased Brownian separable permuton”, the “Baxter permuton” and the "skew Brownian permuton". We finally discuss some recent results that show how permuton limits are useful to investigate the behaviour of certain statistics on random pattern-avoiding permutations, such as the length of the longest increasing subsequence.

Progress towards Nash-Williams’ Conjecture on Triangle Decompositions

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Partitioning the edges of a graph into edge disjoint triangles forms a triangle decomposition of the graph. A famous conjecture by Nash-Williams from 1970 asserts that any sufficiently large, triangle divisible graph on n vertices with minimum degree at least 0.75 n admits a triangle decomposition. In the light of recent results, the fractional version of this problem is of central importance. A fractional triangle decomposition is an assignment of non-negative weights to each triangle in a graph such that the sum of the weights along each edge is precisely one.

We show that for any graph on n vertices with minimum degree at least 0.827327 n admits a fractional triangle decomposition. Combined with results of Barber, Kühn, Lo, and Osthus, this implies that for every sufficiently large triangle divisible graph on n vertices with minimum degree at least 0.82733 n admits a triangle decomposition. This is a significant improvement over the previous asymptotic result of Dross showing the existence of fractional triangle decompositions of sufficiently large graphs with minimum degree more than 0.9 n. This is joint work with Luke Postle.

New results for MaxCut in H-free graphs

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Coauthors: Oliver Janzer, Benny Sudakov

The MaxCut problem asks for the size $\mathrm{mc}(G)$ of a largest cut in a graph G. It is well known that $\mathrm{mc}(G) \geq m/2$ for any m-edge graph G, and the difference $\mathrm{mc}(G) - m/2$ is called the *surplus* of G. The study of the surplus of H-free graphs was initiated by Erdős and Lovász in the 70s, who in particular asked what happens for triangle-free graphs. This was famously resolved by Alon, who showed that in the triangle-free case the surplus is $\Omega(m^{4/5})$, and found constructions matching this bound.

We prove several new results in this area. First, we obtain an optimal bound when H is an odd cycle, adding to the lacunary list of graphs for which such a result is known. Secondly, we extend the result of Alon in the sense that we prove optimal bounds on the surplus of general graphs in terms of the number of triangles they contain. Thirdly, we improve the currently best bounds for K_r-free graphs.

Our proofs combine techniques from semidefinite programming, probabilistic reasoning, as well as combinatorial and spectral arguments.

How does the chromatic number of a random graph vary?

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If we pick an n-vertex graph uniformly at random, how much does its chromatic number vary? In 1987 Shamir and Spencer proved that it is contained in some sequence of intervals of length about $n^{1/2}$. Alon improved this slightly to $n^{1/2}/\log n$. Until recently, however, no non-trivial lower bounds on the fluctuations of the chromatic number of a random graph were known, even though the question was raised by Bollobás many years ago. We will present the main ideas needed to prove that, at least for infinitely many values n, the chromatic number of a uniform n-vertex graph is not concentrated on fewer than $n^{1/2-o(1)}$ consecutive values.

We will also discuss the Zigzag Conjecture, made recently by Bollobás, Heckel, Morris, Panagiotou, Riordan and Smith: this proposes that the correct concentration interval length 'zigzags' between $n^{1/4+o(1)}$ and $n^{1/2+o(1)}$, depending on n.

Joint work with Oliver Riordan.

On graph norms

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The concept of graph norms, introduced by Hatami in 2010 (following suggestions from Lovász and Szegedy), is a convenient tool to work with subgraph densities. The definition is real-analytic, making use of the framework of graphons. I will describe several results obtained by in collaboration with Doležal-Grebík-Rocha-Rozhoň and Garbe-Lee. This in particular includes a characterization norming graphs (and alike) using the 'step-Sidorenko' property and a characterization of disconnected norming graphs.

Extremal density for sparse minors and subdivisions

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We prove an asymptotically tight bound on the extremal density guaranteeing subdivisions of bounded-degree bipartite graphs with a mild separability condition. As corollaries, we prove that: $(1 + o(1))t^2$ average degree is sufficient to force the $t \times t$ grid as a topological minor;

$(3/2 + o(1))t$ average degree forces every t-vertex planar graph as a minor, and the constant $3/2$ is optimal; a universal bound of $(2 + o(1))t$ on average degree forcing every t-vertex graph in any nontrivial minor-closed family as a minor, and the constant 2 is best possible. This is joint work with John Haslegrave and Hong Liu.

Progress on intersecting families

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We say that a family of sets is intersecting, if any two of its sets intersect. The Erdos-Ko-Rado theorem characterizes the largest intersecting families of k-element subets of an n-element set, and a lot of studies have been devoted to the following vague question: what structure could a family have, given that it has size close to extremal? This question has many possible answers, depending on the structure we look for, and I am going to discuss several of them during my talk. These will include junta approximations, diversity and covering number.

Such structural results have implications for other related problems, and I will cover at least one such application to the colorings of Kneser graphs. Recall that a Kneser graph is a graph on the collection of all k-element subsets of an n-element set, with two sets connected by an edge when they are disjoint. One of the famous results of Lovász is the topological proof that such graph has chromatic number $n - 2k + 2$. We discuss the following problem: for which values of $n = n(k)$ we can guarantee that one of the colors is trivial, i.e., it consists of sets containing a fixed element?

Partially based on joint works with Peter Frankl and Sergei Kiselev.

Geometric constructions for Ramsey-Turán theory

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Combining two classical notions in extremal combinatorics, the study of Ramsey-Turán theory seeks to determine, for integers $m \le n$ and $p \le q$, the number $\mathrm{RT}_p(n, K_q, m)$, which is the maximum size of an n-vertex K_q-free graph in which every set of at least m vertices contains a K_p.

Two major open problems in this area from the 80s ask: (1) whether the asymptotic extremal structure for the general case exhibits certain periodic behaviour, resembling that of the special case when $p = 2$; (2) constructing analogues of Bollobás-Erdős graphs with densities other than $1/2$.

We refute the first conjecture by witnessing asymptotic extremal structures that are drastically different from the $p = 2$ case, and address the second problem by constructing Bollobás-Erdős-type graphs using high dimensional complex spheres with *all rational* densities. Some matching upper bounds are also provided.

A solution to Erdős and Hajnal's odd cycle problem

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Coauthor: Hong Liu

This talk will address how to construct cycles of many different lengths in graphs, in particular answering the following two problems on odd and even cycles. Erdős and Hajnal asked in 1981 whether the sum of the reciprocals of the odd cycle lengths in a graph diverges as the chromatic number increases, while, in 1984, Erdős asked whether there is a constant C such that every graph with average degree at least C contains a cycle whose length is a power of 2.

This is joint work with Hong Liu.

Counting transversals in group multiplication tables

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Coauthors: Sean Eberhard, Freddie Manners

Hall and Paige conjectured in 1955 that the multiplication table of a finite group G has a transversal (a set of $|G|$ cells in distinct rows and columns and having different symbols) if and only if G satisfies a straightforward necessary condition. This was proved in 2009 by Wilcox, Evans, and Bray using the classification of finite simple groups and extensive computer algebra. I will discuss joint work with Sean Eberhard and Freddie Manners in which we approach the problem in a more analytic way that enables us to asymptotically count transversals.

Recent advances in Ramsey theory

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For many decades, randomness has been the central idea to address the question of determining growth rates of graph Ramsey numbers. Recently, we proved a theorem which suggests that "pseudo-randomness" and not complete randomness may in fact be a more important concept for this area. Consequently, we reduce one of the main open problems in graph Ramsey theory to the possibly simpler question of constructing certain pseudorandom graphs. This new connection widens the possibility to use tools from algebra, geometry, and number theory to address the fundamental questions in Ramsey theory. This is joint work with Jacques Verstraete.

On a problem of M. Talagrand

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Coauthors: Jeff Kahn, Keith Frankston

We will discuss some special cases of a conjecture of M. Talagrand relating two notions of "threshold" for an increasing family $\mathcal{F}$ of subsets of a finite set X. The full conjecture implies equivalence of the "Fractional Expectation-Threshold Conjecture," due to Talagrand and recently proved by Frankston, Kahn, Narayanan, and myself, and the (stronger) "Expectation-Threshold Conjecture" of Kahn and Kalai.

The conjecture under discussion here says there is a fixed J such that if, for a given increasing family $\mathcal{F}$, $p \in [0,1]$ admits $\lambda : 2^X \to \mathbb{R}^+$ with

$$\sum_{S \subseteq F} \lambda_S \geq 1 \quad \forall F \in \mathcal{F}$$

and

$$\sum_{S} \lambda_S p^{|S|} \leq 1/2,$$

then p/J admits such a λ taking values in $\{0,1\}$.

Talagrand showed this when λ is supported on singletons and suggested a couple of more challenging test cases. In the talk, I will give more detailed descriptions of this problem, and some proof ideas if time allows.

Ramsey properties of randomly perturbed sets of integers and the asymmetric random Rado theorem

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Coauthor: Elad Aigner-Horev

We discuss some results in randomly perturbed sets of integers, i.e. sets of the form $A \cup [n]_p$, and thresholds for asymmetric Rado properties in $[n]_p$, which guarantee existence of monochromatic solutions to certain systems of linear equations.

Packing D-degenerate graphs

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Coauthors: Peter Allen, Julia Böttcher, Jan Hladký, Dennis Clemens, Anusch Taraz

A family $\{H_1, \ldots, H_k\}$ of graphs packs in a host graph G, if there is a colouring of the edges of G with colours $1, \ldots, k$ so that there is a copy of H_i in the subgraph of G induced by colour i. A conjecture of Gyárfás from 1976, now referred to as the Tree Packing Conjecture, says that if T_i is an i-vertex tree for each $1 \leq i \leq n$, then $\{T_1, \ldots, T_n\}$ packs in the complete graph K_n. We shall present a result on packing D-degenerate graphs which has direct implications on the Tree Packing Conjecture.

Erdos-Rademacher Problem

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We will give an overview of recent results on the Erdos-Rademacher Problem which asks for the minimum number of r-cliques that a graph with n vertices and m edges can have.

On Hadwiger's Conjecture

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We discuss recent progress on Hadwiger's conjecture. In 1943, Hadwiger conjectured that every graph with no K_t minor is $(t-1)$-colorable for every $t \geq 1$. In the 1980s, Kostochka and Thomason independently proved that every graph with no K_t minor has average degree $O(t\sqrt{\log t})$ and hence is $O(t\sqrt{\log t})$-colorable. Recently, Norin, Song and I showed that every graph with no K_t minor is $O(t(\log t)^{\beta})$-colorable for every $\beta > 1/4$, making the first improvement on the order of magnitude of the $O(t\sqrt{\log t})$ bound. Here we show that every graph with no K_t minor is $O(t(\log t)^{\beta})$-colorable for every $\beta > 0$; more specifically, they are $O(t(\log \log t)^{6})$-colorable.

Minisymposium

GRAPH POLYNOMIALS (MS-62)

Organized by Saeid Alikhani, *Yazd University, Iran*

- Some new results on independent domination polynomial of a graph, *Saeid Alikhani*
- Normal Subgroup Based Power Graph of a Finite Group,
  Seyed Ali Reza Ashrafi Ghomroodi
- Polynomial Approximation of the Number of Possible Final Positions of a Random Walk for a Certain Class of Metric Digraphs, *Vsevolod Chernyshev*
- Degree Deviation Measure of Graphs, *Ali Ghalavand*

Some new results on independent domination polynomial of a graph

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An independent dominating set of the simple graph $G = (V, E)$ is a vertex subset that is both dominating and independent in G. The independent domination polynomial of a graph G is the polynomial $D_i(G, x) = \sum_A x^{|A|}$, summed over all independent dominating subsets $A \subseteq V$. A root of $D_i(G, x)$ is called an independence domination root. We first enumerate independent dominating sets in several classes of graphs made by a linear or cyclic concatenation of basic building blocks. Explicit recurrences are derived for the number of independent dominating sets of these kind of graphs. We also investigate the independent domination polynomials of some generalized compound graphs. As consequence, construct graphs whose independence domination roots are real.

Normal Subgroup Based Power Graph of a Finite Group

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Let G be a finite group and $N \trianglelefteq G$. The normal subgroup based power graph $\Gamma_N(G)$ is an undirected graph with vertex set $(G \setminus N) \cup \{e\}$ in which two distinct vertices a and b are adjacent if and only if $aN = b^m N$ or $bN = a^n N$, for some positive integers m and n. In this talk, we report on our recent results on the normal subgroup based power graph of a finite group. As a consequence of this talk, a characterization of all pairs (G, N) of a finite group G and a proper non-trivial normal subgroup N of G such that $\Gamma_H(G)$ is split, bisplit or $(n-1)-$bisplit are given. Moreover, all finite groups G with $c-$cyclic normal subgroup based power graph, $c \leq 4$, will be determined.

Polynomial Approximation of the Number of Possible Final Positions of a Random Walk for a Certain Class of Metric Digraphs

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Let us consider a finite compact metric graph (see [1] and references therein), that is, a one-dimensional CW complex and a random walk on it. The main unlikeness with the common graph is that the final position of a walk can be any point on an edge of a metric graph, and not only one of the vertices. Let one point start its move along the graph from a chosen vertex at the initial moment of time. The passage time for each individual edge is fixed. In each vertex, the point with some non-zero probability selects one of the outgoing edges for further movement. Backward turns on the edges are prohibited in this model. Our aim is to analyze an asymptotics of the number of possible final position of such random walk as time increases. The only assumption about the probabilities of choosing an edge is that it is non-zero for all edges, i.e. a situation of a general position. Such random walk could naturally arise while studying the

dynamical systems on various networks. The asymptotics for finite compact metric non-directed graphs was constructed earlier by V.L. Chernyshev, in collaboration with A.I.Shafarevich and A.A. Tolchennikov. A polynomial that asymptotically approximates the number of possible final positions of a random walk on an undirected metric graph with incommensurable edges was described with the help of multiple Barnes-Bernoulli polynomials (see [2] for details).

Recently the asymptotics was constructed for a certain class of strongly connected directed graphs (see [3]). This class has been called one-way Sperner graphs. For them it was proved that the asymptotics of the number of possible final positions of a random walk on a metric graph grows polynomially. The degree of a polynomial is equal to the number of incommensurable oriented cycles minus one. The leading coefficient is determined by the product of the lengths of such cycles and the sum of the lengths of all the edges of the graph. Computer simulations show that this result is valid not only for one-way Sperner graphs.

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Degree Deviation Measure of Graphs

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Let G be a simple graph with n vertices and m edges. The degree deviation measure of G is defined as $s(G) = \sum_{v \in V(G)} |deg_G(v) - \frac{2m}{n}|$. The aim of this talk is to report a positive answer to the Conjecture 4.2 of [J. A. de Oliveira, C. S. Oliveira, C. Justel and N. M. Maia de Abreu, Measures of irregularity of graphs, *Pesq. Oper.* **33** (3) (2013) 383–398].

Minisymposium

GRAPHS AND GROUPS, GEOMETRIES AND GAP – G2G2 (MS-7)

Organized by

Alexander A. Ivanov, *Imperial College London, United Kingdom*

Elena V. Konstantinova, *Sobolev Institute of Mathematics, Siberian Branch of Russian Academy of Science, Russian Federation*

- The p-length of a p-solvable group and its character table, *Neda Ahanjideh*
- On finitely generated quasi-scalar Jordan type algebras, *Ilya Gorshkov*
- On maximal cliques in Paley graphs of square order, *Sergey Goryainov*
- Strongly regular graphs satisfying the 4-vertex condition, *Ferdinand Ihringer*
- Eigenfunctions of the Star graphs for all non-zero eigenvalues, *Vladislav V. Kabanov*
- On spectral properties of the Star graphs, *Ekaterina Khomiakova*
- Discrete Fuglede conjecture on cyclic groups, *Gergely Kiss*
- Strongly Deza graphs, *Elena V. Konstantinova*
- Recent progress in distance-regular graphs, *Jack Koolen*
- Recent results on pronormality of subgroups of odd index in finite groups, *Natalia Maslova*
- Some small progress on the PSV Conjecture, *Luke Morgan*
- On Cayley isomorphism property for abelian groups, *Grigory Ryabov*
- On coverings and perfect colorings of hypergraphs, *Anna Taranenko*
- Vertex-transitive distance-regular antipodal covers of complete graphs, *Ludmila Tsiovkina*
- Minimum supports of eigenfunctions of graphs, *Alexandr Valyuzhenich*
- Closures of solvable permutation groups, *Andrey V. Vasil'ev*
- Computing distance-regular graph and association scheme parameters in SageMath with `sage-drg`, *Janoš Vidali*
- Groups acting with low fixity, *Rebecca Waldecker*

The p-length of a p-solvable group and its character table

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A question in the theory of finite p-solvable groups is to determine a bound for their p-lengths. In several papers, it is shown that this bound can be read on the character table.

For a character χ of G, the number $\chi^c(1) = \frac{[G:\ker\chi]}{\chi(1)}$ is called the co-degree of χ.

In this talk, we obtain a bound of the p-length of a p-solvable group by considering the co-degrees of its irreducible characters.

On finitely generated quasi-scalar Jordan type algebras

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The concept of axial algebras was introduced by Hall, Rehren and Shpectorov. These algebras are commutative, non-associative, and are generated by idempotents. We focus on the class of axial algebras of Jordan type. It is well known that there is a unique bilinear form in algebras of Jordan type such that any axis has length 1. We say that an algebra A with bilinear form is scalar if the bilinear form defines an inner product on A. Quasi-scalarity is a useful generalization of this concept. We say that an axial algebra A of Jordan type is quasi-scalar if for any two axes $a, b \in A$ the equality $(a, b) = 1$ holds if and only if $a = b$. It is easy to show that scalar Jordan type algebras are quasi-scalar. We have studied the structure of quasi-scalar algebras. In particular, it was proved that a finitely generated quasi-scalar algebra has a finite dimension if and only if it is unital.

On maximal cliques in Paley graphs of square order

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In [1], Blokhuis studied maximum cliques in Paley graphs of square order $P(q^2)$. It was shown that a clique of size q in $P(q^2)$ is necessarily a quadratic line in the corresponding affine plane $A(2, q)$.

Let $r(q)$ denote the reminder after division of q by 4. In [2], for any odd prime power q, a maximal (but not maximum) clique in $P(q^2)$ of size $\frac{q+r(q)}{2}$ was constructed.

In [3], for any odd prime power q, a maximal clique in $P(q^2)$ of the same size $\frac{q+r(q)}{2}$ was constructed. This clique was shown to have a remarkable connection with eigenfunctions of $P(q^2)$ that have minimum cardinality of support $q + 1$.

In this talk, we discuss the constructions of maximal cliques from [2] and [3] and establish a correspondence between them.

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Strongly regular graphs satisfying the 4-vertex condition

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A graph Γ satisfies the t-*vertex condition*, when for all triples (T, x_0, y_0) of a t-vertex graph T with two distinct distinguished vertices x_0, y_0, and all pairs of distinct vertices x, y of Γ, where $x \sim y$ if and only if $x_0 \sim y_0$, the number $n(x, y)$ of isomorphic copies of T in Γ, where the isomorphism maps x_0 to x and y_0 to y, does not depend on the choice of the pair x, y. A graph satisfies the 3-vertex condition if and only if it is strongly regular. A graph of order v satisfies the v-vertex condition if and only if it is rank 3. There are not many graphs known which satisfy the 4-vertex condition. We discuss several new families of such graphs related to polar spaces. One of our constructions is prolific and shows that the number of graphs satisfying the 4-vertex condition growths hyperexponentially in the number of vertices.

Eigenfunctions of the Star graphs for all non-zero eigenvalues

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Let G be a finite group and S be a subset of G which does not contain the identity element and is closed under inversion. The Cayley graph $\mathrm{Cay}(G, S)$ is a graph with the vertex set G in which two vertices x, y are adjacent if and only if $xy^{-1} \in S$. For $\Omega = \{1, \ldots, n\}$, $n \geqslant 2$, we consider the symmetric group Sym_Ω and put $S = \{(1\ i) \mid i \in \{2, \ldots, n\}\}$. The *Star graph* $S_n = \mathrm{Cay}(\mathrm{Sym}_\Omega, S)$ is the Cayley graph over the symmetric group Sym_Ω with the generating set S.

A function $f : V(\Gamma) \to \mathbb{R}$ is called an *eigenfunction* of a graph Γ corresponding to an eigenvalue θ if $f \not\equiv 0$ and the equality

$$\theta \cdot f(x) = \sum_{y \in N(x)} f(y) \tag{1}$$

holds for any its vertex x, where $N(x)$ is the neighborhood of x in Γ.

The Star graph S_n, $n \geq 2$, is known to be integral (see [3]), and its spectrum consists of all integers in the range from $-(n-1)$ to $n-1$ (except 0 when $n = 2, 3$). Despite of the fact that spectral properties of the Star graph were studied (see [1, 3, 3, 5]), no explicit construction for the eigenfunctions was known.

In [4], an explicit construction of eigenfunctions of S_n, $n \geq 3$, for all eigenvalues θ with $\frac{n-2}{2} < \theta < n-1$ was presented.

In this work, we generalize ideas from [4] and present eigenfunctions of the Star graph S_n, $n \geq 3$, for all its non-zero eigenvalues.

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On spectral properties of the Star graphs

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The Star graph S_n, $n \geqslant 2$, is the Cayley graph over the symmetric group Sym_n generated by transpositions swapping the ith element of the permutation with the first one. It is a connected bipartite $(n-1)$-regular graph of order $n!$, and diameter $diam(S_n) = \lfloor \frac{3(n-1)}{2} \rfloor$ [1].

In 2012, R. Krakovski and B. Mohar [7] proved that the spectrum of S_n contains all integers in the range from $-(n-1)$ up to $n-1$. Since the Star graph is bipartite, the spectrum is symmetric and multiplicities of eigenvalues of $(n-k)$ and $-(n-k)$ are equal for each integer $1 \leqslant k < n$. Furthermore, $\pm(n-1)$ are simple eigenvalues of S_n. At the same time, G. Chapuy and V. Feray [3] showed that the spectrum of the Star graphs is equivalent to the spectrum of Jucys-Murphy elements in the algebra of the symmetric group. This connection between two kinds of spectra implies that the Star graph is integral.

In 2016, S. V. Avgustinovich, E. N. Khomyakova and E. V. Konstantinova [2] suggested a method for getting explicit formulas for multiplicities of eigenvalues $\pm(n-k)$ and presented such formulas for $2 \leqslant k \leqslant 5$. Moreover, an asymptotic lower bound was obtained. It was proved that for a fixed integer eigenvalue of S_n, its multiplicity is at least $2^{\frac{1}{2}n \log n(1-o(1))}$ for sufficiently large n. In 2018, E. N. Khomyakova [6] investigated the behavior of the eigenvalues multiplicity function of S_n for eigenvalues $\pm(n-k)$ where $1 \leqslant k \leqslant \frac{n+1}{2}$. The main result is given by the following theorem.

Theorem 1. *Let $n, k \in \mathbb{Z}$, $n \geqslant 2$ and $1 \leqslant k \leqslant \frac{n+1}{2}$, then the multiplicity* $\mathrm{mul}(n-k)$ *of eigenvalue $(n-k)$ of the Star graph S_n is calculated by the formula:*

$$\mathrm{mul}(n-k) = \frac{n^{2(k-1)}}{(k-1)!} + P(n),$$

where $P(n)$ is a polynomial of degree $2k-3$.

In 2019, E. N. Khomyakova and E. V. Konstantinova [5] presented explicit formulas for calculating multiplicities of eigenvalues $\pm(n-k)$ where $2 \leqslant k \leqslant 12$ and firstly collected computational results of all eigenvalue multiplicities for $n \leqslant 50$ in a catalogue (`https://link.springer.com/article/10.1007/s40065-019-00271-z`). This exact values show that Theorem 1 holds for any $n \geqslant 2$ and $1 \leqslant k \leqslant n$. Authors used computational results to get diagrams with plotting them on a logarithmic scale with base 2 such that the abscissa corresponds to the eigenvalues of the Star graphs S_n for a fixed n and the ordinate corresponds to the multiplicities [4]. In case k is fixed, diagram looks like a polynomial function by Theorem 1. In case n is fixed, diagram in normal scale contains exponential rises and falls appear. Thus the function may be a straight exponent for sufficiently large n, but it is just conjecture.

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Discrete Fuglede conjecture on cyclic groups

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Fuglede in 1974 conjectured that a bounded domain $S \subset \mathbb{R}^d$ tiles the d-dimensional Euclidean space if and only if the set of functions in $L^2(S)$ admits an orthogonal basis of exponential functions.

In my talk we focus on the discrete version of Fuglede's conjecture that can be formulated as follows. A subset S of a finite abelian group G tiles G if and only if the character table of G has a submatrix, whose rows are indexed by the elements of S, which is a complex Hadamard matrix. Fuglede's original conjecture were disproved first by Tao and the proof is based on a counterexample on elementary abelian p-groups.

On the other hand, it is still an open question whether the discrete Fuglede's conjecture is true on cyclic groups. In my talk I will summarize the known results concerning this question. In particular, I will present our recent result which shows that the conjecture holds on cyclic groups whose order is the product of at most 4 (not necessarily different) primes. I will introduce a geometric technique that we called 'cube-rule' and which is an essential tool of the proof.

Strongly Deza graphs

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A Deza graph G with parameters (n, k, b, a) is a k-regular graph of order n for which the number of common neighbours of two vertices takes just two values, b or a, where $b \geqslant a$. Moreover, G is not the complete graph or the edgeless graph. Deza graphs were introduced in [3], and the name was given in [4], where the basics of Deza graph theory were founded and different constructions of Deza graphs were presented. Strongly regular graphs are a particular case of Deza graphs.

Deza graphs can be considered in terms of matrices. Let G be a graph with n vertices, and M be its adjacency matrix. Then G is a Deza graph with parameters (n, k, b, a) if and only if

$$M^2 = aA + bB + kI$$

for some symmetric $(0, 1)$-matrices A and B such that $A + B + I = J$, where J is the all ones matrix and I is the identity matrix. Graphs G_A and G_B with matrices A and B are called the children of G.

Definition. *A Deza graph is called a strongly Deza graph if its children are strongly regular graphs.*

Theorem 1. [1, Theorem 3.2] *Let G be a Deza graph with parameters (n, k, b, a), $b > a$. Let M, A, B be the adjacency matrices of G and its children, respectively. If $\theta_1 = k, \theta_2, \ldots, \theta_n$ are the eigenvalues of M, then*

(i) *the eigenvalues of* A *are*

$$\alpha = \frac{b(n-1)-k(k-1)}{b-a},\ \alpha_2 = \frac{k-b-\theta_2^2}{b-a},\ \ldots,\ \alpha_n = \frac{k-b-\theta_n^2}{b-a};$$

(ii) *the eigenvalues of* B *are*

$$\beta = \frac{a(n-1)-k(k-1)}{a-b},\ \beta_2 = \frac{k-a-\theta_2^2}{a-b},\ \ldots,\ \beta_n = \frac{k-a-\theta_n^2}{a-b}.$$

By Theorem above, a strongly Deza graph has at most three distinct absolute values of its eigenvalues.

Theorem 2. *Suppose* G *is a strongly Deza graph with parameters* (n, k, b, a). *Then*

(i) G *has at most five distinct eigenvalues.*

(ii) *If* G *has two distinct eigenvalues, then* $a = 0$, $b = k - 1 \geqslant 1$, *and* G *is a disjoint union of cliques of order* $k + 1$.

(iii) *If* G *has three distinct eigenvalues, then* G *is a strongly regular graph with parameters* (n, k, λ, μ), *where* $\{\lambda, \mu\} = \{a, b\}$, *or* G *is disconnected and each component is a strongly regular graph with parameters* (v, k, b, b), *or each component is a complete bipartite graph* $K_{k,k}$ *with* $k \geqslant 2$.

If G is a bipartite graph, then the *halved* graphs of G are two connected components of the graph on the same vertex set, where two vertices are adjacent whenever they are at distance two in G.

The next theorem gives a spectral characterization of strongly Deza graphs.

Theorem 3. *Let* G *be a connected Deza graph with parameters* (n, k, b, a), $b > a$, *and it has at most three distinct absolute values of its eigenvalues.*

(i) *If* G *is a non-bipartite graph, then* G *is a strongly Deza graph.*

(ii) *If* G *is a bipartite graph, then either* G *is a strongly Deza graph or its halved graphs are strongly Deza graphs.*

We also discuss some results on distance-regular strongly Deza graphs.

The main results of the talk are presented in [2].

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Recent progress in distance-regular graphs

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In this talk I report on recent progress in distance-regular graphs. This talk is based on joint work with Meng Yue Cao (Beijing Normal University), Ying Ying Tan (Anhui Jianzhu University), Xiaoye Liang (Anhui University), Jongyook Park (Kyungpook National university), Gary Greaves (Nanyang Technological University), Qianqian Yang (University of Science and Technology of China), Zhi Qiao (Sichuan Normal University)

Recent results on pronormality of subgroups of odd index in finite groups

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We consider only finite groups. A subgroup H of a group G is said to be *pronormal* in G if H and H^g are conjugate in $\langle H, H^g \rangle$ for each $g \in G$. Some of well-known examples of pronormal subgroups are the following: normal subgroups; maximal subgroups; Sylow subgroups; Sylow subgroups of proper normal subgroups; Hall subgroups of solvable groups. Some problems in finite group theory, combinatorics, and permutation group theory were solved in terms of the pronormality, see, for example [1, 11, 12].

In 2012, E. Vdovin and D. Revin [13] proved that the Hall subgroups (when they exist) are pronormal in all simple groups and, guided by the analysis in their proof, they conjectured that any subgroup of odd index of a simple group is pronormal in this group. This conjecture was disproved in [5, 6]. In [4, 5, 6, 7], finite simple groups in which all the subgroups of odd index are pronormal were studied. More detailed surveys of investigations on pronormality of subgroups of odd index in finite (not necessary simple) groups could be found in survey papers [3, 8]. These surveys contain new results and some conjectures and open problems. One such open problem is to complete classification of finite simple groups in which all the subgroups of odd index are pronormal. One more open problem involves the classification of direct products of nonabelian simple groups in which the subgroups of odd index are pronormal. A detailed motivation for these problems was provided in [2]. Note that there are examples of nonabelian simple groups G such that all the subgroups of odd index are pronormal in G, but the group $G \times G$ contains a non-pronormal subgroups of odd index (see [2, Proposition 1]).

In this talk, we discuss a recent progress in investigations on pronormality of subgroups of odd index in finite groups. In particular, we have obtained the complete classification finite simple exceptional groups of Lie type in which the subgroups of odd index are pronormal [9] and have proved that the subgroups of odd index are pronormal in a direct product G of simple symplectic groups over fields of odd characteristics if and only if the subgroups of odd index are pronormal in each direct factor of G [10]; moreover, deciding the pronormality of a given subgroup of odd index in the direct product of simple symplectic groups over fields of odd characteristics is reducible to deciding the pronormality of some subgroup H of odd index in a subgroup of a group $\prod_{i=1}^{t} Z_i \wr Sym_{n_i}$, where $Z_i \in \{1, C_3\}$ for each i, such that H projects onto $\prod_{i=1}^{t} Sym_{n_i}$, and we have obtained a criterium of pronormality of such a subgroup in such a group [10]. All these works were supported by the Russian Science Foundation (project 19-71-10067). These investigations give a rise to researches on effective algorithms (to be implemented in GAP) for deciding the pronormality of a subgroup of odd index in a finite simple group. This is a joint project with Stephen Glasby and Cheryl E. Praeger.

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Some small progress on the PSV Conjecture

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The subject of the talk is the problem of bounding the number of automorphisms of an arc-transitive graph in terms of the valency of the graph. More specifically, we consider the problem for groups acting arc-transitively on graphs such that the local action (that induced on the neighbours of a vertex by the stabiliser of that vertex) is semiprimitive. This problem was first considered by Weiss for the case of primitive local action and then generalised by Praeger to the case of quasiprimitive local action.

I will report on some recent small progress on the first type - that of semiprimitive local action, where the action is that of the symmetric group S_n on the set of $n(n-1)$ ordered pairs. The result is akin to Tutte's famous result on cubic s-arc transitive graphs where the number of automorphisms fixing a vertex is bounded by $3*2^{s-1}$. Tutte's proof was elegant, elementary and self-contained. Our recent progress relies on some group theoretical tools that were developed for use in the Classification of the Finite Simple Groups - and some tricks to allow us to patch things together. I'll try to present these results in a friendly fashion, as well as keeping in mind the "big picture" concerning where progress now stands on these conjectures.

On Cayley isomorphism property for abelian groups

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Coauthor: István Kovács

A finite group G is called a *DCI-group* if every two isomorphic Cayley digraphs over G are Cayley isomorphic, i.e. there exists an isomorphism between these digraphs that is also an automorphism of G. One of the motivations to study DCI-groups comes from the Cayley graph isomorphism problem. Suppose that G is a DCI-group. Then to determine whether two Cayley digraphs $Cay(G, S)$ and $Cay(G, T)$ are isomorphic, we only need to check the existence of $\varphi \in Aut(G)$ with $S^{\varphi} = T$. The latter, usually, is much easier.

The definition of a DCI-group goes back to Ádám who conjectured [1], in our terms, that every cyclic group is DCI. This conjecture was proved to be false. The problem of determining all finite DCI-groups was raised by Babai and Frankl [2]. One of the crucial steps towards the classification of all DCI-groups is to determine abelian DCI-groups. It was proved that every abelian DCI-group is the direct product of groups of coprime orders each of which is elementary abelian or isomorphic to $\mathbb{Z}_4$ (see [7, Theorem 8.8]). However, the classification of abelian DCI-groups is far from complete. In the talk we discuss on new infinite families of abelian DCI-groups and approaches to determining whether a given group is DCI.

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On coverings and perfect colorings of hypergraphs

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In this talk, we consider perfect colorings (also known as equitable partitions) of hypergraphs. A perfect k-coloring of a hypergraph is a coloring of its vertices into k colors such that colors of hyperedges incident to a vertex is defined by its color. A transversal of a hypergraph is one of the simplest examples of perfect 2-colorings. While perfect colorings of graphs are well known and extensively applied, similar objects for hypergraphs are hardly studied. The main aim of the talk is to show that perfect colorings of hypergraphs have most of the nice algebraic properties of colorings of graphs.

Firstly, we define the multidimensional parameter matrix of a perfect coloring of a hypergraph, study its eigenvalues, and connect them with the parameters of the corresponding perfect coloring of the bipartite representation. Next, we introduce coverings of hypergraphs and show that for a vast class of hypergraphs there exist coverings that can be partitioned into perfect matchings. At last, we establish that if two hypergraphs have the same minimal perfect coloring then there is a hypergraph covering both of them.

Vertex-transitive distance-regular antipodal covers of complete graphs

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Distance-regular antipodal covers of complete graphs are closely related to various important algebraic, geometric, or combinatorial objects, such as generalized Hadamard matrices, projective planes, generalized quadrangles, divisible designs, and codes. A recent surge of interest for their study is motivated by their applications in discrete geometry and quantum information theory, since those covers that are abelian turn out to be a potential source of new sets of equiangular lines (ETFs, SIC-POVMs). The general problem of classification of all distance-regular antipodal covers of complete graphs seems to be unsolvable. Nevertheless, a promising task in this direction is to describe vertex-transitive representatives, since they admit group-theoretic characterisations. To date, the following vertex-transitive distance-regular antipodal covers of complete graphs have been classified: (i) covers with distance-transitive automorphism groups (complete description); (ii) covers with arc-transitive automorphism groups (almost complete description). Much less is known in general case.

The aim of this talk is to present a classification of edge-transitive distance-regular antipodal covers of complete graphs. The automorphism group of such a cover is transitive, and by a combination of results of Kantor and Burnside, it induces either a 2-transitive almost simple group, or an affine 2-homogeneous group on the set of fibres. Using the classification of finite 2-transitive permutation groups, we will prove that every such cover with $\mu > 1$ is either arc-transitive, or a Cayley graph whose automorphism group induces a one-dimensional affine permutation group on the set of its fibres. Then we will present a general construction of covers with antipodality index greater than 2 in the almost simple case in terms of graphs of basis relations of some association schemes related to quasi-simple groups.

We will also discuss some recent results on classification of abelian distance-regular antipodal covers of a complete graph which possess a transitive group of automorphisms that induces an almost simple primitive rank 3 permutation group on the set of fibres of the corresponding cover.

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Minimum supports of eigenfunctions of graphs

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Let $G = (V, E)$ be a graph with the adjacency matrix $A(G)$. The set of neighbors of a vertex x is denoted by $N(x)$. Let λ be an eigenvalue of the matrix $A(G)$. A function $f : V \longrightarrow \mathbb{R}$ is called a λ-*eigenfunction* of G if $f \not\equiv 0$ and the equality

$$\lambda \cdot f(x) = \sum_{y \in N(x)} f(y)$$

holds for any vertex $x \in V$. In this talk we focus on the following extremal problem for eigenfunctions of graphs.

Problem 1 (MS-problem). *Let G be a graph and let λ be an eigenvalue of G. Find the minimum cardinality of the support of a λ-eigenfunction of G.*

MS-problem was first formulated by Krotov and Vorob'ev [11] in 2014 (they considered MS-problem for the Hamming graph). During the last six years, MS-problem has been actively studied for various families of distance-regular graphs [6, 4, 3, 1, 2, 7, 8, 9, 10, 11, 12] and Cayley graphs on the symmetric group [5]. In particular, MS-problem is completely solved for all eigenvalues of the Hamming graph [9, 10] and asymptotically solved for all eigenvalues of the Johnson graph [12]. In this talk we will discuss several new results on MS-problem.

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Closures of solvable permutation groups

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Let m be a positive integer and let Ω be a finite set. The *m-closure* $G^{(m)}$ of $G \leq Sym(\Omega)$ is the largest permutation group on Ω having the same orbits as G in its induced action on the Cartesian product Ω^m. Wielandt [5] showed that

$$G^{(1)} \geq G^{(2)} \geq \cdots \geq G^{(m)} = G^{(m+1)} = \cdots = G, \qquad (1)$$

for some $m < |\Omega|$. (Since the stabilizer in G of all but one point is always trivial, $G^{(n-1)} = G$ where $n = |\Omega|$.) In this sense, the m-closure can be considered as a natural approximation of G. It was shown by Praeger and Saxl [2] that for $m \geq 6$, the m-closure $G^{(m)}$ of a primitive permutation group G has the same socle as G. Furthermore, they classified explicitly primitive groups G and H with different socles having the same m-orbits for $m \leq 5$. Unfortunately, their results say very little about closures of solvable permutation groups. The main goal of this talk is to present the results of [1], where we study such closures.

The 1-closure of G is the direct product of symmetric groups $Sym(\Delta)$, where Δ runs over the orbits of G. Thus the 1-closure of a solvable group is solvable if and only if each of its orbits has cardinality at most 4. The case of 2-closure is more interesting. The 2-closure of every (solvable) 2-transitive group $G \leq Sym(\Omega)$ is $Sym(\Omega)$; other examples of solvable G and nonsolvable $G^{(2)}$ appear in [4]. But, as shown by Wielandt [5], each of the classes of finite p-groups and groups of odd order is closed with respect to taking the 2-closure. Currently, no characterization of solvable groups having solvable 2-closure is known.

Seress [3] observed that if G is a primitive solvable group, then $G^{(5)} = G$; so the 5-closure of a primitive solvable group is solvable. Our main result is the following stronger statement.

Theorem 1. *The 3-closure of a solvable permutation group is solvable.*

The corollary below is an immediate consequence of Theorem 1 and the chain of inclusions (1).

Corollary 2. *For every integer $m \geq 3$, the m-closure of a solvable permutation group is solvable.*

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Computing distance-regular graph and association scheme parameters in SageMath with `sage-drg`

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The `sage-drg` package [8] for the SageMath computer algebra system has been originally developed for computation of parameters of distance-regular graphs, and its functionality has later been extended to handle general association schemes. The package has been used to obtain nonexistence results for both distance-regular graphs and Q-polynomial association schemes, mostly using the triple intersection numbers technique, see for example [3, 4, 7].

Recently, checks for two new feasibility conditions have been implemented. The first technique, developed by Kodalen and Martin [5], relies on Schönberg's theorem on positive semidefinite functions in S^{m-1} and its application on the minimal idempotents of an association scheme. The implementation of the relevant checks in `sage-drg` allows us to replicate their nonexistence results for several feasible parameter sets for Q-polynomial association schemes.

The second technique derives from Terwilliger's work on P- and Q-polynomial association schemes [6] and has most recently been used by Gavrilyuk and Koolen [1, 2] to obtain some nonexistence and uniqueness results for Q-polynomial distance-regular graphs. The implementation of the relevant procedures in `sage-drg` allows us to generalize their approach and derive nonexistence for many feasible parameter sets of classical distance-regular graphs.

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Groups acting with low fixity

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This research project started in 2012, joint with Kay Magaard. Stemming from questions about automorphisms of Riemann surfaces, it has lead to many interesting problems in permutation group theory. I will discuss the status of the project and some open questions.

Minisymposium

Graphs, Polynomials, Surfaces, and Knots (MS-49)

Organized by Joanna Ellis-Monaghan, *Saint Michael's College, United States*
Coorganized by Mark Ellingham, *Vanderbilt University, United States*

- Marked Graphs, marked polynomials and relationships with chromatic symmetric functions and W-polynomials, *Jose Aliste Prieto*
- On the symmetry groups of the neighborly polytopes, *Djordje Baralić*
- Limits for embedding distributions, *Yichao Chen*
- On the Gross-Mansour-Tucker conjecture, *Sergei Chmutov*
- Embedded graphs and delta-matroids, *Carolyn Chun*
- Embeddings with Eulerian faces II: degree conditions, *Mark Ellingham*
- Embeddings with Eulerian faces I: context and parities, *Joanna Ellis-Monaghan*
- Formal grammar modeling three-stranded DNA:RNA braids, *Margherita Maria Ferrari*
- Tutte characters for combinatorial coalgebras, *Alex Fink*
- Tutte's dichromate for signed graphs, *Andrew Goodall*
- A colored version of Brylawski's tensor product formula and its applications, *Gábor Hetyei*
- Hopf Algebras in Studying Graph and Embedded Graph Polynomials, *Sergei Lando*
- A new enumerator polynomial with a smart derivative, *Serge Lawrencenko*
- Eulerian and bipartite partial duals, *Metrose Metsidik*
- From matrix pivots to graphs in surfaces: touring combinatorics as guided by partial duals, *Iain Moffatt*
- Framed- and Biframed Knotoids, *Wout Moltmaker*
- Coloring quadrangulations of the projective space, *Atsuhiro Nakamoto*
- The two-variable Bollobás–Riordan polynomial of a connected even delta-matroid is irreducible, *Steven Noble*
- Spanning bipartite subgraphs of triangulations of a surface, *Kenta Noguchi*
- A list orientation of graphs, *Kenta Ozeki*
- Partial Twuality Polynomials, *Thomas W. Tucker*
- Ternary self-distributive cohomology and invariants of framed links and knotted surfaces with boundary, *Emanuele Zappala*

Marked Graphs, marked polynomials and relationships with chromatic symmetric functions and W-polynomials

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We will introduce marked graphs and marked graph polynomials. A marked graph is a graph where each vertex is annotated with two numbers: A weight to keep track of the original number of vertices, and a number of dots, to keep track of the contractions taken in order to create this vertex from the original graph. Marked graph polynomials are polynomials that satisfy a marked deletion-contraction property and a new property called dot-removal formula. This dot removal formula + the marked. deletion-contraction allows us to encode 4T relations of chromatic symmetric functions into these polynomials and hence understanding these new polynomials allows us for a better understanding of the chromatic symmetric function.

On the symmetry groups of the neighborly polytopes

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An n-polytope P is said to be k-*neighborly* if any subset of k or less vertices is the vertex set of a face of P. The polytopes that are $\left[\frac{n}{2}\right]$-neighborly are of particular interests and are called *neighborly* polytopes. They are very important objects in combinatorics because they are solutions of various extremal properties such as the upper bound predicted by Motzkin for maximal number of i-faces of an n-polytope with m vertices. A classical example of a neighborly n-polytope with m vertices is *the cyclic polytope* $C^n(m)$. The cyclic polytope $C^n(m)$ is the convex hull

$$C^n(m) := \operatorname{conv}\{\gamma(t_1), \gamma(t_2), \dots, \gamma(t_m)\},$$

for m distinct points $\gamma(t_i)$ with $t_1 < t_2 < \cdots < t_m$ on the moment curve which is a curve in $\mathbb{R}^n$ defined by $\gamma : \mathbb{R} \to \mathbb{R}^n$, $t \mapsto \gamma(t) = (t, t^2, \dots, t^n) \in \mathbb{R}^n$. The combinatorial class of $C^n(m)$ does not depend on the specific choices of the parameters t_i due to Gale's evenness condition.

If the number of vertices m of a neighborly n-polytope is not grater than $n+3$ then combinatorially the polytope is isomorphic to a cyclic polytope. However, there are many neighborly polytopes which are not cyclic. Barnette in 1981 constructed an infinite family of duals of neighborly n-polytopes by using an operation called 'facet splitting' and Shemer in 1982 introduced a sewing construction that allows to add a vertex to a neighborly polytope in such a way as to obtain a new neighborly polytope. Both constructions show that for a fixed n the number of combinatorially different neighborly polytopes grows superexponentially with the number of vertices m, but our knowledge about the combinatorics of this important objects is considerable small. The number of combinatorial types of neighborly polytopes in dimensions 4, 5, 6 and 7 with 'small' number of vertices is extensively studied in the last decades and following these results we determine their symmetry groups which are found to be very diverse. There exist not only the examples of the neighborly polytopes with trivial symmetry groups or $\mathbb{Z}/2\mathbb{Z}$, but also those with relatively big number of symmetries which rises questions of constructions of the neighborly polytopes with nontrivial symmetry group in arbitrary dimension.

Limits for embedding distributions

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In this paper, we first establish a central limit theorem which is new in probability, then we find and prove that, under some conditions, the embedding distributions of H-linear family of graphs with spiders are asymptotic normal distributions. As corollaries, the asymptotic normality for the embedding distributions of path-like sequence of graphs with spiders and the genus distributions of ladder-like sequence of graphs are given. We also prove that the limit of Euler-genus distributions is the same as that of crosscap-number distributions. The results here can been seen a version of central limit theorem in topological graph theory.

On the Gross-Mansour-Tucker conjecture

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In this presentation I will explain a proof of the Gross, Mansour, Tucker conjecture claiming that any ribbon graph, distinct from explicit family of special exceptions, has a partial dual graph of different genus. This is a joint work with Fabien Vignes-Tourneret based of the preprint arXiv:2101.09319v1 [math.CO].

Embedded graphs and delta-matroids

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Coauthors: Joseph Bonin, Steven Noble, Ralf Rueckriemen, Iain Moffatt, Deboarh Chun

The interplay between embedded graphs and delta-matroids generates useful tools for both research areas. In this talk we explore such results and propose further applications.

Embeddings with Eulerian faces II: degree conditions

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Coauthor: Joanna Ellis-Monaghan

As a natural special case of edge-outer embeddability, we consider the problem of finding maximum genus orientable directed embeddings. We allow some faces to be specified in advance. Digraphs with directed embeddings are necessarily eulerian. If we are given an eulerian digraph and a decomposition of the arcs into edge-disjoint directed walks, then we can regard this as a partial embedding, with the walks as specified face boundaries. If we can complete this to an embedding by adding one more face bounded by an euler circuit, then the embedding will have maximum genus subject to containing the specified faces. We show that this is always possible provided the underlying simple graph of our n-vertex digraph has minimum degree at least

$(8n+1)/9$. This is a broad generalization of results for regular tournaments due to Bonnington et al. (2002) and Griggs, McCourt and Širáň (2020).

Embeddings with Eulerian faces I: context and parities

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Coauthor: Mark Ellingham

Closed walks that cover all the edges of a graph arise in many settings. In DNA self-assembly experiments, they give a route for a special strand of DNA through the assembled molecule. A reporter strand walk covers all the edges of a graph at least once, and if twice then in opposite directions, without doubling back on any edge. A graph is edge-outer embeddable if it has an orientable embedding with a special face whose boundary uses every edge at least once. Thus a reporter strand walk is a facial walk around an outer face of such an edge-outer embedding. While every graph has such an edge-outer embedding, finding one with a minimum size outer face is NP-hard. Furthermore, genus-related questions naturally arise in this new edge-outer embeddability setting. Here we focus on a min-max question: What graphs have edge-outer embeddings with both a minimum size outer face and maximum genus? This question is particularly interesting in the case of Eulerian graphs. Does there always exist an oriented embedding of a given Eulerian graph with two faces, each bounded by an Euler circuit, possibly even with one circuit specified in advance? We answer this by addressing the cases, in order, of yes, no, sometimes, and (at the time of writing) we have a conjecture.

Formal grammar modeling three-stranded DNA:RNA braids

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A formal grammar is a system to generate words; it consists of a set of symbols, partitioned into terminals and non-terminals, and a set of production rules. The production rules specify how to rewrite non-terminal symbols, so that successive applications of those rules yield words formed by only terminals. Adding probabilities to the production rules defines stochastic grammars, which can be used for biological sequence analysis. In this talk, we focus on a "braid grammar" to model R-loops, that are three-stranded structures formed by a DNA:RNA hybrid plus a single strand of DNA, often appearing during transcription. R-loops are described as strings of terminal symbols representing the braiding of the strands in the structure, where each symbol corresponds to a different state of the braided structure. We discuss approaches to develop a stochastic grammar and a probabilistic model for R-loop prediction, as well as refinements of the model by incorporating the effect of DNA topology on R-loop formation.

Tutte characters for combinatorial coalgebras

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The Tutte polynomial is a favourite invariant of matroids and graphs. So when one is working in a generalisation of these settings, for example arithmetic matroids or ribbon graphs, it is a tempting question to find a counterpart of the Tutte polynomial; answers have been given in many cases. Work of Krajewski, Moffatt, and Tanasa found a framework unifying the Tutte-like polynomials arising from graphs in surfaces using Hopf algebras. Our contribution, besides adding some more examples and observing that the machinery is useful for producing convolution formulae in the style of Kook-Reiner-Stanton-Etienne-Las Vergnas, is a generalisation of the formalism using comonoids in linear species.

Joint with Clément Dupont and Luca Moci.

Tutte's dichromate for signed graphs

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A signed graph is a graph with signed edges (positive or negative). Two signed graphs are considered equivalent if their edge signs differ on a cutset of the graph. Proper colourings and nowhere-zero flows of signed graphs are defined analogously to those of graphs. For graphs, these are both enumerated by evaluations of the Tutte polynomial. For signed graphs, Zaslavsky enumerated proper colourings, and recently DeVos–Rollová–Šámal showed that the number of nowhere-zero flows satisfies a deletion-contraction recurrence, and, independently, Qian–Ren and Goodall–Litjens–Regts–Vena gave a subset expansion formula. We construct a trivariate polynomial invariant of signed graphs that contains both the number of proper colourings and the number of nowhere-zero flows as evaluations: for this three variables are needed, giving a "trivariate Tutte polynomial" for signed graphs. Specializations include Zaslavsky's bivariate rank-generating polynomial of the (frame matroid of the) signed graph and the Tutte polynomial of the (cycle matroid of the) underlying graph.

A colored version of Brylawski's tensor product formula and its applications

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The tensor product of a graph and a pointed graph is obtained by replacing each edge of the first graph with a copy of the second. In this talk we outline a simple proof of Brylawski's formula for the Tutte polynomial of the tensor product which can be generalized to the colored Tutte polynomials introduced by Bollobás and Riordan. Consequences include formulas for Jones polynomials of (virtual) knots and for invariants of composite networks in which some major links are identical subnetworks in themselves.

All results presented are joint work with Yuanan Diao, some of them are also joint work with Kenneth Hinson.

Hopf Algebras in Studying Graph and Embedded Graph Polynomials

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Study of Hopf algebra structures on spaces spanned by graphs was initiated by S. Joni and G.-C. Rota in 1979 and was later unified with umbral calculus. Since then, a lot of combinatorial objects similar to graphs were shown to generate natural Hopf algebras. Embedded graphs are not among them, but this is true for closely related to them binary delta-matroids as defined by A. Bouchét in 1987. These general Hopf algebras have interesting Hopf subalgebras the study of which is sometimes easier and leads to effective explicit computations.

Many polynomial invariants of graphs, embedded graphs, and binary delta-matroids demonstrate a nice behavior with respect not only to the multiplicative structure, but to comultiplication as well. Examples include chromatic polynomial, characteristic polynomial, matching polynomial, Stanley's symmetrized chromatic polynomial, and many others.

Invariants of abstract graphs are closely related to those of chord diagrams (which are embedded graphs with a single vertex). In the framework of Vassiliev' theory of finite order knot invariants, chord diagrams serve as a tool to describe the latter. Similarly, certain invariants of binary delta-matroids and embedded graphs produce finite invariants of links. The Hopf algebra point of view leads to unexpected approaches to extending graph invariants to embedded graphs and binary delta-matroids.

The talk will be based on recent results of my students, colleagues, and myself.

A new enumerator polynomial with a smart derivative

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Let S_n be a given set of n-vertex simplicial complexes; e.g., a set of n-vertex paths, cycles, trees, or 2-cell embeddings of graphs, etc. We solve the problem of determining the cardinality $|S_n|$ in a double sense: (1) the labeled sense; all n vertices are mapped bijectively onto the set of labels $\{1, 2, \ldots, n\}$ where different maps (labelings) may produce different complexes, (2) the unlabeled sense, that is, up to isomorphism (labels removed). A new enumerator polynomial, $P_n(x)$, will be introduced. It has interesting properties: The value $P_n(1)$ is equal to $|S_n|$ in the labeled sense while the value of the derivative $P_n'(1)$ is equal to $n!$ times $|S_n|$ in the unlabeled sense. For example, for paths with n vertices $P_n(x) = (n!/2)x^2$. More examples and properties of the enumerator polynomial will be presented.

Eulerian and bipartite partial duals

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Huggett and Moffatt characterized all bipartite partial duals of plane graphs in terms of all-crossing directions of their medial graphs. Then Metsidik and Jin characterized all Eulerian partial duals of plane graphs in terms of semi-crossing directions of their medial graphs. Plane graphs are ribbon graphs with genus 0. In this talk, by introducing the notion of modified medial graphs and using their all-crossing directions, we first extend Huggett and Moffatt's result from plane graphs to ribbon graphs. Then we characterize all Eulerian partial duals of any ribbon graph in terms of crossing-total directions of its medial graph, which are simpler than semi-crossing directions.

From matrix pivots to graphs in surfaces: touring combinatorics as guided by partial duals

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This talk will be accessible to a general mathematical, non-specialist audience.

The concept of the dual of a graph traces back to the very beginnings of graph theory, and can even be found in work of Euler. Roughy speaking, the dual of a graph drawn on the plane is formed by switching its vertices and faces. (For example, the dual of a cube is an octahedron.) Duals are a foundational and well-known concept in graph theory — most undergraduates will meet them at some point in their studies.

Despite their long history, it has only become apparent that you don't have to form the dual of *all* of the edges of a graph at once, you can just take the dual with respect to *some* of its edges. This results in the idea of a "partial dual" — a concept introduced by S. Chmutov in 2009. It arose from work on the Jones polynomial and knot theory.

With such an advance in understanding of such a fundamental construction, it is perhaps unsurprising that partial duals swiftly led to advances in topological graph theory. However, and perhaps more importantly, it turns out that the partial duals reach far beyond graphs in surfaces, are intimately related to several very different areas of combinatorics. Indeed, the concept of the "partial dual" has appeared in very highly disguised forms in various places in the literature over the last 60 years.

In this talk, I'll survey the various appearances and applications of partial duals in graph theory, pointing out the places where the concept has been hiding in the literature for all these years. Along the way we'll encounter various topics in combinatorics such as pivots of matrices, embedded graphs, circle graphs, the Tutte polynomial, knot theory, pivot minors, chord diagrams, and matroids. The emphasis will be on how the various topics fit together, and on what is to be gained by switching between the various perspectives.

Framed- and Biframed Knotoids

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In this talk I will recall the definition of a spherical knotoid and modify this definition to include a framing, in analogy to framed knots. I also define a further modification that includes a secondary 'coframing' to obtain 'biframed' knotoids. Afterwards I will give topological spaces whose ambient isotopy classes are in one-to-one correspondence with framed- and biframed knotoids respectively. Finally I will mention how biframed knotoids allow for the construction of quantum invariants.

Coloring quadrangulations of the projective space

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Coauthor: Kenta Ozeki

A *quadrangulation* on a surface F^2 is a fixed embedding of a simple graph such that each face is quadrilateral. It is known that every quadrangulation on the sphere is bipartite, but every non-spherical surface admits non-bipartite quadrangulations. For the projective plane P^2, Young pointed out an interesting fact that every non-bipartite quadrangulation is 4-chromatic. Kaiser and Stehlík considered a higher dimensional quadrangulations in the projective space, and proved that every non-bipartite d-dimensional quadrangulation in the d-dimensional projective space P^d has chromatic number exactly $d + 2$. In our talk, we will give another proof to Young's result, focusing the dual map of quadrangulations. Moreover, giving a new definition of a higher dimensional quadrangulations different from those by Kaiser and Stehlík, we prove that 3-dimensional quadrangulations of P^3 in a certain class have chromatic number 4, and conjecture that this can be extended to all of our quadrangulations in P^3.

The two-variable Bollobás–Riordan polynomial of a connected even delta-matroid is irreducible

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Coauthors: Joanna Ellis-Monaghan, Andrew Goodall, Iain Moffatt, Lluís Vena

One of the most striking results concerning the Tutte polynomial is that the Tutte polynomial of a matroid is irreducible if and only if the matroid is connected.

The most natural analogue of the Tutte polynomial for an even delta-matroid is perhaps a normalized two variable specialization

$$(x-1)^{w(D)/2} R_D(x, y-1, 1/\sqrt{(x-1)(y-1)}, 1)$$

of the Bollobás–Riordan polynomial. We show that for even delta-matroids this two-variable Bollobás–Riordan polynomial is irreducible if and only if the delta-matroid is connected.

Spanning bipartite subgraphs of triangulations of a surface

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A triangulation (resp., a quadrangulation) of a surface S is an embedded graph (possibly with multiple edges and loops) on S with each face bounded by a closed walk of length 3 (resp., 4). This talk focuses on the relationship between triangulations and quadrangulations of a surface.

(a) Extension of a graph G is the construction of a new graph by adding edges to some pairs of vertices in G. Obviously, every quadrangulation G of any surface can be extended to a triangulation by adding a diagonal to each face of G. If we require some properties for the resulting triangulation, the problem might be difficult and interesting. We prove that every quadrangulation of any surface can be extended to an Eulerian triangulation. Furthermore, we give the explicit formula for the number of distinct Eulerian triangulations extended from a given quadrangulation of a surface. These completely solves the problem raised by Zhang and He [5].

(b) It is easy to see that every loopless triangulation G of any surface has a quadrangulation as a spanning subgraph of G. As well as (a), if we require some properties for the resulting quadrangulation, the problem might be difficult and interesting. Kündgen and Thomassen [1] proved that every loopless Eulerian triangulation G of the torus has a spanning nonbipartite quadrangulation, and that if G has sufficiently large face width, then G also has a bipartite one. We prove that a loopless Eulerian triangulation G of the torus has a spanning bipartite quadrangulation if and only if G does not have K_7 as a subgraph.

This talk is based on the papers [2, 3, 4].

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A list orientation of graphs

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For a list L of a graph G with $L(v) \subseteq \{0, 1, \ldots, \deg_G(v)\}$ for each vertex v, an L-orientation of G is one such that the outdegree of each vertex v is contained in the list $L(v)$. In this talk, we discuss the existence of an L-orientation. In particular, we apply a polynomial method to plane graphs to find an L-orientation if the list L satisfies certain conditions.

Partial Twuality Polynomials

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Poincare duality * and Petrie duality $\times$, as operators on ribbon graphs (cellular graph embeddings), generate a group of operators, or twualities, isomorphic to the symmetric group Σ_3. Any of these twualities T can be restricted to a subset of edges A to give a partial twuality $G^{T|A}$. Recent papers with Jonathan Gross and Toufik Mansour introduce the *partial-T polynomial* of G, the generating function enumerating partial-T twuals of G by euler genus. Interpretation of partial twuals in terms of partial permutations in the monodromy of G allows computation of these polynomials for small examples. Various properties and examples of partial polynomials are discussed with particular attention to interpolating and log-concave behavior, as well as possible connections to Bollobás-Riordan polynomials.

Ternary self-distributive cohomology and invariants of framed links and knotted surfaces with boundary

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Coauthors: Viktor Abramov, Masahico Saito

In this talk I will describe how to construct certain state-sum invariants of framed links that utilize the cohomology groups of ternary self-distributive racks and quandles. I will argue that these invariants can be considered, in an appropriate sense, as quantum invariants and give examples from Hopf algebras and 3-Lie algebras, such as ternary Nambu-Lie algebras. Finally, I will explain how these ideas generalize to the case of (compact and oriented) surfaces with boundary knotted in the 3-space.

Minisymposium

GROUPS, GRAPHS AND NETWORKS (MS-75)

Organized by Yan-Quan Feng, *Beijing Jiaotong University, China*
Coorganized by Rong-Xia Hao, *Beijing Jiaotong University, China*

- Structure connectivity and substructure connectivity of the crossed cube, *Dongqin Cheng*
- Classifications of graphical m-semiregular representation of finite groups, *Jiali Du*
- Fault-tolerance of the data center networks, *Rong-Xia Hao*
- Groups and skew morphisms, *Kan Hu*
- The s-geodesic-transitivity of graphs, *Wei Jin*
- Skew morphisms of finite groups with applications, *István Kovács*
- Symmetric graphs of prime valency, *Zai Ping Lu*
- Symmetric cubic graphs with non-solvable automorphism groups, *Jicheng Ma*
- Symmetric properties, reliabilities and Hamiltonian cycles of some hypercube-like networks, *Da Wei Yang*
- Trivalent dihedrants and bi-dihedrants, *Mimi Zhang*
- Symmetries of bi-Cayley graphs, *Jinxin Zhou*
- Perfect 2-colourings of Cayley graphs, *Sanming Zhou*

Structure connectivity and substructure connectivity of the crossed cube

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Interconnection network is usually represented by a simple graph G. The structure connectivity $\kappa(G;H)$ and substructure connectivity $\kappa^s(G;H)$ are the new proposed indicators to measure network fault tolerance and reliability when the network fails with different structures. As a variant of the popular network hypercube, the crossed cube is also a famous interconnection network in parallel and distributed systems. In this paper, we establish the H-structure connectivity of the n-dimensional crossed cube when $H \in \{K_{1,1}, K_{1,3}, P_k, C_4\}$ and $3 \leq k \leq n$ and H-substructure connectivity of the n-dimensional crossed cube when $H \in \{K_{1,1}, K_{1,3}, P_k, C_m\}$, $3 \leq k \leq n$ and $4 \leq m \leq n$.

Classifications of graphical m-semiregular representation of finite groups

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A graph or digraph is called *regular* if each vertex has the same valency, or, the same out-valency and the same in-valency, respectively. Recently, we extend the classical notion of *digraphical* and *graphical regular representation* of a group. A *(di)graphical m-semiregular representation* (respectively, GmSR and DmSR, for short) of a group G is a regular (di)graph whose automorphism group is isomorphic to G and acts semiregularly on the vertex set with m orbits. When $m = 1$, this definition agrees with the classical notion of GRR and DRR. Finite groups admitting a D1SR were classified by Babai in 1980, and the analogue classification of finite groups admitting a G1SR was completed by Godsil in 1981. Pivoting on these two results, we classify finite groups admitting a GmSR or a DmSR (for arbitrary positive integers m) and also do some work about bipartite (di)graphs.

Fault-tolerance of the data center networks

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The k-dimensional data center network with n-port switches, denoted by $D_{k,n}$, has been proposed for data centers as a server centric network structure. The ℓ spanning trees of a graph G are said to be the completely independent spanning trees (CISTs for short) if for any two vertices $x, y \in V(G)$, the paths joining x and y on the ℓ trees have neither vertex nor edge in common, except x and y.

In this talk, some properties about $D_{k,n}$ such as vertex-pancyclicity and the existence of two completely independent spanning trees are given. Furthermore, we consider fault-tolerance and prove that $D_{k,n}$ is conditional $(2n + 2k - 9)$-edge-fault-tolerant Hamiltonian for any $k \geq 0$ and $n \geq 2$ except $k = 1$ and $n \geq 6$.

Groups and skew morphisms

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In this talk I will give a survey of the theory of skew morphisms, and post several unsolved problems related to symmetric embeddings of graphs into orientable closed surfaces.

The s-geodesic-transitivity of graphs

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Coauthors: Alice Devillers, Caiheng Li, Cheryl Praeger

In a finite graph Γ, a geodesic from a vertex u to a vertex v is one of the shortest paths from u to v, and this geodesic is called an i-geodesic if the distance between u and v is i. The graph Γ is said to be *s-geodesic-transitive* if the graph automorphism group is transitive on the set of s-geodesics. In this talk, I will compare the s-geodesic-transitivity of graphs with other two well-known transitive properties, namely s-arc-transitivity and s-distance-transitivity, and determine the local structure of 2-geodesic-transitive graphs, and also give some results about the family of locally disconnected 2-geodesic-transitive but not 2-arc-transitive graphs.

Skew morphisms of finite groups with applications

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A skew morphism of a finite group G is a bijection $\varphi : G \to G$ fixing the identity element of G and having the property that $\varphi(xy) = \varphi(x)\varphi^{\pi(x)}(y)$ for all $x, y \in G$, where $\pi(x)$ depends only on x. Skew morphisms generalise group automorphisms and were introduced in the context of topological graph theory by Jajcay and Širáň (2002). In this talk, I review some recent results on skew morphisms and also mention some applications. These applications include a connection with the complementary product of a group with a cyclic group, the classification of regular Cayley maps for dihedral groups, and a connection with block transpositions (well-known sorting operations with relevant applications in Bioinformatics).

Symmetric graphs of prime valency

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A graph $\Gamma = (V, E)$ is called a Cayley graph of some group T if the automorphism group $Aut(\Gamma)$ contains a subgroup T which acts on regularly on V. If the subgroup T is normal in $Aut(\Gamma)$ then Γ is called a normal Cayley graph of T. Let r be an odd prime. Fang et al. [On locally primitive Cayley graphs of finite simple groups, J. Combin. Theory Ser. A 118 (2011), 1039-1051] proved that, with a finite number of exceptions for finite simple group T, every connected symmetric Cayley graph of T of valency r is normal. Employing maximal factorizations of finite almost simple groups, we work out a possible list of those exceptions for T.

Symmetric cubic graphs with non-solvable automorphism groups

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A cubic graph Γ is called G-symmetric if a group G of automorphisms of Γ acts transitively on the arcs of Γ, and G-basic if it is G-symmetric and G has no non-trivial normal subgroups with more than two orbits on the vertex set of Γ. We say the graph Γ is basic if it is G-basic for all arc-transitive subgroups of $Aut(\Gamma)$. In this talk, a characterization of basic symmetric cubic graphs with non-solvable automorphism groups will be discussed. This is a joint work with Jin-Xin Zhou.

Symmetric properties, reliabilities and Hamiltonian cycles of some hypercube-like networks

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The class of hypercube-like networks was proposed by Vaidya et al. in 1993, which includes numerous well-known topologies, such as hypercubes, locally twisted cubes, the spined cubes, and crossed cubes. In this talk, I will present our recent works on symmetric properties, reliabilities, and Hamiltonian cycles of some hypercube-like networks.

Trivalent dihedrants and bi-dihedrants

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A Cayley (resp. bi-Cayley) graph on a dihedral group is called a *dihedrant* (resp. *bi-dihedrant*). In 2000, a classification of trivalent arc-transitive dihedrants was given by Marušič and Pisanski, and several years later, trivalent non-arc-transitive dihedrants of order $4p$ or $8p$ (p a prime) were classified by Feng et al. As a generalization of these results, our first result presents a classification of trivalent non-arc-transitive dihedrants. Using this, a complete classification of trivalent vertex-transitive non-Cayley bi-dihedrants is given. As a by-product, we generalize a theorem in [The Electronic Journal of Combinatorics 19 (2012) #P53].

This is joint work with Jin-Xin Zhou.

Symmetries of bi-Cayley graphs

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A graph Γ admitting a group H of automorphisms acting semi-regularly on the vertices with exactly two orbits is called a *bi-Cayley graph* over H. Bi-Cayley graph is a natural generalization of Cayley graph. A large body of research has been developed in recent years to explore the symmetries of bi-Cayley graphs. Much of the work has been focused on the classification and construction of bi-Cayley graphs with specific symmetry properties. In this lecture, I will survey some recent results in this area.

Perfect 2-colourings of Cayley graphs

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Let $\Gamma = (V, E)$ be a graph. A partition $\pi = \{V_1, \ldots, V_m\}$ of V is called an equitable partition or a perfect m-colouring of Γ if there exists an $m \times m$ matrix (b_{ij}), called the quotient matrix of π, such that every vertex in V_i has exactly b_{ij} neighbours in V_j. In particular, if $\{C, V \setminus C\}$ is a perfect 2-colouring of a d-regular graph Γ with quotient matrix $\begin{pmatrix} 0 & d \\ 1 & d-1 \end{pmatrix}$, then C is called a perfect 1-code in Γ. In general, for an integer $t \geq 1$, a perfect t-code in Γ is a subset C of V such that every vertex of Γ is at distance no more than t to exactly one vertex in C. Perfect t-codes in Hamming graph $H(n, q)$ and in the Cartesian product of n copies of cycle C_q are precisely q-ary perfect t-codes of length n under the Hamming and Lee metrics, respectively. Thus perfect codes in Cayley graphs are a generalization of perfect codes in classical coding theory.

I will talk about some recent and not-so-recent results on perfect 2-colourings of Cayley graphs with an emphasis on perfect 1-codes in Cayley graphs.

Minisymposium

Spectral Graph Theory (MS-46)

Organized by Cristina Dalfó, *Universitat de Lleida, Spain*
Coorganized by Francesco Belardo, *University of Naples Federico II, Italy*

- Playing with quaternions unit gain graphs, *Maurizio Brunetti*
- Complementary prisms and their spectra, *Paula Carvalho*
- Spectra and eigenspaces from regular partitions of Cayley (di)graphs of permutation groups, *Cristina Dalfo*
- On the rank of pseudo walk matrices, *Alexander Farrugia*
- The local spectra of a graph and some of their applications, *Miquel Àngel Fiol Mora*
- Maximal cliques in strongly regular graphs, *Gary Greaves*
- Optimal Grid Drawings of Complete Multipartite Graphs and an Integer Variant of the Algebraic Connectivity, *Clemens Huemer*
- Strongly regular signed graphs and association schemes, *Ivana Jovović*
- Classes of strongly regular signed graphs, *Tamara Koledin*
- Systems of equiangular lines, Seidel matrices and adjacency matrices, *Jack Koolen*
- Isospectral magnetic graphs, *Fernando Lledó*
- Almost mixed Moore graphs and their spectra, *Ignacio Lopez Lorenzo*
- Characterizing identifying codes from the spectrum of a graph or digraph, *Berenice Martínez Barona*
- On some classes of signed graphs with small second largest eigenvalue, *Bojana Mihailović*
- Eigenvalues and $[a, b]$-factors in regular graphs, *Suil O*
- Construction of upper bounds of the HOMO-LUMO spectral gaps by semidefinite relaxation techniques, *Soňa Pavlíková*
- On symmetric association schemes and associated quotient-polynomial graphs, *Safet Penjić*

Playing with quaternions unit gain graphs

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A quaternion unit gain graph is a graph where a quaternion unit q is assigned to each oriented edge e_{ij}, and the conjugate $\bar{q}$ is assigned to e_{ji}. In a non-commutative context there exists a “left" spectral theory and a “right" spectral theory. I will show how the latter, but not the former, encapsulates some classical spectral results holding for ordinary graphs, signed graphs and complex unit gain graphs. Bounds for both the left and right eigenvalues of the adjacency and Laplacian matrix are also given, together with some explicit computations.

Complementary prisms and their spectra

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Coauthors: Domingos M. Cardoso, Maria Aguieiras A. de Freitas, Cybele T. M. Vinagre

The complementary prism $G\overline{G}$ of a graph G is obtained from the disjoint union of the graph G and its complement $\overline{G}$ defined on a copy of the vertex set of G, by adding an edge for each pair vertices (v, v'), where v is in G and its copy v' is in $\overline{G}$. The Petersen graph $C_5\overline{C_5}$ and the corona of a complete graph $K_n\overline{K_n}$, with $n \geq 2$, are examples of complementary prisms. In this talk we prove that the Petersen graph is the unique complementary prism which is strongly regular. Furthermore, we compute the eigenpairs of adjacency, signless Laplacian and Laplacian matrix of a complementary prism $G\overline{G}$ in terms of the eigenvalues of adjacency, signless Laplacian and Laplacian matrix of G, respectively. In particular, to the complementary prisms of regular graphs are given special attention.

Spectra and eigenspaces from regular partitions of Cayley (di)graphs of permutation groups

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Coauthor: Miquel Àngel Fiol Mora

In this talk, we present a method to obtain regular (or equitable) partitions of Cayley (di)graphs (that is, graphs, digraphs, or mixed graphs) of permutation groups on n letters. We prove that every partition of the number n gives rise to a regular partition of the Cayley graph. By using representation theory, we also obtain the complete spectra and the eigenspaces of the corresponding quotient (di)graphs. More precisely, we provide a method to find all the eigenvalues and eigenvectors of such (di)graphs, based on their irreducible representations. As examples, we apply this method to the pancake graphs $P(n)$ and to a recent known family of mixed graphs $\Gamma(d, n, r)$ (having edges with and without direction). As a byproduct, the existence of perfect codes in $P(n)$ allows us to give a lower bound for the multiplicity of its eigenvalue -1.

On the rank of pseudo walk matrices

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In the literature, the walk matrix $\mathbf{W_b}$ associated with a graph G having vertex set $\mathcal{V}(G)$ is the matrix with columns $\mathbf{b}, \mathbf{Ab}, \mathbf{A}^2\mathbf{b}, \ldots, \mathbf{A}^{r-1}\mathbf{b}$ that enumerates the number of all possible walks on G of length $0, 1, 2, \ldots, r-1$ starting from each vertex of G and ending at any of the vertices indicated by $\mathbf{b}$. We generalize walk matrices further to obtain pseudo walk matrices $\mathbf{W_v}$ having any walk vector $\mathbf{v}$. For any subset S of $\mathcal{V}(G) \times \mathcal{V}(G)$, the total number of walks $N_0(S), N_1(S), N_2(S), \ldots$ of length $0, 1, 2, \ldots$ in G that start from vertex i and end at vertex j for all $(i, j) \in S$ is considered. Various results on such pseudo walk matrices are presented, particularly related to their rank. The matrix rank of pseudo walk matrices allows the consideration of controllable and recalcitrant pairs.

The local spectra of a graph and some of their applications

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Given a graph Γ with vertex set V, the local spectrum of a vertex subset $C \subset V$ is constituted by the eigenvalues of Γ with local multiplicities, which are nonnegative real numbers (possibly zero). The local spectrum of C gives information on the structure of the graph Γ when it is 'seen' from C. In particular, when C consists of a single vertex u, the local multiplicities of u sum up to one, while the local multiplicities of a given eigenvalue λ, when added over all vertices, gives the (standard) multiplicity of λ. The aim of this talk is to describe some applications of the local spectra. For instance, they have been used in the characterization of distance-regular graphs, completely regular codes, existence of some subgrafs of a distance-regular graph, identifying vital nodes in complex networks, possible graph automorphisms, etc.
Keywords: Graph; Distance-regular graph, Complex network, Adjacency matrix, Adjacency spectrum, Laplacian spectrum, Local eigenvalues; Local multiplicities.
2010 Mathematics Subject Classification: 05C50, 05C82, 05E30, 15A18, 94B25.

Maximal cliques in strongly regular graphs

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In this talk, I will introduce a cubic polynomial that can be associated to a strongly regular graph Γ. The roots of this polynomial give rise to upper and lower bounds for the size of a maximal clique in Γ. I will explain how we can use this cubic polynomial to rule out the existence of strongly regular graphs that correspond to an infinite family of otherwise feasible parameters. This talk is based on joint work with Jack Koolen and Jongyook Park.

Optimal Grid Drawings of Complete Multipartite Graphs and an Integer Variant of the Algebraic Connectivity

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We use spectral graph theory to show how to draw the vertices of a complete multipartite graph G on different points of a bounded d-dimensional integer grid, such that the sum of squared distances between vertices of G is (i) minimized or (ii) maximized. For both problems we provide a characterization of the solutions. For the particular case d = 1, our solution for (i) also settles the minimum-2-sum problem for complete bipartite graphs; the minimum-2-sum problem was defined by Juvan and Mohar in 1992. Weighted centroidal Voronoi tessellations are the solution for (ii). Such drawings are related with Laplacian eigenvalues of graphs. This motivates us to study which properties of the algebraic connectivity of graphs carry over to the restricted setting of drawings of graphs with integer coordinates.

Strongly regular signed graphs and association schemes

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We consider relations between strongly regular signed graphs and symmetric association schemes. Our results include constructions of new examples of such signed graphs, relations between their structure and spectra, and their classifications.

We also propose definitions of Johnson signed graphs and Hamming signed graphs that arise from Johnson and Hamming schemes and act as the ‘signed’ counterparts to the well-known Johnson and Hamming graphs. We compute the eigenvalues of these signed graphs and provide necessary and sufficient conditions for their strong regularity. We also provide some results concerning strongly regular signed graphs that naturally arise from Johnson and Hamming schemes and have a comparatively small number of eigenvalues. Some constructions of strongly regular Johnson and Hamming signed graphs with at most five eigenvalues are provided.

Classes of strongly regular signed graphs

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We consider a concept of strong regularity defined for signed graphs – a generalization of strong regularity of the unsigned ones. We say that the signed graph $\dot{G}$ is *strongly regular* (for short, $\dot{G}$ is a *SRSG*) whenever it is regular, neither homogeneous complete nor totally disconnected, and if its adjacency matrix $A_{\dot{G}}$ satisfies

$$A_{\dot{G}}^2 = \frac{a}{2}(A_{\dot{G}} + A_G) - \frac{b}{2}(A_{\dot{G}} - A_G) + cA_{\overline{G}} + rI,$$

where G and $\overline{G}$ are the underlying graph of $\dot{G}$ and its complement and r is the vertex degree of G (and $\dot{G}$).

We establish certain basic structural and spectral properties of such signed graphs, and suggest a natural way to divide all SRSGs into five classes according to the relations among their defining parameters, which allows us to better perceive their properties.

Next, we consider walk-regularity of SRSGs with a relatively small number of distinct eigenvalues, belonging to some of those specified classes. In the end, we investigate the relationship between SRSGs with three or four distinct eigenvalues and three-class symmetric association schemes.

Systems of equiangular lines, Seidel matrices and adjacency matrices

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It is known that large systems of equiangular lines with common angle $\arccos 1/\alpha$ are closely related with Seidel matrices S with smallest eigenvalue $-\alpha$ such that $S + \alpha I$ has low rank. In this talk I will introduce the notion of a switching root and show how we can use the switching root to relate adjacency matrices and Seidel matrices. If time permits I will also discuss some new maximal connected graphs with minimal eigenvalue -3, that is, any proper connected supergraph of such graph has smallest eigenvalue less than -3. This is based on joint work with Meng-Yue Cao (Beijing Normal University), Akihiro Munemasa (Tohoku University), Kiyoto Yoshino (Tohoku University) and Brhane Gebremichel (University of Science and Technology of China).

Isospectral magnetic graphs

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We present a new geometrical construction leading to an infinite collection of families of graphs, where all the elements in each family are (finite) isospectral non-isomorphic graphs for the discrete magnetic Laplacian with normalised weights (in particular for standard weights). The construction is based on the notion of isospectral frames which, together with the s-partition of a natural number r, define the isospectral families of graphs by contraction of distinguished vertices. The isospectral frames have high symmetry and we use a spectral preorder of graphs studied in [2,3] to control the spectral spreading of the eigenvalues under elementary perturbations of the graph like vertex contraction and vertex virtualisation.

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Almost mixed Moore graphs and their spectra

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Almost mixed Moore graphs appear in the context of the *Degree/Diameter problem* as a class of extremal mixed graphs, in the sense that their order is one less than the Moore bound for mixed graphs. In this talk we will give some necessary conditions for the existence of almost mixed Moore graphs derived from the factorization in $\mathbb{Q}[x]$ of their characteristic polynomial. In this context, we deal with the irreducibility of certain polynomials $\Phi_i(x) \circ f(x)$, where $\Phi_i(x)$ denotes the i-th cyclotomic polynomial.

Characterizing identifying codes from the spectrum of a graph or digraph

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A $(1, \leq \ell)$-identifying code in digraph D is a dominating subset C of vertices of D, such that all distinct subsets of vertices of D with cardinality at most ℓ have distinct closed in-neighborhoods within C. In this talk we give a new method to obtain an upper bound on ℓ for digraphs. The results obtained here can also be applied to graphs. As far as we know, it is the first time that the spectral graph theory has been applied to the identifying codes.

On some classes of signed graphs with small second largest eigenvalue

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In previous descriptions of some classes of maximal cacti for the property $\lambda_2 \leq r$ (λ_2 being the second largest eigenvalue of the corresponding adjacency matrix), certain graph transformations that preserve the sign of $\lambda_2 - r$ have been used. Now, a possibility of applying such transformations to some classes of signed graphs is examined, especially for $r = 1$.

Eigenvalues and $[a, b]$-factors in regular graphs

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For positive integers, $r \geq 3, h \geq 1$, and $k \geq 1$, Bollobás, Saito, and Wormald proved some sufficient conditions for an h-edge-connected r-regular graph to have a k-factor in 1985. Lu gave an upper bound for the third-largest eigenvalue in a connected r-regular graph to have a k-factor in 2010. Gu found an upper bound for certain eigenvalues in an h-edge-connected r-regular graph to have a k-factor in 2014.

For positive integers $a \leq b$, an even (or odd) $[a, b]$-factor of a graph G is a spanning subgraph H such that for each vertex $v \in V(G)$, $d_H(v)$ is even (or odd) and $a \leq d_H(v) \leq b$. In this talk, we provide best upper bounds (in terms of a, b, and r) for certain eigenvalues (in terms of a, b, r, and h) in an h-edge-connected r-regular graph G to guarantee the existence of an even $[a, b]$-factor or an odd $[a, b]$-factor. This result extends the one of Bollbás, Saito, and Wormald, the one of Lu, and the one of Gu.

Construction of upper bounds of the HOMO-LUMO spectral gaps by semidefinite relaxation techniques

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An important application of graph theory in quantum chemistry is based on the fact that the energy of the highest occupied molecular orbital (HOMO) and of the lowest unoccupied molecular orbital (LUMO) of a molecule correspond, respectively, to the smallest positive and the largest negative eigenvalue of a graph representing the molecule; the difference of these eigenvalues is known as the HOMO-LUMO spectral gap.

In our contribution we study the HOMO - LUMO spectral gap of weighted graphs. In particular, we focus constructions of new graphs from old by 'bridging' two input graphs over a common bipartite subgraph, with the aim to maximize the spectral gap with respect to the structure of the ingredients. Among the tools we use estimates of the spectrum of the inverse of a block matrix consisting of adjacency matrices of input graphs on its block diagonal and the adjacency matrix of the bridging graph as off-diagonal blocks; maximization of the HOMO-LUMO gap turns out to be equivalent to minimization of the sum of largest and smallest eigenvalues of the inverse of the block matrix. An upper bound on the gap is then be obtained by semidefinite programming.

This a joint work with Daniel Sevcovic.

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On symmetric association schemes and associated quotient-polynomial graphs

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Let Γ denote an undirected, connected, regular graph with vertex set X, adjacency matrix A, and $d+1$ distinct eigenvalues. Let $\mathcal{A} = \mathcal{A}(\Gamma)$ denote the subalgebra of $\mathrm{Mat}_X(\mathbb{C})$ generated by A. We refer to $\mathcal{A}$ as the *adjacency algebra* of Γ. In this talk we investigate algebraic and combinatorial structure of Γ for which the adjacency algebra $\mathcal{A}$ is closed under Hadamard multiplication. In particular, under this simple assumption, we show the following: (i) $\mathcal{A}$ has a standard basis $\{I, F_1, \ldots, F_d\}$; (ii) for every vertex there exists identical distance-faithful intersection diagram of Γ with $d+1$ cells; (iii) the graph Γ is quotient-polynomial; and (iv) if we pick $F \in \{I, F_1, \ldots, F_d\}$ then F has $d+1$ distinct eigenvalues if and only if $\mathrm{span}\{I, F_1, \ldots, F_d\} = \mathrm{span}\{I, F, \ldots, F^d\}$. We describe the combinatorial structure of quotient-polynomial graphs with diameter 2 and 4 distinct eigenvalues. As a consequence of the technique from the paper we give an algorithm which computes the number of distinct eigenvalues of any Hermitian matrix using only elementary operations. When such a matrix is the adjacency matrix of a graph Γ, a simple variation of the algorithm allow us to decide whether Γ is distance-regular or not. In this context, we also propose an algorithm to find which distance-i matrices are polynomial in A, giving also these polynomials.

Minisymposium

Symmetry of Graphs, Maps and Polytopes (MS-9)

Organized by Primož Potočnik, *University of Ljubljana, Faculty of Mathematics and Physics, Slovenia*

Coorganized by Primož Šparl, *University of Ljubljana, Slovenia,* and *University of Primorska, Slovenia*

- Some observations about regular maps, *Marston Conder*
- On the Asymmetrizing Cost and Density of Graphs, *Wilfried Imrich*
- Non-Cayley regular maps and generalizations of skew-morphisms, *Robert Jajcay*
- Realisation of groups as automorphism groups of maps and hypermaps, *Gareth A. Jones*
- On rainbow domination in regular and symmetric graphs, *Boštjan Kuzman*
- Cyclotomic Association Schemes of Broad Classes and Applications to the Construction of Combinatorial Structures, *Luis Martínez*
- Intersection densities of transitive permutation groups, *Dragan Marušič*
- On automorphisms of direct products of abelian Cayley graphs, *Dave Witte Morris*
- Base sizes for the symmetric and alternating groups, *Joy Morris*
- A family of non-Cayley cores that are constructed from vertex-transitive or strongly regular self-complementary graphs, *Marko Orel*
- Having fun with designs, *Cheryl Praeger*
- Edge-biregular maps, *Olivia Reade*
- Distinguishing discrete groups by their finite quotients, *Alan Reid*
- On compact Riemann surfaces and hypermaps of genus $p+1$ where p is prime, *Sebastián Reyes-Carocca*
- Geometry and Combinatorics of Semiregular Polytopes, *Egon Schulte*
- Regular and 'half-regular' maps of negative prime Euler characteristic, *Jozef Širáň*
- Proper locally spherical hypertopes of hyperbolic type, *Asia Ivić Weiss*

Some observations about regular maps

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A map on an orientable surface is called 'orientably-regular' if its automorphism group has a single orbit on arcs (incident vertex-edge pairs), and is then called 'reflexible' or 'chiral' depending on whether or not it admits reflections (for example, fixing an arc but swapping the two faces incident with it). In every such map with a high degree of symmetry, all vertices have the same valency, say k, and all faces have the same size, say m, and then the (ordered) pair $\{m, k\}$ is called the 'type' of the map.

Writing a book with Gareth Jones, Jozef Širáň and Tom Tucker) on regular maps has prompted us to review and extend what is known about them. Various questions have arisen naturally, including what I believe is the most important unanswered one, namely whether chirality is more prevalent than reflexibility. Other questions include these: What types $\{m, k\}$ occur the most frequently among orientably-regular maps on hyperbolic surfaces? What kinds of groups are the most prevalent as the group of orientation-preserving automorphisms? (Simple groups? insoluble groups? soluble groups? 2-groups?) Is chirality is more prevalent than reflexibility for a given type? I will give some partial answers to these questions, with reference to computational evidence. These answers may be surprising.

On the Asymmetrizing Cost and Density of Graphs

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A set S of vertices in a graph G with nontrivial automorphism group is *asymmetrizing* if the identity mapping is the only automorphism of G that preserves S as a set. If such sets exist, then their minimum cardinality is the *asymmetrizing cost* $\rho(G)$ of G. For finite graphs the *asymmetrizing density* $\delta(G)$ of G is the quotient of the size of S by the order of G. For infinite graphs $\delta(G)$ is defined by a limit process.

The talk discusses bounds on the asymmetrizing cost, classes of graphs with asymmetrizing density zero, and infinite graphs with finite asymmetrizing cost.

It is easy to construct graphs with positive asymmetrizing density, unless they are vertex transitive. Hitherto no infinite vertex transitive graphs with $\delta(G) > 0$ seem to have been known. Here we construct connected, infinite vertex transitive cubic graphs of asymmetrizing density $\delta(G) = n^{-1}2^{-n-1}$ for each $n \geq 1$.

We also construct finite vertex transitive cubic graphs of arbitrarily large asymmetrizing cost. The examples are Split Praeger–Xu graphs, for which we provide another characterization.

Non-Cayley regular maps and generalizations of skew-morphisms

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A Cayley map $CM(G, X, p)$ is a 2-cell embedding of a connected Cayley graph $C(G, X)$ into an orientable surface having the property that for each $a \in G$ the graph automorphism of $C(G, X)$ induced by the left multiplication by a, $g \mapsto a \cdot g$, 'lifts' into a map automorphism of the embedding. Hence, all Cayley maps admit automorphism groups acting regularly on the vertices of the map, namely, the groups $G_L \cong G$ of automorphisms induced by left multiplications by the elements of G. As is well-known, an embedding of a Cayley graph $C(G, X)$ into an orientable surface is a Cayley map if and only the rotation system of the embedding has the property that all local permutations of the neighbors gx, $x \in X$, of the vertices $g \in G$ determined by the embedding are equal to a constant cyclic permutation p of X.

A 2-cell embedding of a connected graph into an (orientable or non-orientable) surface, a map $\mathcal{M}$, is said to be regular if the full automorphism group of $\mathcal{M}$ acts regularly on its flags. An orientable map is said to be orientably regular if it admits an orientation preserving automorphism group acting transitively on its arcs. A Cayley map $CM(G, X, p)$ is known to be orientably regular if and only if there exists a skew-morphism of G that agrees on X with p. In our talk, we will focus on orientably regular and regular maps which are not Cayley, but are in some sense 'close to being Cayley' and which admit a 'partial' skew-morphism generating a vertex-stabilizer of the map. The ideas stem from a recent article on generalized Cayley maps written jointly by the presenter, J. Siran and Y. Wang, and from the concept of half-regular Cayley maps introduced by the presenter and R. Nedela.

Realisation of groups as automorphism groups of maps and hypermaps

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I will show that in various categories, including many consisting of maps or hypermaps, oriented or unoriented, of a given hyperbolic type, or of coverings of a suitable topological space, every countable group A is isomorphic to the automorphism group of uncountably many non-isomorphic objects, infinitely many of which are finite if A is finite. In particular, the latter applies to dessins d'enfants, regarded as finite oriented hypermaps. The objects realising A are obtained as regular coverings by A of certain basic objects with primitive monodromy groups, corresponding to maximal subgroups of triangle groups. The constructions of these generalise results of Bernhard Neumann on maximal subgroups of infinite index in the modular group, and of Marston Conder on maximal subgroups of finite index in various cocompact triangle groups.

On rainbow domination in regular and symmetric graphs

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The k-rainbow domination function of a graph is a function that assigns a subset of $\{1, 2, \ldots, k\}$ to each vertex of a graph, such that each non-colored vertex has a complete k-rainbow of neighbours, that is, $f(v) = \emptyset$ implies $\cup_{u \sim v} f(u) = \{1, \ldots, k\}$. The k-rainbow domination number $\gamma_{rk}(G)$ of a graph is the minimal possible value of weight $w(f) = \sum |f(v)|$ over all k-rainbow domination funtions f on G.

Recently, we have shown that the k-rainbow domination number $\gamma_{rk}(G)$ of a d-regular graph for $d \leq k \leq 2d$ is bounded below by $\lceil kn/2d \rceil$, where n is the order of a graph, and determined some necessary conditions for regular graphs to attain this bound. This enabled us to find simpler proofs of some known results on kRD-number for specific graph families and determine exact kRD-numbers for all cubic Cayley graphs over abelian groups, as well as opened several questions on finding k-rainbow-domination-regular graphs that will be presented in a talk.

Cyclotomic Association Schemes of Broad Classes and Applications to the Construction of Combinatorial Structures

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In 2010, G. Fernández, R. Kwashira and L. Martínez gave a new cyclotomy on $A = \prod_{i=1}^{n} \mathbb{F}_{q_i}$, where $\mathbb{F}_{q_i}$ is a finite field with q_i elements. They defined a certain subgroup H of the group of units of this product ring A for which the quotient is cyclic. The orbits of the corresponding multiplicative action of the subgroup on the additive group of A are of two types:

- The cyclotomic cosets of the quotient of the group of units of A over the subgroup H.
- The n-tuples with arbitrary non-zero elements in positions indicated by a proper subset S of $\{1, \ldots, n\}$ and zeroes elsewhere.

In this talk, we introduce and study a fusion of a class of association schemes derived from the mentioned cyclotomy. The association schemes that we are proposing correspond with a fusion of orbits associated to subsets S of $\{1, \ldots, n\}$ of the same cardinality. We call cyclotomic association schemes of broad classes to these association schemes. The fusion corresponds to the operation of adding the permutations of A induced by the permutations of the symmetric group S_n to the transitive permutation group that determines the original association scheme.

We use these association schemes to obtain sporadic examples and infinite families of difference sets and partial difference sets.

Intersection densities of transitive permutation groups

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Two elements g and h of a permutation group G acting on a set V are said to be *intersecting* if $g(v) = h(v)$ for some $v \in V$. More generally, a subset $\mathcal{F}$ of G is an *intersecting set* if every pair of elements of $\mathcal{F}$ is intersecting. The *intersection density* $\rho(G)$ of a transitive permutation group G is the maximum value of the quotient $|\mathcal{F}|/|G_v|$ where $\mathcal{F}$ runs over all intersecting sets in G and G_v is a stabilizer of $v \in V$.

In this talk intersection densities of transitive permutation groups of certain degrees are determined, thus settling some of the problems and conjectures raised in [K. Meagher, A. S. Razafimahatratra and P. Spiga, On triangles in derangement graphs, *J. Combin. Theory, Ser. A* **180** (2021), 105390.] and [A. S. Razafimahatratra, On multipartite derangement graphs, *Ars Math. Contemp.* (2021), doi: 10.26493/1855-3974.2554.856.].

On automorphisms of direct products of abelian Cayley graphs

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The direct product of two graphs X and Y is denoted $X \times Y$. (It is also known as the "tensor product" or "categorical product" or "Kronecker product" or "conjunction" of X and Y.) This is a natural construction, so any isomorphism from X to X' can be combined with any isomorphism from Y to Y' to obtain an isomorphism from $X \times Y$ to $X' \times Y'$. Therefore, the automorphism group $\mathrm{Aut}(X \times Y)$ contains a copy of $(\mathrm{Aut}\, X) \times (\mathrm{Aut}\, Y)$. It is not known when this inclusion is an equality, even for the special case where $Y = K_2$ is the complete graph with only 2 vertices. (The direct product $X \times K_2$ is also known as the "canonical bipartite double cover" of X. The graph X is said to be "stable" if equality holds in this special case.)

When X is a circulant graph with an odd number of vertices (and $Y = K_2$), recent work of B. Fernandez and A. Hujdurović shows that equality holds if and only if X is connected and no two vertices of X have exactly the same neighbours. We will present a short, elementary proof that generalizes this theorem to the case where X is a Cayley graph on an abelian group of odd order.

Base sizes for the symmetric and alternating groups

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A base of a permutation group is a set of the elements on which it is acting, that is only fixed by the identity element of that permutation group. The base size of a permutation group is the minimum possible size for a base.

These concepts have been rediscovered and studied in the context of automorphism groups of combinatorial objects: if a permutation group is the automorphism group of an object, then a distinguishing set is a collection of "points" of the object that are only fixed by the identity automorphism. The distinguishing number is the smallest size of such a set.

I will present new results about the base sizes for two non-standard actions of the symmetric group Sym(n): its action on partitions, and its action on subsets of a fixed cardinality. For the first of these, I will also present results about the corresponding base sizes for the alternating group.

A family of non-Cayley cores that are constructed from vertex-transitive or strongly regular self-complementary graphs

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Let Γ be a finite simple graph on n vertices. In the talk I will consider the graph $\Gamma \equiv \bar{\Gamma}$ on $2n$ vertices, which is obtained as the disjoint union of Γ and its complement $\bar{\Gamma}$, where we add a perfect matching such that each its edge joins two copies of the same vertex in Γ and $\bar{\Gamma}$. The graph $\Gamma \equiv \bar{\Gamma}$ generalizes the Petersen graph, which is obtained if Γ is the pentagon. It is a non-Cayley graph if $n > 1$, and is vertex-transitive if and only if Γ is vertex-transitive and self-complementary. In this case $\Gamma \equiv \bar{\Gamma}$ is Hamiltonian-connected whenever $n > 5$. It is shown that the fraction between the cardinalities of the automorphism groups of $\Gamma \equiv \bar{\Gamma}$ and Γ can attain only values 1, 2, 4, or 12, and the corresponding four classes of graphs are described. The spectrum of the adjacency matrix of $\Gamma \equiv \bar{\Gamma}$ is computed whenever Γ is regular. The main results involve the endomorphisms of $\Gamma \equiv \bar{\Gamma}$. It is shown that the graph $\Gamma \equiv \bar{\Gamma}$ is a core, i.e. all its endomorphisms are automorphisms, whenever Γ is strongly regular and self-complementary. The same type of a result is obtained for many cases, where Γ is vertex-transitive and self-complementary.

Having fun with designs

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The kind of design we explore is a finite 2-design: a point-block incidence structure where each block is a k-subset of points, and each pair of points lies in a constant number λ of blocks. We ask that the design admits a block-transitive group G of automorphisms which preserves also a nontrivial partition of the point set. One famous study of these designs, by Delandtsheer and Doyen in 1989 introduced two parameters, now called Delandtsheer–Doyen parameters, that linked the design structure with the point-partition. Another even earlier study, by Davies in 1987, showed that, if G is transitive on flags (incident point-block pairs) then the number of points is bounded above by some function of λ (but no function was specified).

Recently, with Alice Devillers, we have been exploring these two results.

For flag-transitive designs: we showed that $4\lambda^6$ could be taken for the Davies function – though this is not a tight upper bound. Moreover, while investigating possible examples with small parameters, we found a rather beautiful flag-transitive design where the point set is a 6×6 grid, the block size is 8, $\lambda = 4$, and the full automorphism group is the symmetric group S_6.

Then, with Carmen Amarra and Alice Devillers, while exploring bounds and several extreme cases of the Delandtsheer–Doyen parameters, we constructed a (probably infinite) new family of designs. Whether the family is infinite depends on the Bunyakovsky Conjecture about the prime numbers, and the number of examples in a certain range seems to be related to another number theoretic conjecture, the Bateman—Horn Conjecture. We are grateful to Sasha Zvonkin and Gareth Jones for their interest in the number theoretic puzzles our construction spawned: their computer enumerations related to these conjectures showed that there are $12,357,532$ designs in the family where the point-partition has classes of prime cardinality less than 10^8.

Edge-biregular maps

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A regular map is an embedding of a graph on a surface such that the automorphism group acts regularly, that is semi-regularly and transitively, on the set of flags of the map. A map with an alternate edge colouring is such that every edge is coloured with one of two colours, and the edges are arranged so that any pair of edges which are adjacent with respect to the embedding have one edge of each colour. An edge-biregular map has an assigned alternate edge colouring and is such that the colour-preserving automorphism group acts regularly on the corners of the map. This poster presents a classification of these maps for the torus and the Klein bottle as well as a classification for when the colour-preserving automorphism group is dihedral.

Distinguishing discrete groups by their finite quotients

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We will survey work on when certain discrete groups arising from low-dimensional topology and geometry can be distinguished by the set of their finite quotients, or equivalently, when the profinite completion of a discrete group determines the group up to isomorphism.

On compact Riemann surfaces and hypermaps of genus $p + 1$ where p is prime

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In this talk we shall discuss a classification of compact Riemann surfaces of genus g, where $g - 1$ is a prime p, which have a group of automorphisms of order $\rho(g - 1)$ for some integer $\rho \geqslant 1$. We also provide isogeny decompositions of their Jacobian varieties. As a consequence, a classification of orientably regular hypermaps of genus $p + 1$ with automorphism group of order divisible by the prime p is obtained.

Geometry and Combinatorics of Semiregular Polytopes

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Traditionally, a polyhedron or polytope is semiregular if its facets are regular and its symmetry group is transitive on vertices. We briefly review the semiregular convex polytopes, and then discuss semiregular abstract polytopes, which have abstract regular facets, still with combinatorial automorphism group transitive on vertices. Our focus is on alternating semiregular polytopes, with two kinds of regular facets occurring in an alternating fashion. The cuboctahedron is a familiar example in rank 3. We then describe recent progress on the assembly problem for alternating semiregular polytopes: which pairs of regular n-polytopes can occur as facets of a semiregular (n+1)-polytope? If time permits, we brief discuss semiregularity in the context of skeletal polyhedra in 3-space. Most work is joint with Barry Monson.

Regular and 'half-regular' maps of negative prime Euler characteristic

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In 2005 A. Breda, R. Nedela and the presenter classified the (fully) regular maps on surfaces with negative prime Euler characteristic; this was the first such classification for an infinite family of surfaces. Extending a 2005 result of M. Belolipetsky and G. Jones, in a 2010 paper by M. Conder, T. Tucker and the presenter a corresponding orientable version of the classification was given for orientably-regular maps of genus $p+1$ for any prime p.

Algebraically, fully regular maps of valency k and face length ℓ correspond to normal quotients of the full $(2,k,\ell)$-triangle groups presented in the form $\Delta(2,k,\ell) = \langle r_0, r_1, r_2;\ r_0^2, r_1^2, r_2^2, (r_0r_2)^2, (r_2r_1)^k, (r_1r_0)^\ell \rangle$. Orientably-regular maps then arise from normal quotients of the orientation-preserving subgroup $\langle r_0r_1, r_1r_2\rangle$ of index two in $\Delta(2,k,\ell)$. Depending on the parity of k and ℓ, however, the group $\Delta(2,k,\ell)$ may contain up to 7 subgroups of index two, giving rise to further families of 'half-regular' maps in addition to the orientably-regular ones.

The above classification results have generated interest in a similar investigation of the remaining families of 'half-regular' maps. The first family studied from this point of view appears to be the one of *bi-rotary maps* which arise from the index-two subgroup $\langle r_0, r_1r_2\rangle$ of $\Delta(2,k,\ell)$ for ℓ even. A classification of bi-rotary maps of negative prime Euler characteristic was completed in 2019 by A. Breda, D. Catalano and the presenter. Recently, another such family of maps, called *edge-biregular* and generated by the subgroup $\langle r_0, r_2, r_1r_0r_1, r_1r_2r_1\rangle$ of index two in $\Delta(2,k,\ell)$ for both k,ℓ even, have been investigated in detail by O. Reade (2021). Moreover, in a joint 2021 paper by O. Reade and the presenter we have classified edge-biregular maps on surfaces of negative prime Euler characteristic.

In the talk we will review the previous classification results and present details on the new ones for edge-biregular maps.

Acknowledgment: This research was supported by the APVV Research Grants 17-0428 and 19-0308, and by the VEGA Research Grants 1/0238/19 and 1/0206/20.

Proper locally spherical hypertopes of hyperbolic type

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Given any irreducible Coxeter group C of hyperbolic type with non-linear diagram and rank at least 4, whose maximal parabolic subgroups are finite, we construct an infinite family of locally spherical regular hypertopes of hyperbolic type whose Coxeter diagram is the same as that of C.

Minisymposia in

DIFFERENTIAL GEOMETRY AND APPLICATIONS

- Differential Geometry: Old and New (MS-15)
- Geometric analysis and low-dimensional topology (MS-59)
- Geometries Defined by Differential Forms (MS-44)

Minisymposium

DIFFERENTIAL GEOMETRY: OLD AND NEW (MS-15)

Organized by Claudiu C. Remsing, *Rhodes University, South Africa*
Coorganized by Zlatko Erjavec, *University of Zagreb, Croatia*

- Characterization of manifolds of constant curvature by spherical curves and ruled surfaces, *Luiz C. B. Da Silva*
- Differential geometry of submanifolds in flag varieties via differential equations, *Boris Doubrov*
- J-trajectories in Sol_0^4, *Zlatko Erjavec*
- Similarlity geometry revisited: Differential Geometry and CAGD, *Jun-ichi Inoguchi*
- Contact CR submanifolds in odd-dimensional spheres: new examples, *Marian Ioan Munteanu*
- Cauchy-Riemann geometry of Legendrian curves in the 3-dimensional Sphere, *Emilio Musso*
- On the Bishop frame of a partially null curve in Minkowski spacetime, *Emilija Nešović*
- Topologically Embedded Pseudospherical Surfaces, *Lorenzo Nicolodi*
- New results in the study of magnetic curves in quasi-Sasakian manifolds of product type, *Ana Irina Nistor*
- On composition of geodesic and conformal mappings between generalized Riemannian spaces preserving certain tensors, *Miloš Petrović*
- Null scrolls, B-scrolls and associated evolute sets in Lorentz-Minkowski 3-space, *Ljiljana Primorac Gajčić*
- Non-holonomic equations for sub-Riemannian extremals and metrizable parabolic geometries, *Jan Slovak*
- Simple closed geodesics on regular tetrahedra in spaces of constant curvature, *Darya Sukhorebska*

Characterization of manifolds of constant curvature by spherical curves and ruled surfaces

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Coauthor: José D. Da Silva

Space forms, i.e., Riemannian manifolds of constant sectional curvature, play a prominent role in geometry and an important problem consists of finding properties that characterize them. In this talk, we report results from [1], where we show that the validity of some theorems concerning curves and surfaces can be used for this purpose. For example, it is known that the so-called rotation minimizing (RM) frames allow for a characterization of geodesic spherical curves in Euclidean, hyperbolic, and spherical spaces through a linear equation involving the coefficients that dictate the RM frame motion [2]. Here, we shall prove the converse, i.e., if all geodesic spherical curves on a manifold are characterized by a certain linear equation, then all the geodesic spheres with a sufficiently small radius are totally umbilical, and consequently, the ambient manifold is a space form. (We also present an alternative proof, in terms of RM frames, for space forms as the only manifolds where all geodesic spheres are totally umbilical [3].) In addition, we furnish two other characterizations in terms of (i) an inequality involving the mean curvature of a geodesic sphere and the curvature function of their curves and (ii) the vanishing of the total torsion of closed spherical curves in the case of 3d manifolds. (These are the converse of previous results [4].) Finally, we introduce ruled surfaces and show that if all extrinsically flat surfaces in a 3d manifold are ruled, then the manifold is a space form.

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Differential geometry of submanifolds in flag varieties via differential equations

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We give a unified method for the general equivalence problem osculating embeddings

$$\varphi\colon (M, \mathfrak{f}) \to \mathrm{Flag}(V, \phi)$$

from a filtered manifold $(M, \mathfrak{f})$ to a flag variety $\mathrm{Flag}(V, \phi)$. We establish an algorithm to obtain the complete systems of invariants for the osculating maps which satisfy the reasonable regularity condition of constant symbol of type $(\mathfrak{g}_-, \mathrm{gr}\, V)$. We show the categorical isomorphism between the extrinsic geometries in flag varieties and the (weightedly) involutive systems of linear differential equations of finite type. Therefore we also obtain a complete system of invariants for a general involutive systems of linear differential equations of finite type and of constant symbol.

The invariants of an osculating map (or an involutive system of linear differential equations) are proved to be controlled by the cohomology group $H^1_+(\mathfrak{g}_-, \mathfrak{gl}(V)/\,\mathrm{Prol}(\mathfrak{g}_-))$, which is defined algebraically from the symbol of the osculating map (resp. involutive system), and which, in many cases (in particular, if the symbol is associated with a simple Lie algebra and its irreducible representation), can be computed by the algebraic harmonic theory, and the vanishing of which gives rigidity theorems in various concrete geometries.

J-trajectories in Sol_0^4

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J-trajectories are arc length parameterized curves in almost Hermitian manifold which satisfy the equation $\nabla_{\dot\gamma}\dot\gamma = qJ\dot\gamma$. J-trajectories are 4-dim analogon of 3-dim magnetic trajectories, curves which satisfy the Lorentz equation $\nabla_{\dot\gamma}\dot\gamma = q\phi\dot\gamma$.

In this talk J-trajectories in the 4-dimensional solvable Lie group Sol_0^4 are considered. Moreover, the first and the second curvature of a non-geodesic J-trajectory in an arbitrary 4-dimensional LCK manifold whose anti Lee field has constant length are examined. In particular, the curvatures of non-geodesic J-trajectories in Sol_0^4 are characterized.

Similarlity geometry revisited: Differential Geometry and CAGD

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Similarity geometry is a Klein geometry whose transformation group is the similarity transformation group. The similarity transformation group is generated by Euclidean isometries and scalings.

One can develop differential geometry of plane curves under similarity transformation group.

In particular we obtain similarity curvature, similarity Frenet formula and fundamental theorem of plane curves in similarity geometry.

On the other hand, in industrial design (CAGD), log-aesthetic curves are studied extensively. In this talk, we give a similarity geometric reformulation of log-aesthetic curves and discuss relations to curve flows derived form Burgers flows.

Contact CR submanifolds in odd-dimensional spheres: new examples

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University Alexandru Ioan Cuza of Iasi, Romania

The notion of CR-submanifold in Kähler manifolds was introduced by A. Bejancu in 70's, with the aim of unifying two existing notions, namely complex and totally real submanifolds in Kähler manifolds. Since then, the topic was rapidly developed, mainly in two directions:

• Study CR-submanifolds in other almost Hermitian manifolds.

• Find the odd dimensional analogue of CR-submanifolds. Thus, the notion of semi-invariant submanifold in Sasakian manifolds was introduced. Later on, the name was changed to contact CR-submanifolds.

A huge interest in the last 20 years was focused on the study of CR-submanifolds of the nearly Kähler six dimensional unit sphere. Interesting and important properties of such submanifolds were discovered, for example, by M. Antic, M. Djoric, F. Dillen, L. Verstraelen, L. Vrancken. As the odd dimensional counterpart, contact CR-submanifolds in odd dimensional spheres were, recently, intensively studied. In this talk we focus on those proper contact CR-submanifolds, which are as closed as possible to totally geodesic ones in the seven dimensional spheres endowed with its canonical structure of a Sasakian space form. We give a complete classification for such a submanifold having dimension 4 and describe the techniques of the study. We present also some very recent developments concerning dimension 5 and 6 and propose further problems in this direction.

This presentation is based on some papers in collaboration with M. Djoric and L. Vrancken.

Keywords: (contact) CR-submanifold, Sasakian manifolds, minimal submanifolds, mixed and nearly totally geodesic CR-submanifolds

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Cauchy-Riemann geometry of Legendrian curves in the 3-dimensional Sphere

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Let S^3 be the unit 3-sphere with its standard Cauchy–Riemann (CR) structure. We consider the CR geometry of Legendrian curves in S^3, thought of as a 3-dimensional homogeneous CR manifold. We introduce the two main local invariants : a line element (the cr-infinitesimal strain) and the cr-bending. Integrating the invariant line element we get the simplest cr-invariant variational problem for Legendrian curves in S^3 . We discuss Liouville integrability and the existence of closed critical curves.

On the Bishop frame of a partially null curve in Minkowski spacetime

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The Bishop frame $\{T, N_1, N_2\}$ (relatively parallel adapted frame) of a regular curve in Euclidean space E^3 contains the tangent vector field T of the curve and two relatively parallel vector fields N_1 and N_2 whose derivatives in arc length parameter s make minimal rotations along the curve. In Minkowski spaces E_1^3 and E_1^4, the Bishop frame of a non-null curve and a null Cartan curve has analogous property.

In this talk, we present a method for obtaining the Bishop frame (rotation minimizing frame) of a partially null curve α lying in the lightlike hyperplane of Minkowski spacetime. We show that α has two possible Bishop frames, one of which coincides with its Frenet frame. By using spacetime geometric algebra, we derive the Darboux bivectors of Frenet and Bishop frame and give geometric interpretation of the Frenet and the Bishop curvatures in terms of areas obtained by projecting the Darboux bivector onto a spacelike or a lightlike plane.

Topologically Embedded Pseudospherical Surfaces

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It is known that the class of traveling wave solutions of the sine-Gordon equation is in 1-1 correspondence with the class of (necessarily singular) pseudospherical helicoids, i.e., pseudospherical surfaces in Euclidean space with screw-motion symmetry. We illustrate our solution to the problem of explicitly describing all pseudospherical helicoids posed by A. Popov in [Lobachevsky Geometry and Modern Nonlinear Problems, Birkhäuser, Cham, 2014]. As an application, countably many continuous families of topologically embedded pseudospherical helicoids are constructed. This is joint work with Emilio Musso.

New results in the study of magnetic curves in quasi-Sasakian manifolds of product type

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This presentation is based on the joint paper with M.I. Munteanu entitled "Magnetic curves in quasi-Sasakian manifolds of product type" which was accepted for publication in "New Horizons in Differential Geometry and its Related Fields", Eds. T. Adachi and H. Hashimoto, 2021.

The main result represents a positive answer to sustain our conjecture about the order of a magnetic curve in a quasi-Sasakian manifold. More precisely, we show that the magnetic curves in quasi-Sasakian manifolds, obtained as the product of a Sasakian and a Kähler manifold, have maximum order 5.

Next, we study the magnetic curves in $\mathbb{S}^3 \times \mathbb{S}^2$. First, we find the explicit parametrizations of such curves. Then, we find a necessary and sufficient condition for a magnetic curve in $\mathbb{S}^3 \times \mathbb{S}^2$ to be periodic. Finally, we conclude with some examples of magnetic curves in $\mathbb{S}^3 \times \mathbb{S}^2$.

On composition of geodesic and conformal mappings between generalized Riemannian spaces preserving certain tensors

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Recently, O. Chepurna, V. Kiosak and J. Mikeš studied geodesic and conformal mappings betweeen two Riemannian spaces preserving the Einstein tensor and among other things proved that in that case Yano's tensor of concircular curvature is also invariant with respect to these mappings. On the other hand I. Hinterleitner and J. Mikeš recently investigated composition of geodesic and conformal mappings between Riemannian spaces that is at the same time harmonic. In the present paper we connect these results and consider it in the settings of generalized Riemannian spaces in Eisenhart's sense.

Null scrolls, B-scrolls and associated evolute sets in Lorentz-Minkowski 3-space

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Coauthors: Željka Milin Šipuš, Ivana Protrka

In classical differential geometry in Euclidean space, the Bonnet's theorem states that there are two surfaces of constant mean curvature parallel a surface of constant positive Gaussian curvature. These two constant mean curvature surfaces are so-called harmonic evolutes of each other. In this short presentation, we present results of the analogous investigation in Lorentz-Minkowski 3-space, however, restricted to the case of surfaces that have no Euclidean counterpart, the quasi-umbilical surfaces, [1]–[4]. These surfaces are characterized by the property that their shape operator is not diagonalizable, and they can be parametrized as null scrolls or B-scrolls, [6]. In [5] we have shown that they are the only surfaces whose evolute set degenerates to a curve. The curve is of either null or spacelike causal character, and we analyse them respectively.

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Non-holonomic equations for sub-Riemannian extremals and metrizable parabolic geometries

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I will report several recent attempts linking sub-Riemannian geometries to the rich geometry of filtered manifolds, particularly the parabolic ones. After touching on some relation between the canonical Cartan geometries, I shall present an approach to sub-Riemannian extremals motivated by the tractor calculus. Finally, I will explore some implications of the BGG machinery to the (sub-Riemannian) metrizability of parabolic geometries. All that will be based on joined work with D. Alekseevsky, A. Medvedev, R. Gover, D. Calderbank, V. Soucek.

Simple closed geodesics on regular tetrahedra in spaces of constant curvature

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Coauthor: Alexander Borisenko

Since regular triangles form a regular tiling of Euclidean plane it easily follows the full classification of closed geodesics on a regular tetrahedron in Euclidean space.

We described all simple (without self-intersections) closed geodesics on regular tetrahedra in three-dimensional hyperbolic and spherical spaces. In these spaces the tetrahedron's curvature is concentrated not only into its vertices but also into its faces. The value α of the faces' angle for hyperbolic space's tetrahedron satisfies $0 < \alpha < \pi/3$ and for a tetrahedron in spherical space the faces' angle is measured α that $\pi/3 < \alpha < 2\pi/3$. The intrinsic geometry of such tetrahedra depends on the value of its faces' angles.

A simple closed geodesic on a tetrahedron has the type (p, q) if it has p points on each of two opposite edges of the tetrahedron, q points on each of another two opposite edges, and there are $(p + q)$ points on each edges of the third pair of opposite one.

We prove that on a regular tetrahedron in hyperbolic space for any coprime integers (p, q), $0 \leq p < q$, there exists unique, up to the rigid motion of the tetrahedron, simple closed geodesic of type (p, q). These geodesics exhaust all simple closed geodesics on a regular tetrahedron in hyperbolic space. The number of simple closed geodesics of length bounded by L is asymptotic to constant (depending on α) times L^2, when L tending to infinity [1].

On a regular tetrahedron in spherical space there exists the finite number of simple closed geodesic. The length of all these geodesics is less than 2π. For any coprime integers (p, q) we presented the numbers α_1 and α_2 depending on p, q and satisfying the inequalities $\pi/3 < \alpha_1 < \alpha_2 < 2\pi/3$ such that on a regular tetrahedron in spherical space with the faces' angle of value $\alpha \in (\pi/3, \alpha_1)$ there exists unique, up to the rigid motion of the tetrahedron, simple closed geodesic of type (p, q) and on a regular tetrahedron with the faces' angle of value $\alpha \in (\alpha_2, 2\pi/3)$ there is no simple closed geodesic of type (p, q).

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Minisymposium

Geometric analysis and low-dimensional topology (MS-59)

Organized by Paul Feehan, *Rutgers University, United States*
Coorganized by
Raphael Zentner, *University of Regensburg, Germany*
Natasa Sesum, *Rutgers University, United States*

- A Local Singularity Analysis for the Ricci Flow, *Reto Buzano*
- Spaces of constrained positive scalar curvature metrics, *Alessandro Carlotto*
- Foliation of Asymptotically Schwarzschild Manifolds by Generalized Willmore Surfaces, *Alexander Friedrich*
- A two-valued Bernstein theorem in dimension four, *Fritz Hiesmayr*
- Characterizing slopes for Legendrian knots, *Marc Kegel*
- Large area-constrained Willmore spheres in initial data sets, *Thomas Koerber*
- A Positive Mass Theorem for Fourth-Order Gravity, *Paul Laurain*
- Applications of virtual Morse–Bott theory to the moduli space of $\mathrm{SO}(3)$ Monopoles, *Thomas Leness*
- Recent progress in Lagrangian mean curvature flow of surfaces, *Jason Lotay*
- Applications of Floer homology to clasp number vs genus, *Tomasz Mrowka*
- Wedge theorems for ancient mean curvature flows, *Niels Martin Møller*
- Alternating links, rational balls, and tilings, *Brendan Owens*
- Negatively curved Einstein metrics on quotients of 4-dimensional hyperbolic manifolds, *Bruno Premoselli*
- The Witten Conjecture for homology $S^1 \times S^3$, *Nikolai Saveliev*
- Free boundary minimal surfaces in the unit ball, *Mario Schulz*
- Disoriented homology of surfaces and branched covers of the 4-ball, *Sašo Strle*
- A Brakke type regularity for the Allen-Cahn flow, *Shengwen Wang*
- Algebraic and geometric classification of Yang-Mills-Higgs flow lines, *Graeme Wilkin*

A Local Singularity Analysis for the Ricci Flow

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The Ricci Flow is the most famous and most successful geometric flow, having led to resolutions of the Poincaré and Geometrisation Conjectures, as well as proofs of the Differentiable Sphere Theorem and the Generalised Smale Conjecture. For many of these applications, it is important to understand precisely how singularities form along the flow - which is a notoriously difficult task, in particular in dimensions strictly greater than three. In this talk, we develop a new and refined singularity analysis for the Ricci Flow by investigating curvature blow-up rates locally. We introduce general definitions of Type I and Type II singular points and show that these are indeed the only possible types of singular points in a Ricci Flow. In particular, near any singular point the Riemannian curvature tensor has to blow up at least at a Type I rate, generalising a result previously obtained with Enders and Topping under a global Type I assumption. We also prove analogous results for the Ricci tensor, as well as a localised version of Sesum's result, namely that the Ricci curvature must blow up near every singular point of a Ricci flow, again at least at a Type I rate. If time permits, we will also see some applications of the theory to Ricci flows with bounded scalar curvature.

Spaces of constrained positive scalar curvature metrics

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In this lecture, I will present a collection of results concerning the interplay between the scalar curvature of a Riemannian manifold and the mean curvature of its boundary, with special emphasis on dimension-dependent phenomena. Our work is motivated by a network of far-reaching conjectures by Gromov on the one hand, and by the study of the space of admissible initial data sets for the Einstein field equation in general relativity on the other.

Foliation of Asymptotically Schwarzschild Manifolds by Generalized Willmore Surfaces

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From the perspective of general relativity asymptotically Schwarzschild, or more generally asymptotically flat, manifolds represent isolated systems. Here the idea is that in the absents of classical energy the spacetime should resemble the Minkowski space. The Hawing energy is a quasi local energy of general relativity that strong relation to the Willmore functional. It was introduced by S.W. Hawking in order to measure the gravitational energy of spacetimes that are classically empty.

Starting from the Hawking energy we develop a notion of generalized Willmore functionals. Further, we construct a foliation of the asymptotically flat end of an asymptotically Schwarzschild manifold by large, area constrained spheres which are critical with respect to a

generalized Willmore functional. This achieved via a perturbation argument starting from round centered spheres in Schwarzschild space. These foliations can be interpreted as a center of mass as measured by the generalized Willmore functional.

Time permitting we will discuss the existence and regularity of critical points of generalized Willmore functionals in a more general setting.

The results presented are based on the a corresponding analysis for the Willmore functional by T. Lamm, J. Metzger and F. Schulze, and are part of the authors Ph.D. thesis.

A two-valued Bernstein theorem in dimension four

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The Bernstein theorem is a classical result in geometric analysis, which states that entire minimal graphs are linear in all dimensions up to eight. Here we present a generalisation to so-called two-valued minimal graphs. These are graphs of two-valued functions, which are singular geometric objects that model the behaviour of minimal hypersurfaces near branch points. The two-valued Bernstein theorem we present proves that entire two-valued minimal graphs are linear in dimension four.

Characterizing slopes for Legendrian knots

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From a given Legendrian knot K in the standard contact 3-sphere, we can construct a symplectic 4-manifold W_K by attaching a Weinstein 2-handle along K to the 4-ball. In this talk, we will construct non-equivalent Legendrian knots K and K' such that W_K and W'_K are equivalent. On the other hand, we will discuss an example of a Legendrian knot K that is characterized by its symplectic 4-manifold W_K. This is based on joint work with Roger Casals and John Etnyre.

No previous knowledge of contact geometry is assumed. We will discuss all relevant notions.

Large area-constrained Willmore spheres in initial data sets

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Coauthor: Michael Eichmair

Area-constrained Willmore spheres are surfaces that are particularly well-adapted to the Hawking mass, a local measure of the gravitational field of initial data sets for isolated gravitational systems. In this talk, I will present recent results (joint with M. Eichmair) on the existence and uniqueness of large area-constrained Willmore surfaces in such initial data sets. In particular, I will describe necessary and sufficient conditions on the scalar curvature of the initial data set that guarantee the existence of a unique asymptotic foliation by large area-constrained Willmore spheres. I will also discuss recent results on the geometric center of mass of this foliation.

A Positive Mass Theorem for Fourth-Order Gravity

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Coauthors: Rodrigo Avalos, Jorge Lira

In classical Einstein gravity, the metric is a critical point of the Einstein-Hilbert functional, i.e. $g \mapsto \int_M R_g \, dv$. It has been proved that there exists a conserved quantity along space-like hypersurface, called the ADM mass. In a celebrated work Schoen and Yau proved that if the scalar curvature of the hypersurface is none-negative then the mass is none-negative, with rigidity if the mass vanishes. After remembering some facts about this result, I will introduce a new mass associated to a fourth order functional, namely $g \mapsto \int_M \alpha R_g^2 + \beta |\mathrm{Ric}_g|^2 \, dv_g$. Then I will explain how we obtain an analogue of the positive mass theorem for this new mass by replacing the none-negativity of the scalar curvature by the one of the Q-curvature.

Applications of virtual Morse–Bott theory to the moduli space of $\mathrm{SO}(3)$ Monopoles

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Coauthor: Paul Feehan

We describe some recent joint work with P. Feehan on the Morse theory of a function on the moduli space of $\mathrm{SO}(3)$ monopoles on a smooth four-manifold and sketch some applications to the topology of smooth four-manifolds.

Recent progress in Lagrangian mean curvature flow of surfaces

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Lagrangian mean curvature flow is potentially a powerful tool for tackling several important open problems in symplectic topology. The first non-trivial and important case is the flow of Lagrangian surfaces in 4-manifolds. I will describe some recent progress in understanding the Lagrangian mean curvature flow of surfaces, including general results about ancient solutions and the study of the Thomas-Yau conjecture in explicit settings.

Applications of Floer homology to clasp number vs genus

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A basic question in four dimensional topology is determining the smooth four-ball genus of a knot in the the three sphere. A closely related question is determining the minimal number of double points of a disk smoothly immersed bounding the knot. We'll survey recent advances on this problem using tools coming from gauge theory and related invariant. This circle of ideas is related to interesting questions in algebraic geometry.

Wedge theorems for ancient mean curvature flows

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We show that so-called "wedge theorems" hold for all properly immersed, not necessarily compact, ancient solutions to the mean curvature flow in $\mathbb{R}^{n+1}$. Such nonlinear parabolic Liouville-type results add to a long story, generalizing recent results for self-translating solitons, which in turn imply the minimal surface case (Hoffman-Meeks, '90) that contains the classical cases of cones (Omori '67) and graphs (Nitsche, '65). As an application we classify the convex hulls of the spacetime tracks of all proper ancient flows, without any of the usual curvature assumptions. The proofs make use of a linear parabolic Omori-Yau maximum principle for (non-compact) ancient flows. This is joint work with F. Chini.

Alternating links, rational balls, and tilings

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Coauthor: Josh Greene

If an alternating knot is a slice of a knotted 2-sphere, then it follows from work of Greene and Jabuka, using Donaldson's diagonalisation combined with Heegaard Floer theory, that the flow lattice L of its Tait graph admits an embedding in the integer lattice $\mathbb{Z}^n$ of the same rank which is "cubiquitous": every unit cube in $\mathbb{Z}^n$ contains an element of L. The same is true more generally for an alternating link whose double branched cover bounds a rational homology 4-ball. This results in an upper bound on the determinant of such links. In this talk I will describe recent joint work with Josh Greene in which we classify alternating links for which this upper bound on determinant is realised. This makes use of Minkowski's conjecture, proved by Hajós in 1941, which states that every lattice tiling of $\mathbb{R}^n$ by cubes has a pair of cubes which share a facet.

Negatively curved Einstein metrics on quotients of 4-dimensional hyperbolic manifolds

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Few examples of closed Einstein manifolds with negative scalar curvature are known in dimensions larger than 4, and until recently it was believed that the only negatively curved ones were the trivial ones, ie closed quotients of (complex)-hyperbolic space. We will construct in this talk new examples of non-trivial closed negatively curved Einstein 4-manifolds.

More precisely, we will show that Einstein metrics with negative sectional (and scalar) curvature can be found on quotients of "large" closed hyperbolic 4-manifolds with symmetries. The proof is via a glueing procedure, starting from an approximate Einstein metric that is obtained as the interpolation between a "black-hole – type" model metric near the symmetry locus and the hyperbolic metric at large distances. This is a joint work with J. Fine (ULB, Brussels).

The Witten Conjecture for homology $S^1 \times S^3$

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The Witten conjecture (1994) poses that the Seiberg–Witten invariants contain all of the topological information of the Donaldson polynomials. The natural domain of this conjecture comprises closed simply connected oriented smooth 4-manifolds with $b_+ > 1$, where the Seiberg–Witten invariants are obtained by a straightforward count of irreducible solutions to the Seiberg–Witten equations. The Seiberg-Witten invariants have also been extended to manifolds with $b_+ = 1$ using wall-crossing formulas. In our work with Mrowka and Ruberman (2009) we defined the Seiberg–Witten invariant for a class of manifolds X with $b_+ = 0$ having homology of $S^1 \times S^3$. The usual count of irreducible solutions in this case depends on metric and perturbation but we succeeded in countering this dependence by a correction term to obtain a diffeomorphism invariant of X. In the spirit of the Witten conjecture, we conjectured that the degree zero Donaldson polynomial of X can be expressed in terms of this invariant. I will describe the special cases in which the conjecture has been verified, together with some applications. This is a joint project with Jianfeng Lin and Daniel Ruberman.

Free boundary minimal surfaces in the unit ball

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Coauthors: Alessandro Carlotto, Giada Franz

Free boundary minimal surfaces arise naturally in partitioning problems for convex bodies, in capillarity problems for fluids and in the study of extremal metrics for Steklov eigenvalues on manifolds with boundary. The theory has been developed in various interesting directions, yet many fundamental questions remain open. One of the most basic ones can be phrased as follows: Can a surface of any given topology be realised as an embedded free boundary minimal surface in the 3-dimensional Euclidean unit ball? We will answer this question affirmatively for surfaces with connected boundary and arbitrary genus.

Disoriented homology of surfaces and branched covers of the 4-ball

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Coauthor: Brendan Owens

An often used construction in low-dimensional topology is to associate to a properly embedded surface $F \subset B^4$ the branched double cover X of B^4 with branch set F. If the surface is obtained by pushing the interior of an embedded surface in S^3 into the interior of B^4, a classical result of Gordon and Litherland states that $H_2(X;\mathbb{Z})$ is isomorphic to $H_1(F;\mathbb{Z})$ and that the intersection pairing of X may be described in terms of a pairing on $H_1(F;\mathbb{Z})$ which is determined by the embedded surface in S^3.

We generalize this result to any surface F by defining a non-standard homology theory $DH_*(F)$ that depends on a description of F in S^3. This homology captures the homological information of X and may be equipped with a pairing on $DH_1(F)$ that corresponds to the intersection pairing of X. This construction also works for closed surfaces.

A Brakke type regularity for the Allen-Cahn flow

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Coauthor: Huy Nguyen

We will talk about an analogue of the Brakke's local regularity theorem for the ϵ parabolic Allen-Cahn equation. In particular, we show uniform $C_{2,\alpha}$ regularity for the transition layers converging to smooth mean curvature flows as ϵ tend to 0 under the almost unit-density assumption. This can be viewed as a diffused version of the Brakke regularity for the limit mean curvature flow. This talk is based on joint work with Huy Nguyen.

Algebraic and geometric classification of Yang-Mills-Higgs flow lines

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The Yang-Mills-Higgs flow on a compact Riemann surface is modelled on a nonlinear heat equation, and therefore existence of the reverse flow is problematic in general. In this talk I will explain how the existence of a certain filtration (analogous to the Harder-Narasimhan filtration, but with the opposite inequality on the slopes) means that one can gauge the initial condition so that the reverse flow exists for all time. This leads to an algebraic classification of flow lines between pairs of critical points. A refinement of these ideas leads to a geometric classification of flow lines in terms of certain secant varieties, which can then be used to distinguish between broken and unbroken flow lines.

Minisymposium

Geometries Defined by Differential Forms (MS-44)

Organized by Sergey Grigorian, *University of Texas Rio Grande Valley, United States*
Coorganized by
Mahir Bilen Can, *Tulane University, United States*
Sema Salur, *University of Rochester, United States*

- On Fiber Bundle Property of a Schubert Variety, *Mahir Bilen Can*
- Closed G_2-structures on compact locally homogeneous spaces, *Anna Fino*
- Smooth loops, *Sergey Grigorian*
- G_2- and $Spin(7)$-structures by means of vector cross products, *Hông Vân Lê*
- Special Lagrangians and Bridgeland Stability Conditions, *Goncalo Oliveira*
- Complex volume forms, totally real submanifolds and convexity, *Tommaso Pacini*
- The heterotic G_2 system on contact Calabi–Yau 7-manifolds, *Henrique Sá Earp*
- The Obata first eigenvalue theorem on a seven dimensional quaternionic contact manifold, *Dimiter Vassilev*

On Fiber Bundle Property of a Schubert Variety

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Tulane University, United States

The Schubert varieties are essential geometric objects for studying the projective homogenous spaces. Many Schubert varieties admit Zariski-locally trivial fiber bundle structures over certain Schubert subvarieties. In this talk we will discuss various geometric and representation theoretic consequences of the existence of a fiber bundle structure on a Schubert variety. In particular, we will present our recent results on the spherical Schubert varieties.

Closed G_2-structures on compact locally homogeneous spaces

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Università di Torino, Italy

G_2-structures defined by a closed positive 3-form constitute the starting point in various known and potential methods to obtain holonomy G_2-metrics on 7-manifolds. Although linear, the closed condition for a G_2-structure is very restrictive, and no general results on the existence of closed G_2-structures on compact 7-manifolds are known. In the seminar I will present some results for compact locally homogeneous spaces admitting closed G_2-structures.

Smooth loops

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A loop is a rather general algebraic structure that has an identity element and division, but is not necessarily associative, and thus generalizes the notion of a group. A smooth loop is a manifold that is also a loop with smooth multiplication and division operations, and is hence a direct generalization of a Lie group. A key example of a non-associative smooth loop is the 7-dimensional sphere regarded as the loop of octonions of unit length. Given a smooth loop, the tangent space at identity then inherits an algebra structure that generalizes a Lie algebra structure. In this talk we will first overview the key properties of loops and their "pseudoautomorphisms" in general, and will then specialize to smooth loops, their associated tangent algebras, and will show the key differences and similarities with Lie theory.

G_2- and $Spin(7)$-structures by means of vector cross products

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Institute of Mathematics, Academy of Sciences of Czech Republic, Czech Republic

The notion of a multilinear vector cross product (VCP) has been introduced by Gray as a natural generalization of the notion of an almost complex structure. Gray also associated G_2-structures on 7-manifolds and $Spin(7)$-structures on 8-manifolds with VCPs on the underlying manifolds. In my talk I shall present recent results on G_2-structures and $Spin(7)$-structures using VCPs:

1) the correspondence between parallel VCPs on a Riemannian manifold M and parallel almost complex structures on a higher dimensional knot space over M endowed with a L^2-metric, which generalizes Brylinski's, LeBrun's, Henrich's and Verbitsky's results for the case that S is a codimension 2 submanifold in M, and $S = S^1$ or M is a torsion-free G_2-manifold, respectively; 2) the similarities between integrable complex structures on one hand and torsion-free G_2-and Spin(7)-structures on the other hand; 3) the construction of CR-twistor spaces over G_2-manifolds, due to Verbitsky, and its extension to $Spin(7)$-manifolds. My lecture is based on my joint works with Domenico Fiorenza, Kotaro Kawai, Lorenz Schwachhöfer and Luca Vitagliano.

Special Lagrangians and Bridgeland Stability Conditions

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Coauthor: Jason Lotay

Building on conjectures of Richard Thomas and Shing-Tung-Yau, together with the definition of Bridgeland stability conditions, Dominic Joyce proposed to use Lagrangian mean curvature flow to "decompose" certain Lagrangian submanifolds into a combination of special Lagrangians. I will report on joint work in progress with Jason Lotay to prove parts of Joyce's program for certain symmetric hyperKahler 4-manifolds. Our work provides concrete geometric interpretations for many algebraic notions related to Bridgeland stability conditions.

Complex volume forms, totally real submanifolds and convexity

Tommaso Pacini, tommaso.pacini@unito.it
University of Torino, Italy

Totally real submanifolds play a natural role at the intersection between symplectic and complex geometry. I will survey recent results in this direction, emphasizing relationships with the (anti-)canonical bundle and with pluri-subharmonic functions.

The heterotic G_2 system on contact Calabi–Yau 7-manifolds

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Unicamp, Brazil

We obtain non-trivial solutions to the heterotic G_2 system, which are defined on the total spaces of non-trivial circle bundles over Calabi–Yau 3-orbifolds. By adjusting the S^1 fibres in proportion to a power of the string constant α', we obtain a cocalibrated G_2-structure the torsion of which realises an arbitrary constant (trivial) dilaton field and an H-flux with nontrivial Chern–Simons defect. We find examples of connections on the tangent bundle and a non-flat G_2-instanton induced from the horizontal Calabi–Yau metric which satisfy together the anomaly-free condition, also known as the heterotic Bianchi identity. The connections on the tangent bundle are G_2-instantons up to higher order corrections in α'.

The Obata first eigenvalue theorem on a seven dimensional quaternionic contact manifold

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University of New Mexico, United States

Coauthor: Abdelrahman Mohamed

We give an Obata type rigidity result for the first eigenvalue of the sub-Laplacian. on a compact seven dimensional quaternionic contact manifold which satisfies a Lichnerowicz-type bound on its quaternionic contact Ricci curvature and has a non-negative Paneitz P-function. In particular, under the stated conditions, the lowest possible eigenvalue of the sub-Laplacian is achieved if and only if the manifold is qc-equivalent to the standard 3-Sasakian sphere.

Minisymposia in

Dynamical Systems and Ordinary Differential Equations and Applications

- Rational approximation for data-driven modeling and complexity reduction of linear and nonlinear dynamical systems (MS-69)
- Spectral Theory and Integrable Systems (MS-57)
- Topological methods in dynamical systems (MS-65)

Minisymposium

RATIONAL APPROXIMATION FOR DATA-DRIVEN MODELING AND COMPLEXITY REDUCTION OF LINEAR AND NONLINEAR DYNAMICAL SYSTEMS (MS-69)

Organized by Sanda Lefteriu, *IMT Lille Douai, France*
Coorganized by Ion Victor Gosea, *Max Planck Institute, Magdeburg, Germany*

- The Loewner Framework: An overview and recent results, *Athanasios C. Antoulas*
- Model order reduction approach for problems with moving discontinuous features, *Harshit Bansal*
- Inf-Sup-Constant-Free State Error Estimator for Model Order Reduction of Parametric Systems in Electromagnetics, *Sridhar Chellappa*
- On improving the data collection step for data driven modeling methods, *Karim Cherifi*
- Physics-based reduced basis methods for CAD in time-harmonic Maxwell's equations, *Valentin de la Rubia*
- Numerical aspects of the Koopman and the dynamic mode decomposition for model reduction, *Zlatko Drmač*
- Barycentric Hermite interpolation and its application to data-driven model reduction, *Ion Victor Gosea*
- Algorithms for identification and reduction of nonlinear dynamical systems from time-domain data, *Dimitrios S. Karachalios*
- Rational interpolation and model order reduction for data-driven controller design, *Pauline Kergus*
- Comparison of greedy-type approaches involving the Loewner matrix for rational modeling, *Sanda Lefteriu*
- Input-tailored moment matching – a system-theoretic model reduction method for nonlinear systems, *Björn Liljegren Sailer*
- The use of rational approximation for linearization of models that are nonlinear in the frequency, *Karl Meerbergen*
- Mixed interpolatory and inference for non-intrusive reduced order nonlinear modelling, *Charles Poussot-Vassal*
- Dynamic neural networks and model order reduction for the simulation of electronic circuits, *Wil Schilders*
- Structured Realization Based on Time-Domain Data, *Philipp Schulze*
- Structure Preserving Model Order Reduction by Parameter Optimization, *Paul Schwerdtner*
- Spurious poles, *Nick Trefethen*
- Interpolatory Model Reduction in $\mathcal{H}_\infty$-Controller Design, *Matthias Voigt*
- Structure-Preserving Interpolation for Bilinear Systems, *Steffen W. R. Werner*

The Loewner Framework: An overview and recent results

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Rice University, United States

In this talk we will present an overview of the Loewner framework for the reduction of dynamical systems both linear and nonlinear. Both time-domain and frequency-domain methods will be discussed, followed by sevarl numerical examples.

Model order reduction approach for problems with moving discontinuous features

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Coauthors: Stephan Rave, Laura Iapichino, Wil Schilders, Nathan van de Wouw

The motivation of this work is to enable the usage of multi-phase hydraulic models, such as the Drift Flux Model (DFM) [1], in developing automation strategies for real-time down-hole pressure management in drilling systems. The DFM is a system of multi-scale non-linear hyperbolic Partial Differential Equations (PDEs) and its response is dominated by wave propagation characteristics. The central aim of this work is to accurately capture wave-front propagation (and wave interaction) phenomena (induced by slow or fast transients) in a reduced-order modeling framework.

Moving discontinuities (shock-fronts) are representative features of the models governed by hyperbolic PDEs. Such features pose a major hindrance to obtain effective reduced-order model representations [2]. This motivates us to investigate and propose efficient, advanced, and automated approaches to obtain reduced models, while still guaranteeing the accurate approximation of wave propagation phenomena.

We propose a new model order reduction (MOR) approach to obtain an effective reduction for transport-dominated problems or hyperbolic PDEs. The main ingredient is a novel decomposition of the solution into a function that tracks the evolving discontinuity and a residual part that is devoid of shock features. This decomposition ansatz is then combined with Proper Orthogonal Decomposition applied to the residual part only to develop an efficient reduced-order model representation for problems with multiple moving and possibly merging discontinuous features. Numerical case studies show the potential of the approach in terms of computational accuracy compared with standard MOR techniques.

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Inf-Sup-Constant-Free State Error Estimator for Model Order Reduction of Parametric Systems in Electromagnetics

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Coauthors: Lihong Feng, Valentin de la Rubia, Peter Benner

In this talk, we discuss efficient, sharp *a posteriori* state error estimation for reduced-order models of general linear parametric systems. Standard *a posteriori* state error estimation for model order reduction relies on the inf-sup constant. The *a posteriori* error estimation for systems with very small or vanishing inf-sup constant poses a challenge, since it is inversely proportional to the inf-sup constant, resulting in overly pessimistic error estimators. Such systems appear in electromagnetics since the inf-sup constant values are zero or close to zero, at or near resonant frequencies. We propose a novel *a posteriori* state error estimator which avoids the calculation of the inf-sup constant. The proposed state error estimator is compared with the standard error estimator and a recently proposed one in the literature. It is shown that our proposed error estimator outperforms both existing estimators. Furthermore, our new estimator is integrated within an adaptive greedy algorithm that iteratively builds the reduced-order model. Numerical experiments are performed on real-life microwave devices such as narrowband and wideband antennas, as well as a dual-mode waveguide filter. These examples show the capabilities and efficiency of the proposed methodology.

References

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On improving the data collection step for data driven modeling methods

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Data-driven modeling methods have gained in popularity in recent years. However, to obtain good reliable models, one generally needs to process a huge amount of data. In this work, we present a heuristic algorithm to improve the data collection step such that the data used is only the data that holds the most information about the system that we are trying to model. This algorithm is applied on the Loewner framework where interpolation points are chosen adaptively in order to obtain a reliable low order realization of the system at hand. It is designed for frequency domain data then extended to time domain data. The operation of the algorithm is illustrated by applications and well known benchmarks.

Physics-based reduced basis methods for CAD in time-harmonic Maxwell's equations

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Coauthor: Alvaro Martin-Cortinas

Electromagnetics today underpins all modern information and communication technologies. Increasing deployment of telecommunication services is urging RF industry to carry out better and better electrical designs, where a single device is no longer conceived to perform a single functionality, but rather multiple tasks at the same time.

Unfortunately, much electrical design activity is still based on brute-force computational simulations to predict the actual physical behavior of electromagnetic devices. These time-consuming simulations are repeated, changing the device characteristics until satisfying stricter specifications of emerging information and communications technologies (5G and IoT).

In our work we use computational electromagnetics as an actual design tool, rather than just an analysis one. This can be achieved by shriking computational electromagnetics into a simple parameterized equivalent circuit form, from which an electrical engineer can get actionable design insights.

Our efforts stand upon model order reduction techniques, such as the reduced-basis method [1, 2, 3]. However, a new physics-based strategy is carried out to provide further physical insight of the electromagnetic device under analysis and, ultimately, extremely valuable design information.

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Numerical aspects of the Koopman and the dynamic mode decomposition for model reduction

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The Dynamic Mode Decomposition (DMD) is a tool of trade in computational data driven analysis of complex dynamical systems, e.g. fluid flows, where it can be used to decompose the flow field into component fluid structures, called DMD modes, that describe the evolution of the flow. The DMD is deeply connected with the analysis of nonlinear dynamical systems in terms of the spectral data of the associated Koopman operator. The main tasks of the analysis are approximations of eigenfunctions and eigenvalues of the operator, and computation of a spatio-temporal representation of the data snapshots, based on the computed eigenvalues and eigenfunctions. The latter includes possibly ill-conditioned least squares problem.

Exceptional performances motivated developments of several modifications that make the DMD an attractive method for identification, analysis and model reduction of nonlinear systems in data driven settings.

In this talk, we will present our recent results on the numerical aspects of the DMD/Koopman analysis. We show how the state of the art numerical linear algebra can be deployed to improve the numerical performances in cases that are usually considered notoriously ill-conditioned. The numerical framework is based on algorithms that also apply e.g. to matrix rational approximations in modeling by Vector Fitting.

Barycentric Hermite interpolation and its application to data-driven model reduction

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The barycentric representation of rational interpolants offers some advantages over other classical rational formulations, one of which being the numerical stability of the barycentric formula. Here, we concentrate on the Hermite interpolation problem, for which not only measurements of the underlying function are available, but also of the function's derivatives. We revisit two model reduction algorithms based on rational approximation, i.e., the Loewner framework (Mayo/Antoulas '07) and IRKA/TF-IRKA (Gugercin/Antoulas /Beattie '08, Beattie/Gugercin '12); we show how Hermitian interpolation is connected to these methods. Moreover, we present an extension of the recent AAA algorithm (Nakatsukasa/Sete/Trefethen '18) that is adapted to satisfy Hermitian conditions (interpolating not only functions values, as the original AAA, but also values of derivatives). The new variant also uses least-squares fitting on the data set to construct a rational interpolant in barycentric representation.

Algorithms for identification and reduction of nonlinear dynamical systems from time-domain data

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We propose a comparison of algorithms that use time-domain data to fit models of dynamical systems that explain the available measurements. The eigensystem realization algorithm (ERA) and the Loewner framework (LF) constitute data-driven methods for identifying and reducing linear and nonlinear dynamical systems. The ERA uses the system's invariants known as Markov parameters into a matrix of Hankel structure. Similarly, the LF encodes the invariants of a system into the Loewner matrix structure through frequency-domain measurements. These can be estimated from time-domain data by employing spectral transforms. For both methods under consideration, the singular value decomposition (SVD) provides a trade-off between the complexity and the accuracy of the reduced model. These two approaches are compared first for the case of fitting linear systems and then, for fitting systems with nonlinear dynamics, such as bilinear systems. We discuss recent extensions for the latter case.

Rational interpolation and model order reduction for data-driven controller design

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In many control applications, a mathematical description of the system, derived from physical laws, is not available. In this case, the controller has to be designed on the basis of experimental measurements. This work presents a data-driven control strategy based on the Loewner framework, where a reduced-order controller is directly obtained from the available experimental data. Rational interpolation is also used to build achievable specifications and to ensure closed-loop stability for the controlled system. No parametric model of the system is used, allowing to handle applications in which the model of the system might be too complicated or too difficult to obtain for traditional model-based strategies. This technique is particularly appealing to control infinite dimensional systems, such as the ones described by linear partial differential equations. Such an example, a crystallizer (common in the chemical industry), is tackled in this work.

Comparison of greedy-type approaches involving the Loewner matrix for rational modeling

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The Loewner framework has established itself as a popular choice for building rational approximations in barycentric form. The Loewner together with the shifted Loewner matrices are built from measurements and, together with the data matrices, yield a high-order rational model for a potentially non-rational function. To eliminate the redundancy in the data, an SVD step of the Loewner matrix pencil is involved. However, as the size of these matrices is equal to half of the number of measurements, this step is rather costly for large data sets when the traditional SVD is employed.

This talk compares several greedy-type approaches which use the same principle: starting from an order 1 approximant, points from the available data set are added in a greedy fashion by minimizing the error measure of choice. These approaches considered in the comparison are: AAA [1], the CUR decomposition [2], DEIM-CUR decomposition [3], as well as the adaptive and recursive approaches [4].

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Input-tailored moment matching – a system-theoretic model reduction method for nonlinear systems

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Coauthor: Nicole Marheineke

We suggest a new moment matching method for quadratic-bilinear dynamical systems. Most system-theoretic model order reduction methods for nonlinear systems rely on multivariate frequency representations derived from the Volterra series expansion of the solution. Similarly, our approach relies on variational expansions, but we consider instead univariate frequency representations tailored towards user-pre-defined families of inputs. Then moment matching corresponds to a one-dimensional interpolation problem, not to multi-dimensional interpolation as for the multivariate approaches, i.e., it also involves fewer interpolation frequencies to be chosen. The resulting moment matching problems are approached exploiting the inherent low-rank tensor structure.

In addition, our approach allows for the incorporation of more general input relations in the state equations – not only affine-linear ones as in existing system-theoretic methods – in an elegant way.

The use of rational approximation for linearization of models that are nonlinear in the frequency

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Coauthors: Elke Deckers, Stijn Jonckheere

Finite element models for the analysis of vibrations typically have a quadratic dependency on the frequency. This makes the finite element method suitable for eigenvalue computations and time integration, by a formulation as a first order system, which we call a linearization.

The study of new damping materials often leads to nonlinear frequency dependencies, sometimes represented by a rational functions but, often, by truly nonlinear functions. In classical analyses, vibrations are studied in the frequency domain. The nonlinear frequency dependency is an issue for algorithms for fast frequency sweeping. In the context of numerical algorithms for digital twins, time integration of mathematical models is required, which is not straightforward for models that are not linear or polynomial in the frequency.

We will discuss rational approximation and linearization of nonlinear frequency dependencies and their use for fast frequency sweeping and time integration. In particular, we use the AAA rational approximation and the associated linearization based on the barycentric Lagrange formulation of rational functions, which is successfully used for solving nonlinear eigenvalue problems. We show how real valued matrices can be obtained. We also show how the parameters can be tuned to obtain a stable linear model.

Mixed interpolatory and inference for non-intrusive reduced order nonlinear modelling

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Based on input-output time-domain raw data collected from a complex simulator, the Mixed Interpolatory Inference (**MII**) process approach allows to infer a reduced-order linear or nonlinear (e.g. bilinear or quadratic) time invariant dynamical model of the form

$$\begin{cases} \dot{\hat{\mathbf{x}}} &= \hat{A}\hat{\mathbf{x}} + \hat{B}\mathbf{u} + \hat{\mathbf{f}}(\hat{\mathbf{x}}, \mathbf{u}) \\ \hat{\mathbf{y}} &= \hat{C}\hat{\mathbf{x}} + \hat{D}\mathbf{u} + \hat{\mathbf{g}}(\hat{\mathbf{x}}, \mathbf{u}) \end{cases}, \tag{1}$$

that accurately reproduces the underlying phenomena dictated by the raw data. In (1), $\hat{\mathbf{x}}(\cdot) \in \mathbb{R}^r$, $\mathbf{u}(\cdot) \in \mathbb{R}^{n_u}$ and $\hat{\mathbf{y}}(\cdot) \in \mathbb{R}^{n_y}$, denote the reduced internal, input and approximated output variables respectively. Moreover, $\hat{\mathbf{f}}$ and $\hat{\mathbf{g}}$ denote either quadratic or bilinear functions.

The approach is essentially based on the sequential combination of **rational interpolation** (e.g. Pencil, Loewner, AAA) with a **linear least square** resolution.

With respect to intrusive methods, no prior knowledge on the operator is needed. In addition, compared to the traditional non-intrusive operator inference approaches, the proposed rationale alleviates the need of measuring and storing the original full-order model internal variables. It is thus applicable to a wider range of applications than the standard intrusive and non-intrusive methods. It is therefore very close to the identification field.

The **MII** is successfully applied on different numerically challenging application related to pollutant dispersion. First *(i)* a large eddy simulation of a pollutants dispersion case over an airport area, and second *(ii)* a flow simulation over a building, both involving multi-scale and multi-physics dynamical phenomena.

Despite the simplicity of the resulting low complexity model, the proposed approach shows satisfactory results to predict the pollutants plume pattern while being significantly faster to simulate.

Dynamic neural networks and model order reduction for the simulation of electronic circuits

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In recent years, there is a strong drive towards the use of artificial neural networks combined with more traditional simulation techniques based upon physical modelling. Karniadakis and his team at Brown University are frontrunners in this field, using so-called Physics Informed Neural Networks (PINNs). At Philips Research, we worked on dynamic neural networks for the simulation of electronic circuits. We were able to establish a 1-1 connection between the specific networks used, with special neuron activation functions, and state space models. On the one hand, this connection enabled us to predict the topology of the artificial neural network. For example, the number of hidden layers turned out to depend on the multiplicity of eigenvalues of the matrix A in the state space model. On the other hand, this connection could potentially also lead us to a theory of model order reduction for neural networks. In the presentation, we will review the artificial networks, establish the 1-1 connection and discuss the resulting implications.

Structured Realization Based on Time-Domain Data

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In this talk we present a method for constructing a continuous-time linear time-invariant system based on discrete samples of an input/output trajectory of the system. Especially, our approach allows to impose different structures on the constructed system including structures like second-order systems, systems with time-delay in the state, and fractional systems. The proposed method consists of first using the measured time-domain data to estimate the transfer function at selected points in the frequency domain by using a modified version of the empirical transfer function estimation presented in [Peherstorfer, Gugercin, Willcox, SIAM J. Sci. Comput., 39(5):2152–2178, 2017]. Afterward, we construct a structured realization based on the estimated frequency data such that the transfer function of the obtained realization interpolates the frequency data. The effectiveness of this new approach is illustrated by means of a numerical example.

Structure Preserving Model Order Reduction by Parameter Optimization

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We present a framework for structure-preserving model order reduction (MOR) based on direct parameter optimization as introduced in [1]. We explain how our method can be applied to compute reduced order models for linear port-Hamiltonian (pH) systems. For that, we first describe how we fully parametrize pH systems such that all structural constraints regarding the system matrices are automatically satisfied. After that, we give insights into the specific optimization problem we set up to find a good approximation with respect to the H-infinity norm. Finally, we highlight the effectiveness of our method by comparing it to other structure preserving MOR algorithms [2, 3] on a pH benchmark system from [2].

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Spurious poles

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Spurious poles of rational approximations are fascinating in theory and troublesome in practice. This talk reviews the mathematical and computational sides of this subject and explores the use of the two-step method of AAA-least squares approximation for avoiding the problem.

Interpolatory Model Reduction in $\mathcal{H}_\infty$-Controller Design

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In engineering problems, it is often desired to attenuate the influence of external noise and to deal with uncertainties in a dynamical system. Classically, this amounts to the design of so-called $\mathcal{H}_\infty$-controllers which results in very difficult nonconvex and nonsmooth optimization problems. The corresponding numerical methods require multiple evaluations of the $\mathcal{H}_\infty$-norm and its gradient with respect to the controller variables. In this talk we address new efficient methods for the computation of the $\mathcal{H}_\infty$-norm of large-scale dynamical systems with possibly irrational transfer functions. We discuss a subspace projection approach for solving this problem using interpolatory techniques that are well-known in model reduction. More precisely, after performing the reduction, we compute the $\mathcal{H}_\infty$-norm of the reduced transfer function and choose the point at which the $\mathcal{H}_\infty$-norm is attained as a new interpolation point to update the projection matrices. We will discuss convergence properties of this procedure and illustrate it by various examples. One focus of this talk will be on delay systems which are reduced by employing the Loewner framework, This is useful in order to get a sequence of linear reduced models whose $\mathcal{H}_\infty$-norm can be evaluated more efficiently. Finally, we show how to apply these techniques in the context of controller design.

Structure-Preserving Interpolation for Bilinear Systems

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The modeling of natural processes as population growth, mechanical structures and fluid dynamics, or stochastic modeling often results in bilinear time-invariant dynamical systems

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + \sum_{j=1}^{k} N_j x(t) u_j(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

with $E, A, N_j \in \mathbb{R}^{n\times n}$, for $j = 1, \ldots, m$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$. The aim of model reduction for (1) is the reduction of related computational resources, like time and memory for the simulation of (1), by the reduction of internal states n, while approximating the input-to-output behavior of the system. Often related to the underlying applications, bilinear systems (1) can have special structures that one wants to preserve in the reduced-order model as, e.g., in case of bilinear mechanical systems

$$\begin{aligned} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) &= \sum_{j=1}^{m} N_{\mathrm{p},j} q(t) u_j(t) + \sum_{j=1}^{m} N_{\mathrm{v},j} \dot{q}(t) u_j(t) + B_{\mathrm{u}} u(t), \\ y(t) &= C_{\mathrm{p}} q(t) + C_{\mathrm{v}} \dot{q}(t), \end{aligned} \tag{2}$$

with $M, D, K, N_{\mathrm{p},j}, N_{\mathrm{v},j} \in \mathbb{R}^{n\times n}$, for $j = 1, \ldots, m$, $B_{\mathrm{u}} \in \mathbb{R}^{n\times m}$ and $C_{\mathrm{p}}, C_{\mathrm{v}} \in \mathrm{R}^{p\times n}$.

In case of linear systems, structured-preserving interpolation of the underlying transfer function in the frequency domain can be used to efficiently construct reduced-order models with the same structure as the original system [1].

We present an extension of the structure-preserving interpolation framework to the bilinear system case, which we describe in the frequency domain by general multivariate transfer functions

$$\begin{aligned} G_k(s_1, \ldots, s_k) = \mathcal{C}(s_k)\mathcal{K}(s_k)^{-1} &\left(\prod_{j=1}^{k-1} (I_{m^{j-1}} \otimes \mathcal{N}(s_{k-j}))(I_{m^j} \otimes \mathcal{K}(s_{k-j})^{-1}) \right) \\ &\times (I_{m^{k-1}} \otimes \mathcal{B}(s_1)), \end{aligned} \tag{3}$$

for $k \geq 1$ and where $\mathcal{N}(s) = \begin{bmatrix} \mathcal{N}_1(s) & \ldots & \mathcal{N}_m(s) \end{bmatrix}$ with the matrix functions $\mathcal{C} : \mathbb{C} \to \mathbb{C}^{p\times n}$, $\mathcal{K} : \mathbb{C} \to \mathbb{C}^{n\times n}$, $\mathcal{B} : \mathbb{C} \to \mathbb{C}^{n\times m}$, $\mathcal{N}_j : \mathbb{C} \to \mathbb{C}^{n\times n}$ for $j = 1, \ldots, m$. We develop a projection-based, structure-preserving interpolation framework for bilinear systems associated with (3) that allows the efficient construction of reduced-order structured bilinear systems.

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Minisymposium

Spectral Theory and Integrable Systems (MS-57)

Organized by Gerald Teschl, *University of Vienna, Austria*
Coorganized by Aleksey Kostenko, *University of Ljubljana, Slovenia*

- An almost periodic model for general reflectionless spectral data, *Roman Bessonov*
- Beyond the Strong Szegö Limit Theorem, *Maurice Duits*
- Control of eigenfunctions on negatively curved surfaces, *Semyon Dyatlov*
- Stahl–Totik regularity for continuum Schrödinger operators, *Benjamin Eichinger*
- On a weighted inequality for fractional integrals, *Giorgi Imerlishvili*
- On the Lax operator of the Benjamin-Ono equation and Tao's gauge transform, *Thomas Kappeler*
- Eigenfunctions on random hyperbolic surfaces of large genus, *Etienne Le Masson*
- A scalar Riemann–Hilbert problem on the torus, *Mateusz Piorkowski*
- Computing Eigenvalues of the Laplacian on Rough Domains, *Frank Rösler*
- A note on the multiple fractional integrals defined on the product of quasi-metric measure spaces, *Tsira Tsanava*

An almost periodic model for general reflectionless spectral data

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A basic feature of second order almost periodic differential and finite-difference operators is the reflectionless property of their Weyl functions. Conversely, each regular enough pair of reflectionless Nevanlinna functions generate an almost periodic operator on the real line whose half-line Weyl functions coincide with the given pair. Until recently, for operators with unbounded spectra, this scheme worked under quite restrictive assumptions on the "quality" of the spectrum. We extend it to cover all homogeneous spectra, and, more generally, to all (possibly, unbounded) spectra satisfying Widom and DCT conditions. Joint work with M. Lukic and P. Yuditskii.

Beyond the Strong Szegö Limit Theorem

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In random matrix theory it is well-established that the Strong Szegö Limit Theorem for Toeplitz determinants implies a CLT for linear statistics for eigenvalues of a CUE matrix. The purpose of this talk will be to discuss an extension of the Strong Szegö Limit Theorem to determinants of truncated exponentials of banded matrices such as Jacobi and CMV matrices. This extension shows that the second term in the Strong Szegö Limit Theorem is universal, providing a general CLT for more general classes of determinantal point processes including orthogonal polynomial ensembles on the real line and unit circle. A time-dependent analogue can be used tot establish Gaussian Free Field fluctuations in certain non-colliding process and random tilings of planar domains. The talk aims to present an overview of various results based on this idea.

Control of eigenfunctions on negatively curved surfaces

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Coauthors: Long Jin, Stéphane Nonnenmacher

Given an L^2-normalized eigenfunction with eigenvalue λ^2 on a compact Riemannian manifold (M, g) and a nonempty open set subset Ω of M, what lower bound can we prove on the L^2-mass of the eigenfunction on Ω? The unique continuation principle gives a bound for any Ω which is exponentially small as λ goes to infinity. On the other hand, microlocal analysis gives a λ-independent lower bound if Ω is large enough, i.e. it satisfies the geometric control condition.

This talk presents a λ-independent lower bound for any set Ω in the case when M is a negatively curved surface, or more generally a surface with Anosov geodesic flow. The proof uses microlocal analysis, the chaotic behavior of the geodesic flow, and a new ingredient from harmonic analysis called the Fractal Uncertainty Principle. Applications include control for Schrodinger equation and exponential decay of damped waves. Joint work with Jean Bourgain, Long Jin, and Stéphane Nonnenmacher.

Stahl–Totik regularity for continuum Schrödinger operators

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We develop a theory of Stahl–Totik regularity for half-line Schrödinger operators $-\partial_x^2 + V$ with bounded potentials (in a local L^1 sense). We prove a universal thickness result for the essential spectrum, E, in the language of potential theory. Namely, E is an Akhiezer-Levin set and the Martin function of the complementary domain at ∞ obeys an asymptotic expansion $\sqrt{-z} + \frac{a_E}{\sqrt{-z}} + o(\frac{1}{\sqrt{-z}})$ as $z \to -\infty$. The constant a_E plays the role of a Robin constant suited for Schödinger operators. Stahl-Totik regularity is characterized in terms of the behavior of the averages $\frac{1}{x}\int_0^x V(t)dt$ and root asymptotics of the Dirichlet solutions as $x \to \infty$. Moreover, it is connected to the zero counting measure for finite truncations. Applications to decaying and ergodic potentials will be discussed.

On a weighted inequality for fractional integrals

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Coauthor: Alexander Meskhi

The weighted inequality

$$\|I_\alpha f\|_{L^q_V} \le C\|f\|_{L^p} \tag{1}$$

for the Riesz potential I_α, $0 < \alpha < n$, plays an important role in the theory of PDEs. It is worth mentioning its applications to the theory of Sobolev embeddings (see, e.g., [3]), its connection with eigenvalue estimates for the Schrödinger operator $H = -\Delta - V$ with a potential V (see, e.g., [1], PP. 91-94), etc. In 1972 D. Adams proved that the above mentioned weighted inequality holds for $1 < p < q < \infty$ if and only if there is a positive constant C such that for all balls B in $\mathbb{R}^n$,

$$V(B) \le C|B|^{(1/p-\alpha/n)q}.$$

In the diagonal case, i.e., when $p = q$, necessity of this condition remains valid, however, it is not sufficient for (1) (see, e.g., [2]). We proved that the condition

$$V(B) \le C|B|^{1-p\alpha/n}$$

is simultaneously necessary and sufficient for the boundedness of I_α from the Lorentz space $L^{p,1}$ to the weighted Lebesgue space L^p_V. Some other related results are also derived.

Acknowledgment. The work was supported by the Shota Rustaveli National Foundation grant of Georgia (Project No. DI-18-118).

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On the Lax operator of the Benjamin-Ono equation and Tao's gauge transform

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Coauthors: Patrick Gérard, Petar Topalov

I will report on recent results on the spectral theory of the Lax operator of the Benjamin-Ono equation, discuss applications to the analysis of this equation, and explain the role of Tao's gauge transform in all this.

Eigenfunctions on random hyperbolic surfaces of large genus

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High frequency eigenfunctions in chaotic systems such as hyperbolic surfaces are known to exhibit some universal behaviour of delocalisation and randomness. We will introduce to some results on the behaviour of eigenfunctions on random compact hyperbolic surfaces, in the limit where the genus (or equivalently the volume) tends to infinity, and the frequency is in a fixed window. These results suggest that in the large scale limit we can expect a similar universal behaviour. We will focus on the Weil-Petersson model of random surfaces introduced by Mirzakhani. One advantage of this point of view is the analogy with eigenvectors of random regular graphs, about which there has been very strong developments in the recent years. Based on joint works with Tuomas Sahlsten and Joe Thomas.

A scalar Riemann–Hilbert problem on the torus

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In this talk we will present a case study of a scalar Riemann–Hilbert problem on the torus. This work has been motivated by the analysis of the KdV equation with steplike initial data via the nonlinear steepest descent method. It turns out that the model problem for the transition region can be naturally formulated as a scalar Riemann–Hilbert problem on the torus. This approach does not only lead to the explicit Riemann–Hilbert solutions given in terms of Jacobi theta functions, but also illuminates the uniqueness and ill-posedness issues raised in an earlier paper.

Computing Eigenvalues of the Laplacian on Rough Domains

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We discuss a recent work in which we prove a general Mosco convergence theorem for bounded Euclidean domains satisfying a set of mild geometric hypotheses. For bounded domains, this notion implies norm-resolvent convergence for the Dirichlet Laplacian which in turn ensures spectral convergence. A key element of the proof is the development of a novel, explicit Poincaré-type inequality, which is of independent interest.

These results are applied construct a universal algorithm capable of computing the eigenvalues of the Dirichlet Laplacian on a wide class of rough domains. This immediately leads to new classifications in the so-called "Solvability Complexity Index Hierarchy" recently inroduced by Hansen et al.

A note on the multiple fractional integrals defined on the product of quasi-metric measure spaces

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A complete characterization of a vector-measure $\overrightarrow{\mu} = (\mu_1, \cdots, \mu_n)$ governing the boundedness of the multiple fractional integral operator

$$I^{\overrightarrow{\gamma}} f(x_1, \ldots, x_n) = \int\limits_{X_1} \cdots \int\limits_{X_n} \frac{f(y_1, \ldots, y_n) d\mu_1(y_1) \cdots d\mu_n(y_n)}{\prod\limits_{j=1}^{n} (d_j(x_j, y_j))^{1-\gamma j}}, \quad \overrightarrow{\gamma} = (\gamma_1, \ldots, \gamma_n)$$

from one mixed norm Lebesgue space $L^{\overrightarrow{p}}_{\overrightarrow{\mu}}$ to another one $L^{\overrightarrow{q}}_{\overrightarrow{\mu}}$ is obtained, where (X_i, d_i, μ_i), $i = 1, \ldots, n$, are quasi-metric measure spaces (spaces of nonhomogeneous type).

Acknowledgement. The work was supported by the Shota Rustaveli National Foundation grant of Georgia (Project No. DI-18-118).

Minisymposium

TOPOLOGICAL METHODS IN DYNAMICAL SYSTEMS (MS-65)

Organized by Grzegorz Graff, *Gdansk University of Technolgy, Poland*
Coorganized by Piotr Oprocha, *AGH University, Kraków, Poland*

- Čech cohomology and the region of influence of non-saddle sets, *Héctor Barge*
- Densely branching trees as models for Hénon-like and Lozi-like attractors, *Jan Boroński*
- Pseudo-arc in measurable Dynamical Systems, *Jernej Činč*
- Attractors of dissipative homeomorphisms of the infinite surface homeomorphic to a punctured sphere, *Grzegorz Graff*
- Magnetic Helicity and the Calabi Invariant, *Gunnar Hornig*
- On the boundary of the basins of attraction for the secant method applied to polynomials, *Xavier Jarque*
- Wecken property and boundary preserving coincidences, *Michael Kelly*
- The topological conjugacy criterion for surface Morse-Smale flows with a finite number of moduli, *Vladislav Kruglov*
- Dynamics and bifurcations of a map-based neuron model, *Frank Fernando Llovera Trujillo*
- Topological estimates of the number of vertices of minimal triangulation, *Wacław Marzantowicz*
- On the set of periods for the Morse-Smale diffeomorphisms, *Adrian Myszkowski*
- Zero topological entropy and invariant measures in dimension one, *Piotr Oprocha*
- Dissipative flows and bifurcations of global attractors, *José M. R. Sanjurjo*
- Dynamical mechanisms of Type III responses in a nonlinear hybrid neuron model, *Justyna Signerska-Rynkowska*

Čech cohomology and the region of influence of non-saddle sets

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This talk is devoted to show some properties of a class of isolated invariant compacta known as non-saddle sets. We shall see that while the local dynamics near these sets is fairly easy, flows having this kind of invariant sets may exhibit a great deal of global complexity. As we will see, this global complexity is closely related to the way in which the non-saddle set sits in the phase space at the cohomological level.

Densely branching trees as models for Hénon-like and Lozi-like attractors

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Inspired by a recent work of Crovisier and Pujals on mildly dissipative diffeomorphisms of the plane, we show that Hénon-like and Lozi-like maps on their strange attractors are conjugate to natural extensions (a.k.a. shift homeomorphisms on inverse limits) of maps on metric trees with dense set of branch points. In consequence, these trees very well approximate the topology of the attractors, and the maps on them give good models of the dynamics. To the best of our knowledge, these are the first examples of canonical two-parameter families of attractors in the plane for which one is guaranteed such a 1-dimensional locally connected model tying together topology and dynamics of these attractors. For Hénon maps this applies to Benedicks-Carleson positive Lebesgue measure parameter set, and sheds more light onto the result of Barge from 1987, who showed that there exist parameter values for which Hénon maps on their attractors are not natural extensions of any maps on branched 1-manifolds. For Lozi maps the result applies to an open set of parameters given by Misiurewicz in 1980. Our result, that extends to certain attractors that are homoclinic classes for mildly dissipative diffeomorphisms, can be seen as a generalization to the non-uniformly hyperbolic world of a classical result of Williams from 1967.

Pseudo-arc in measurable Dynamical Systems

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Pseudo-arc is besides the arc the only planar continuum (i.e. compact connected metric space) so that every of its proper subcontinua is homeomorphic to itself. Its first description appeared in the literature about hundred years ago and due to many of its remarkable properties it is an object of much interest in several branches of mathematics. There are results indicating that pseudo-arc appears as a generic continuum in very general settings. For instance, Bing has proven that in any manifold M of dimension at least 2, the set of subcontinua homeomorphic to the pseudo-arc is a dense residual subset of the set of all subcontinua of M (equipped with

the Vietoris topology). In this talk I will present a result which reveals that pseudo-arc is a generic object also in a measure theoretical setting; namely, I will show that the inverse limit of the generic Lebesgue measure preserving interval map is the pseudo-arc. Building on this result I will construct a parametrised family of planar homeomorphisms with attractors being the pseudo-arc and several interesting topological and measure-theoretical properties.

Attractors of dissipative homeomorphisms of the infinite surface homeomorphic to a punctured sphere

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Coauthors: Rafael Ortega, Alfonso Ruiz-Herrera

A class of dissipative orientation preserving homeomorphisms of the infinite annulus, pairs of pants, and generally infinite surface homeomorphic to a punctured sphere is considered. We prove that in some isotopy classes the local behavior of such homeomorphisms at a fixed point, namely the existence of so-called inverse saddle, impacts the topology of the attractor - it cannot be arcwise connected.

This work was supported by the National Science Centre, Poland within the grant Sheng 1 UMO-2018/30/Q/ST1/00228 (G. Graff).

Magnetic Helicity and the Calabi Invariant

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Magnetic helicity (also called Asymptotic Hopf invariant, V.I Arnold 1974) is an important tool in the study of both astrophysical and laboratory plasmas. Helicity is an integral over the helicity density $h = A \wedge B$, $dA = B$, that measures the average asymptotic linkage of magnetic flux B in a given domain. It is a topological invariant of the magnetic field and provides a lower bound for the energy. However, the use of the helicity integral has been hampered by the fact that it only measures the average linkage over the whole domain and does not provide any more detailed information about linkage within parts of the domain. Attempts to extract more information about the linkage of flux, for instance by considering the helicity density, or line integrals over the helicity density (field line helicity), encounter the problem that h is not gauge invariant. In this talk we introduce the Calabi invariant, an integral quantity closely associated with helicity (Calabi 1970, Gambaudo et al. 2000) and show that this leads to interesting new ways to interpret the helicity integral and allows to calculate a gauge invariant asymptotic field line helicity.

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On the boundary of the basins of attraction for the secant method applied to polynomials

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We investigate the discrete dynamical system S defined on $\mathbb{R}^2$ given by the secant method applied to a real polynomial p. Every simple root α of p has associated its basin of attraction $\mathcal{A}(\alpha)$ formed by the set of points converging under S towards α and $\mathcal{A}^*(\alpha)$ its immediate basin of attraction.

We focus on the structure and dynamical behaviour of the boundary of the immediate basin of attraction of a root of p. We call *external* roots of p the smallest and largest value and *internal* all the rest. If α is an internal root of p then $\partial\mathcal{A}^*(\alpha)$ is given by the stable manifold of a 4-cycle. Moreover we show that, under some hypothesis, those internal basins are simply connected.

Wecken property and boundary preserving coincidences

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A result of B. Jiang divides all surfaces into two groups depending on the realizability of the Nielsen number. A surface is said to be Wecken if each homotopy class of self-maps has a representative which realizes the Nielsen number. It turns out that all non-Wecken surfaces admit a sequence of maps for which the difference tends to infinity. The setting of boundary preserving self-maps has a relative Nielsen number. The analogous problem was then studied in a series of papers by B. Brown, B Sanderson and M. Kelly, resulting in a slightly different classification. Here, there are two non-Wecken surfaces for which the difference remains bounded.

This talk considers this same problem, but now in the setting of coincidences for a pair of maps using the corresponding Nielsen numbers for coincidence. The results obtained are joint work with Leticia Silva and Joao Vieira (UNESP-Rio Claro, Brasil). We focus on the boundary preserving case and as a result produce a class of pairs of bounded surfaces which satisfy the Wecken property.

The topological conjugacy criterion for surface Morse-Smale flows with a finite number of moduli

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The Morse-Smale flows have been classified in sense of topological equivalence for several times during the last century. The most known invariants for such flows are invariants by E. Leontovich and A. Mayer [1], [2], M. Peixoto [3], A. Oshemkov and V. Sharko [4]. Besides, the Ω-stable flows on surfaces have been classified in such sense too by D. Neumann and T. O'Brien [5] and V. Kruglov, D.Malyshev and O. Pochinka [6]. Attempts were also made to classify Morse-Smale flows in sense of topological conjugacy: in particular, V. Kruglov [7] proved that the classes of topological equivalence and topological conjugacy for gradient-like flows on surfaces coincide.

J. Palis [8] considered a flow in a neighbourhood of a separatrix which connects two saddle points. He showed that in each topological equivalence class there is continuum of topological conjugacy classes, that is a flow with a separatrix-connection has analitical conjugacy invariants called *moduli of stability* or *moduli of topological conjugacy*. Each limit cycle likewise generates at least one modulus associated with its period. V. Kruglov, O. Pochinka and G. Talanova [9] proved that non-singular flows on an annulus with only two limit cycles on the annulus's boundary components have infinite number of moduli, expressed by a function.

The first result of this report is the following.

Theorem 1. *A Morse-Smale surface flow has finite number of moduli iff it has no a trajectory going from one limit cycle to another.*

To construct the topological classification in sense of conjugacy we use the complete topological classifications in sense of equivalence from [4], [6]. Namely, there is one-to-one correspondence between equivalent classes of a Morse-Smale flow ϕ^t on a surface and isomorphic classes of the *equipped graph* $\Upsilon^*_{\phi^t}$, which contains an information about partition the ambient manifold into cells with similar trajectories behaviour and the limit cycles types.

To distinguish topological conjugacy classes we add to the equipped graph an information on the periods of the limit cycles. It gives a new equipped graph $\Upsilon^{**}_{\phi^t}$, and here is the second result.

Theorem 2. *Morse-Smale surface flows ϕ^t, ϕ'^t without trajectories going from one limit cycle to another one are topologically conjugate iff their equipped graphs $\Upsilon^{**}_{\phi^t}$ and $\Upsilon^{**}_{\phi'^t}$ are isomorphic.*

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Dynamics and bifurcations of a map-based neuron model

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The Chialvo Model is a well known 2D discrete scheme to describe the dynamics of single neurons. However the existing analytical and numerical results do not yield complete picture of its dynamics. In some parameters regime this system can be viewed as singularly perturbed

(nonlinear) discrete system. Observing, that the fast-subsystem is given by the S- unimodal map, we are able to precisely describe the dynamics of the reduced 1D model

and extend some of these results to the full model. In particular we study flip and saddle-node bifurcations and chaotic behaviours in the model, interpreting them in terms of Izhikevich-Hoppensteadt classification of bursting mappings which is important from the point of neuron's electrophysiology and excitability properties. Our analytical findings are illustrated numerically.

Topological estimates of the number of vertices of minimal triangulation

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Coauthors: Dejan Govc, Petar Pavešić

From the beginning of the algebraic topology, then also called the combinatorial topology, i.e. from the beginning of the 20th century, its basic object is the simplicial complex. The representation of a given topological space X as a simplicial complex (i.e. a homeomorphism with it)

is called triangulation. The name comes from the fact that when space X is a two-dimensional surface, triangulation means representing it as the union of adjacent triangles with edges meeting at the vertices. In the case of higher dimensions, the basic cells are the "i" - dimensional simplices. One of the natural questions is to find a triangulation with the minimum number of vertices, respectively of all simplexes (or estimate these numbers). This lecture will be devoted to this problem. We will present a new method based on the notion of covering type estimating from below the number of vertices by the weighted length of the elements in the cohomology ring $H^*(X)$, or the weighted Lusternik-Schirelmann category theory. As a consequence, we got not only a unified method of proof of estimates of the number of vertices of the minimal triangulations derived originally by ad hock combinatorial methods, but also sharper estimates, or estimates for the families of manifolds not studied earlier.

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On the set of periods for the Morse-Smale diffeomorphisms

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We apply the representation of Lefschetz numbers of iterates in the form of so-called periodic expansion to determine the minimal set of Lefschetz periods $\mathrm{MPer}_L(f)$. Applying this approach we present an algorithmic method for finding minimal Lefschetz periods for Morse-Smale diffeomorphisms on M_g and N_g, respectively an orientable and non-orientable compact surface without boundary of genus g. Llibre and Sirvent calculated $\mathrm{MPer}_L(f)$ for several genuses $g \leqslant 9$ of M_g and N_g. Our approach makes it possible to find easily $\mathrm{MPer}_L(f)$ for much bigger values of g.

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Zero topological entropy and invariant measures in dimension one

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In this talk I will survey some recent results on special properties of maps with zero entropy on one-dimensional continua (mostly interval and topological graphs). I will be interested in relations between Li-Yorke pairs, sequence entropy and how they influence properties of invariant measures. The talk will be based, in part, on recent joint works with Jian Li, Guohua Zhang and Xianjuan Liang.

Dissipative flows and bifurcations of global attractors

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We study bifurcations of dissipative flows in which a global attractor becomes a non-global attractor after a small perturbation of the flow. Using Conley's index theory, we examine the mechanism which produces this phenomenon and we identify a dynamical property which is responsible for this bifurcation and analyze the topological features of some isolated invariant sets generated throughout this process. On the other hand we show that global attractors continue to global attractors if and only if the family of flows is uniformly dissipative. We illustrate this property with the flow induced by the Lorenz equations and their global attractors.

Dynamical mechanisms of Type III responses in a nonlinear hybrid neuron model

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Coauthors: Jonathan Rubin, Jonathan Touboul

The dynamic mechanisms shaping neurons' responses to transient inputs can bear significant physiological relevance and is connected, among others, with such phenomena as *post-inhibitory facilitation* (PIF), where an otherwise subthreshold excitatory input can induce a spike if it is applied with proper timing after an inhibitory pulse, and *slope detection*, in which a neuron spikes to a transient input only when the input's rate of change is in a specific, bounded range. These phenomena have been previously associated with so-called Type III neurons (in Hodgkin's classification) which are those neurons that never exhibit continuous firing in response to sustained excitatory currents.

In our study we analyse responses to transient inputs in nonlinear adaptive hybrid models and provide a geometric characterization of dynamical structures associated with PIF and an analytical study of slope detection for tent inputs. In particular, our proofs show that PIF and slope-detection do not always require pure Type III regime.

Minisymposia in

MATHEMATICAL PHYSICS

- Analysis on Graphs (MS-48)
- Multicomponent diffusion in porous media (MS-42)

Minisymposium

ANALYSIS ON GRAPHS (MS-48)

Organized by Aleksey Kostenko, *University of Ljubljana, Slovenia*
Coorganized by Pavel Exner, *Czech Technical University, Prague, Czech Republic*

- Neumann domains on metric graphs, *Ram Band*
- Universality of nodal count statistics for large quantum graphs, *Gregory Berkolaiko*
- The density of states for periodic Jacobi matrices on trees, *Jonathan Breuer*
- Effects of time-reversal asymmetry in the vertex coupling of quantum graphs, *Pavel Exner*
- Steklov eigenvalues on graphs, *Bobo Hua*
- Stochastic completeness and uniqueness class for graphs, *Xueping Huang*
- Multifractal eigenfunctions in a singular quantum billiard, *Jon Keating*
- Subordinacy theory on star-like graphs, *Netanel Levi*
- Asymptotics of Green functions: Riemann surfaces and Graphs, *Noema Nicolussi*
- Solitons for the KdV equation on metric graphs, *Christian Seifert*

Neumann domains on metric graphs

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Coauthor: Lior Alon

The Neumann points of an eigenfunction f on a quantum (metric) graph are the interior zeros of f'. The Neumann domains of f are the sub-graphs bounded by the Neumann points. Neumann points and Neumann domains are the counterparts of the well-studied nodal points and nodal domains.

We present the following three main properties of Neumann domains: their count, wavelength capacity and spectral position. We study their bounds and probability distributions and use those to investigate inverse spectral problems.

The relevant probability distributions are rigorously defined in terms of selected random variables for quantum graphs. To this end, we provide conditions for considering spectral functions of quantum graphs as random variables with respect to the natural density on $\mathbb{N}$.

The talk is based on joint work with Lior Alon.

Universality of nodal count statistics for large quantum graphs

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Coauthors: Lior Alon, Ram Beand

An eigenfunction of the Laplacian on a metric (quantum) graph has an excess number of zeros due to the graph's notrivial topology. This number, called the nodal surplus, is an integer between 0 and the first Betti number of the graph. We study the value distribution of the nodal surplus within the countably infinite spectrum of the graph. We conjecture that this distribution converges to Gaussian in any sequence of graphs of growing Betti number. We prove this conjecture for several special graph sequences and test it numerically for some other well-known types of graphs. An accurate computation of the distribution is made possible by a formula expressing the distribution as an integral over a high-dimensional torus with uniform measure.

The density of states for periodic Jacobi matrices on trees

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The Hebrew University of Jerusalem, Israel

Periodic Jacobi matrices on trees are a natural generalization of one dimensional periodic Schroedinger operators. While the one dimensional theory is very well developed, very little is known about the general tree case. In this talk we will review some of the few known results with a focus on the question of convergence of the appropriately defined finite volume approximations. If time permits, we will also discuss some open problems. Based on joint works with Nir Avni, Gil Kalai and Barry Simon.

Effects of time-reversal asymmetry in the vertex coupling of quantum graphs

Pavel Exner, exner@ujf.cas.cz
Czech Academy of Sciences, Nuclear Physics Institute, Czech Republic

The talk concerns the effects coming from a violation of time-reversal symmetry in the vertex coupling of quantum graph, focusing on the situation when the asymmetry is maximal at a fixed energy. It is shown that such a coupling has a topological property: the transport behaviour of such a vertex in the high-energy regime depend substantially on the vertex parity. We explore consequences of this fact for several classes of graphs, both finite and infinite periodic ones. The results come from a common work with Marzieh Baradaran, Jiří Lipovský, and Miloš Tater.

Steklov eigenvalues on graphs

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Fudan University, China

The eigenvalues of the Dirichlet-to-Neumann operator are called Steklov eigenvalues, which are well studied in spectral geometry. In this talk, we introduce Steklov eigenvalues on graphs, and estimate them using geometric quantities, based on joint works with Wen Han, Zunwu He, Yan Huang, and Zuoqin Wang.

Stochastic completeness and uniqueness class for graphs

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Nanjing University of Information Science and Technology, China

Uniqueness class for the heat equation on a weighted graph is closely related to stochastic completeness of the corresponding minimal continuous time random walk. For a class of so called globally local graphs, we obtain essentially sharp criteria which are in the same form as for manifolds. Sharp volume growth type criteria for stochastic completeness then follow, after a reduction to the globally local case.

This talk is based on joint work with M. Keller and M. Schmidt.

Multifractal eigenfunctions in a singular quantum billiard

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University of Oxford and London Mathematical Society, United Kingdom

Spectral statistics and questions relating to quantum ergodicity in star graphs are closely related to those of Šeba billiards (rectangular billiards with a singular point scatterer) and other intermediate systems. In intermediate systems, it has been suggested in the Physics literature that quantum eigenfunctions should exhibit mulifractal properties. I shall discuss the proof that this is the case for Šeba billiards. The work I shall report on was done jointly with Henrik Ueberschaer.

Subordinacy theory on star-like graphs

Netanel Levi, netanel.levi@mail.huji.ac.il
Hebrew University of Jerusalem, Israel

The notion of subordinacy was introduced by Gilbert and Pearson, and it enables one to separate the singular and absolutely continuous parts of the spectrum of Schroedinger operators on the line via asymptotic properties of solutions to the eigenvalue equation. Informally speaking, a solution is called subordinate if it decays faster than any other linearly independent solution. We present a generalization of the Gilbert-Pearson subordinacy theory to Schroedinger operators on star-like graphs, which are graphs that consist of a compact component C, to which a finite number of half-lines are attached. We use our result to draw conclusions on the multiplicity of the singular spectrum of such operators.

Asymptotics of Green functions: Riemann surfaces and Graphs

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Analysis on graphs and Riemann surfaces admits many interesting parallels. Both settings admit a canonical measure (the Arakelov–Bergman and Zhang measures) and an associated canonical Green function reflecting crucial geometric information.

Motivated by the question of describing the limit of the Green function on degenerating families of Riemann surfaces, we introduce new higher rank versions of metric graphs and their Laplace operators. We discuss how these limit objects describe the asymptotic of the Green function on metric graphs and Riemann surfaces close to the boundary of their respective moduli spaces.

Based on joint work with Omid Amini (École Polytechnique).

Solitons for the KdV equation on metric graphs

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Technische Universität Hamburg and CAU Kiel, Germany

We consider the Korteweg-de Vries equation on metric star graphs. After reviewing suitable coupling conditions at the vertex for the linearised equation, i.e. an Airy-type evolution equation, we investigate coupling conditions at the vertex such that the system allows for solitary waves.

This is joint work with Delio Mugnolo (Hagen) and Diego Noja (Milan).

Minisymposium

MULTICOMPONENT DIFFUSION IN POROUS MEDIA (MS-42)

Organized by Nicola Zamponi, *Charles University of Prague, Czech Republic*
Coorganized by
Esther S. Daus, *Vienna University of Technology, Austria*
Josipa Pina Milišić, *University of Zagreb, Croatia*

- Gradient flow techniques for multicomponent diffusion-reaction, *Martin Burger*
- Well-posedness results for mixed-type PDE systems modelling pressure-driven multicomponent flows, *Pierre Etienne Druet*
- Evolution nonlocal diffusion problems with Lipschitz-continuous diffusion kernels, *Gonzalo Galiano*
- Modeling of ion transport by a Maxwell-Stefan approach and numerical results, *Rüdiger Müller*
- A bi-fluid model for a mixture of two compressible non interacting fluids with general boundary data, *Sarka Necasova*
- Existence analysis of a stationary compressible fluid model for heat-conducting and chemically reacting mixtures, *Milan Pokorny*
- Uniqueness for a cross-diffusion system issuing from seawater intrusion problems, *Carole Rosier*
- Variational methods for fluid-structure interaction: Bulk elastic solids interacting with the Navier Stokes equation, *Sebastian Schwarzacher*
- Nonisothermal Richards flow in porous media with cross diffusion, *Nicola Zamponi*
- The Boltzmann gas equation in relation to Onsager–Stefan–Maxwell diffusion for Lennard-Jones gas mixtures, *Maxim Zyskin*

Gradient flow techniques for multicomponent diffusion-reaction

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FAU Erlangen-Nürnberg, Germany

In this talk we will discuss some gradient flow approaches to reaction-diffusion systems with nonlinear cross-diffusion, including the derivation from microscopic systems and the analysis of the macroscopic systems in terms of renormalized solutions. We will also discuss the case of coupled bulk-surface systems.

Well-posedness results for mixed-type PDE systems modelling pressure-driven multicomponent flows

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Weierstrass Institute Berlin, Germany

In this talk we consider the purely convective mass transport in such isothermal multicomponent fluids for which the velocity field is negative proportional to the gradient of the thermodynamic pressure. The equations of motion formally reduce to the Darcy law, and the main driving mechanism is volume filling. Thus, this type of flow is mathematically related to the theory of transport in porous media. We shall introduce a special equation of state, which allows to define the thermodynamic pressure in a consistent way. This constitutive choice results into a system of PDEs which, after an appropriate change of variables, consists of N-1 first-order transport equations for the volume fractions, and one parabolic second-order equation of porous-medium-type for the volume. We show that this system admits a unique classical or, in less smooth geometrical settings, a unique weak solution. We shall also report on ongoing work concerning the optimal control of the PDE–system. These results are obtained in the context of joined work with A. Jüngel (TU Vienna) and J. Sprekels (WIAS Berlin).

Evolution nonlocal diffusion problems with Lipschitz-continuous diffusion kernels

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University of Oviedo, Spain

We present some recent results on the well-posedness of problems formulated as scalar or systems of scalar equations involving nonlocal (cross) diffusion and reaction terms. Lipschitz continuous diffusion kernels are considered.

Modeling of ion transport by a Maxwell-Stefan approach and numerical results

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Coauthor: Wolfgang Dreyer

Electro-thermodynamics provides a consistent framework to derive continuum models for the description of electrochemical systems on the device level, e.g. for batteries or fuel cells. These models must be equipped with two additional ingredients: (i) a free energy model to calculate the chemical potentials and (ii) a kinetic model for the kinetic coefficients. Suitable free energy models for liquid electrolytes incorporating ion–solvent interaction, finite ion sizes and solvation already exist and have been validated against experimental measurements.

In this work, we apply a Maxwell–Stefan setting for multicomponent transport in order to derive mobility coefficients. However, instead of these mobility coefficients, often other transport parameters are more insightful for the interpretation of measurements. In the context of energy conversion systems, the electric conductivity of electrolytes has naturally attracted most interest. Further relevant parameters are the transference numbers and diffusion coefficients. Contrary to the equilibrium properties, some of these mentioned transport properties depend on a combination of chemical potentials and kinetic coefficients, while others depend solely on the kinetic coefficients.

In a numerical study, we analyze the impact of ion solvation and incomplete salt dissociation on the transport parameters of a non-dilute electrolyte.

A bi-fluid model for a mixture of two compressible non interacting fluids with general boundary data

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We prove global existence of weak solutions for a version of one velocity Baer-Nunziato system with dissipation describing a mixture of two non interacting viscous compressible fluids in a piecewise regular Lipschitz domain with general inflow/outflow boundary conditions. The geometrical setting is general enough to comply with most current domains important for applications as, for example, (curved) pipes of picewise regular and axis-dependent cross sections. As far as the existence proof is concerned, we adapt to the system the nowaday's classical Lions-Feireisl approach to the compressible Navier-Stokes equations which is combined with a generalization of the theory of renormalized solutions to the transport equations. The results related to the families of transport equations presented in this paper extend/improve some of statements of the theory of renormalized solutions, and they are therefore of independent interest.

Existence analysis of a stationary compressible fluid model for heat-conducting and chemically reacting mixtures

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Coauthors: Miroslav Bulicek, Ansgar Juengel, Nicola Zamponi

The existence of large-data weak solutions to a steady compressible Navier–Stokes–Fourier system for chemically reacting fluid mixtures is proved. General free energies are considered satisfying some structural assumptions, with a pressure containing a γ-power law. The model is thermodynamically consistent and contains the Maxwell–Stefan cross-diffusion equations in the Fick–Onsager form as a special case. Compared to previous works, a very general model class is analyzed, including cross-diffusion effects, temperature gradients, compressible fluids, and different molar masses. A priori estimates are derived from the entropy balance and the total energy balance. The compactness for the total mass density follows from an estimate for the pressure in L^p with $p > 1$, the effective viscous flux identity, and uniform bounds related to Feireisl's oscillations defect measure. These bounds rely heavily on the convexity of the free energy and the strong convergence of the relative chemical potentials.

Uniqueness for a cross-diffusion system issuing from seawater intrusion problems

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Coauthor: Catherine Choquet

This work is devoted to the mathematical analysis of the Cauchy problem for cross-diffusion systems without any assumption about its entropic structure. A global existence result of non-negative solutions is obtained by applying a classical Schauder fixed point theorem. The proof is upgraded for enhancing the regularity of the solution, namely its gradient belongs to the space $L^r((0,T)\times\Omega)$ for some $r > 2$. To this aim, the Schauder's strategy is coupled with an extension of Meyers regularity result for linear parabolic equations. We show how this approach allows to prove the well-posedness of the problem using only assumptions prescribing and admissibility range for the *ratios* between the diffusion and cross-diffusion coefficients. Finally, the question of the maximal principle is also addressed, especially when source terms are incorporated in the equation in order to ensure the confinement of the solution.

Variational methods for fluid-structure interaction: Bulk elastic solids interacting with the Navier Stokes equation

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Coauthors: Barbora Benesova, Malte Kampschulte

We introduce a two time-scale scheme which allows to extend the method of minimizing movements to hyperbolic problems. This method is used to show the existence of weak solutions to a fluid-structure interaction problem between a nonlinear, visco-elastic, n-dimensional bulk

solid governed by a hyperbolic evolution and an incompressible fluid governed by the (n-dimensional) Navier-Stokes equations for $n \geq 2$.

Nonisothermal Richards flow in porous media with cross diffusion

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Coauthors: Esther Daus, Pina Milišić

The existence of large-data weak solutions to a nonisothermal immiscible compressible two-phase unsaturated flow model in porous media is proved. The model is thermodynamically consistent and includes temperature gradients and cross-diffusion effects. So-called variational entropy solutions, which involve the integrated total energy balance, are considered in order to overcome the lack of integrability of some terms in the total energy flux. A priori estimates are derived from the entropy balance and the total energy balance. A sequence of approximated solutions is built via a time semi-discretization and several regularizations. The compactness of the sequence of approximated solutions is achieved by using the Div-Curl lemma.

The Boltzmann gas equation in relation to Onsager–Stefan–Maxwell diffusion for Lennard-Jones gas mixtures

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The Enskog method is an asymptotic method that provides solutions of the Boltzmann equation close to local equilibrium. The method predicts kinetic properties of gases, such as Stefan–Maxwell diffusivities, in terms of certain collision integrals. We describe a method to compute collision integrals, and a method of molecular dynamics simulation of Stefan–Maxwell diffusivities based on Onsager's regression hypothesis. We investigate how analytical predictions compare with molecular dynamics simulations for mixtures of Lennard-Jones gases and experiments with mixtures of monatomic gases. We apply the Enskog method to derive continuum-level model equations. Within the limitations of Enskog's method, these results identify with a set of multi-species transport equations that Goyal and Monroe recently derived using the alternative theory of irreversible thermodynamics.

Minisymposia in

Mathematics Education and History of Mathematics

Minisymposium

MATHEMATICS IN EDUCATION (MS-19)

Organized by Amalija Žakelj, *University of Primorska, Faculty of Education, Slovenia*

- Using the CLIL approach in teaching maths in the fourth grade, *Silva Bratož*
- Approaches of gifted pupils to solving algebraic word problems, *Irena Budínová*
- Mathematical reasoning: which are the issues?, *Daniel Doz*
- Mathematical Representations and Inconsistencies in Communication, *Darjo Felda*
- Computational/algorithmic thinking in the school mathematics, *Djordje Kadijevich*
- Making a rhombic 1080-hedron, *Andreja Klančar*
- How sixth graders' represent some mathematics concepts with drawings, *Alenka Lipovec*
- Lessons learned from pre-service teachers' narratives of math failure, *Sonja Lutovac*
- The role of Fermi problems in the concept of developing mathematical literacy among students, *Vida Manfreda Kolar*
- A Contextual Approach to Teaching Algebra in Elementary Education, *Sanja Maričić*
- Five Mathematicians from MacTutor History of Mathematics, *Marko Razpet*
- Graph Theory Between The World Wars, *Antonín Slavík*
- Software generation of images using mathematics, *Bogdan Soban*
- Generalizations and the history of the Butterfly theorem, *Martina Škorpilová*
- Interactive teaching tools and mathematical student competency: Pedagogical experiment in higher mathematics courses teaching, *Daria Termenzhy*
- Cross-Curricular Integration of Knowledge in Mathematics at the Primary School Level, *Marina Volk*
- Peano- and Hilbert curve, *Jan Zeman*
- Students' achievements in solving geometric problems using visual representations in a virtual learning environment, *Amalija Žakelj*

Using the CLIL approach in teaching maths in the fourth grade

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The present paper focuses on teaching English and mathematics in primary school using the content and language integrated learning (CLIL) approach. In practice, the CLIL approach implies that learners use a foreign language as a medium to study subjects, such as natural sciences, geography, or mathematics which are usually taught in the first language. There are several benefits of using the CLIL approach in teaching a foreign language in primary school. For the purpose of our research the most important benefits can be seen in the fact that CLIL enables the learner to access subject-specific target terminology, it provides opportunities to study content through different perspectives and at the same time allows learners to gain advanced understandings of subject-specific content (Coyle et al 2010, Klimova 2012). Another important advantage is that it enhances a higher level of cross-curricular integration. In order to explore the efficiency of using the CLIL approach in teaching English and mathematics, we carried out an experiment in which we compared the control group of pupils who had a mathematics lesson in Slovenian with the experimental group in which the same lesson was conducted in the English language. The sample consisted of 133 primary school pupils in the fourth grade (aged 9/10) from three different schools in Slovenia. The results of the post test showed that the classes which were taught in Slovene were generally more successful than the CLIL classes. However, in the majority of the items tested, there were no statistically significant differences and the aims have been achieved by both, the control and experimental groups. This suggests that CLIL can be seen as an efficient approach in teaching selected mathematical content through English.

Keywords: Content and Language Integrated Learning (CLIL), English as a foreign language, mathematics, cross-curricular integration

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Approaches of gifted pupils to solving algebraic word problems

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Gifted students, and especially mathematically gifted students, differ from other students in their approach to solving mathematical problems. As the complexity of mathematical problems increases in pupils of the 2nd grade of primary school, it is difficult to monitor these differences in pupils only from the success in the test. There are even tasks in which gifted pupils may be less successful than other pupils. The paper will briefly discuss the issue of educating gifted students in mathematics and one algebraic word problem will illustrate how more complex thinking of gifted students can affect their success in the test task.

Mathematical reasoning: which are the issues?

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Mathematical reasoning is believed to play an important role in mathematics, since many mathematical processes require it. Despite its importance, many students have difficulties with mathematical reasoning. Current literature has shown that students face several issues while proving by counterexamples and formal proving. It is not however clear if such reasoning problems are present also among Italian students. In this paper we analyzed the answers of 17 grade-10 Italian students that solved the Italian national assessment of mathematical knowledge in 2011. The analysis shows that the majority of students struggled with correct reasoning. Several incorrect reasoning processes have been used; hence educators and school authorities should focus even more on the development of students' reasoning skills.

Mathematical Representations and Inconsistencies in Communication

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Mathematical notions and conceptions are the outcome of abstraction of immediate experience in physical world and are usually transmitted through a special language, which makes mathematical discourse complex and in many respects unusual. It is thus necessary to use adequate representations that enable learners to relate mathematical notions to what they already know and to learn with understanding. In this process of learning mathematics, communication has an important role and the language has a crucial influence on the correct construction of mathematical concepts. Some inconsistencies in presenting mathematical facts or descriptions will be discussed.

Computational/algorithmic thinking in the school mathematics

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Due to the globalization and internationalization of the mathematics curriculum, there is, for example, a rapidly developing interest in including computational/algorithmic thinking in mathematics education. By focusing on this thinking, this lecture comprises two parts. In the first part I summarize what computational/algorithmic thinking is, discuss recent research outcomes regarding it, and examine a place for this thinking in school mathematics, with an emphasis on recent international trends and emerging implications for mathematics education. In the second part I present a way to cultivate computational thinking in a mathematical context through data practice based upon the use of interactive displays, which, embedded in another context, could contribute the learning of statistics or computer science (informatics), for example. The content of the first part is based upon a joint research with Dr Max Stephens, University of Melbourne, Australia, whereas the content of the second part is based upon my own recent research.

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Making a rhombic 1080-hedron

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Paper models of polyhedra are attractive but difficult to make. That is why in schools we prefer to make polyhedra from plastic parts (Polydron, Zometool). We are interested in golden rhombic solids. There are only five convex golden polyhedra: prolate and oblate rhombohedron, rhombic triacontahedron (Kepler, 1611), rhombic icosahedron (Fedorov, 1885), and Bilinski dodecahedron (1960). Our project is to make a polyhedron that has 1080 golden rhombuses as faces and has the symmetry of an icosahedron. Such a polyhedron has 2160 edges and 1062 vertices. So we need 2160 sticks and 1062 Zometool balls. Theoretically, we could describe the fabrication by starting with Kepler's triacontahedron. Each rhombus is divided into 36 smaller ones. We then do an inversion on the vertices of valence 3 until no two adjacent rhombuses are in same plane. The orthogonal projection of such polyhedron along an axes of fivefold rotation form a Penrose tiling.

How sixth graders' represent some mathematics concepts with drawings

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Visual representations enable teachers and researchers to interpret the meanings of mathematical concepts, relations and processes; therefore, they play an important role in mathematics education. In the present study, we analysed the students' understanding of various mathematical concepts using drawings. Numerical expressions $17 - 9$, $3 \cdot (4 + 5)$, $\frac{3}{5}$ of 15 and 2^3 were provided to sixth grade elementary students (*N* = 1595). Students were asked to draw one picture for each numerical expression that describes it. We were interested in whether pictures adequately represent the concept underlying the expression. The data were analysed with a combination of qualitative and quantitative methods. The results show that the participants represented the concepts quite adequately. Expectedly, less abstract concepts were depicted more adequately. In the qualitative content analysis, two themes emerged. Those themes illustrate two ways of mathematical understanding (instrumental and relational) and two types of mathematical knowledge (procedural and conceptual). Procedurally oriented images predominated;

they were also more prone to arithmetic errors. The research results can help researchers to create new research tools to measure students' mathematical understanding and help teachers to find new approaches that give them insight into students' mathematical understanding.

Keywords: visualisation, representation, understanding, subtraction, parenthesis, fraction, exponentiation.

Lessons learned from pre-service teachers' narratives of math failure

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This presentation will discuss the findings of the 'Narrated Failures' project (Academy of Finland, project ID 307672), where pre-service teachers' narratives of math failure were analysed. Several studies were conducted in which both pre-service elementary school and pre-service mathematics teachers were instructed to write an essay on the topic of 'math failure and identity'. They were asked to recall their autobiographical experiences of past failure, to define math failure, and reflect on how their failure experiences have shaped them as students, and as future teachers. The narrative analysis of individual cases, as well as cross-case comparison were performed in attempt to deepen understanding of what math failure is from the perspective of the two mentioned cohorts of future teachers of mathematics and how failure shapes their identity development as teachers. In this presentation, the findings of four distinct yet entwined investigations are discussed to shed light on how narratives of math failure can inform teacher education pedagogies. The need to understand math failure as an autobiographical experience is highlighted to better prepare both generalist and specialist future teachers for the variety of ways failure will manifest in their classrooms.

Keywords: math failure, identity, narrative, pre-service teachers, teacher education

The role of Fermi problems in the concept of developing mathematical literacy among students

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The article deals with mathematical literacy in relation to mathematical knowledge and mathematical problems. It presents also the Slovenian project NA-MA POTI, which started in 2018 and aims to develop mathematical literacy at the national level, from kindergarten to secondary education. The concept of mathematical literacy identifies two cornerstones of mathematical literacy: 1) mathematical thinking, that is, understanding and using mathematical concepts, procedures, strategies and communication as a basis for mathematical literacy; and 2) problem solving in different contexts (personal, social, professional, scientific) that enable a mathematical approach. The latter also highlights mathematical modelling, which generally involves interpreting real-world observations using conceptual (mathematically structured) language. Fermi problems represent a special type of mathematical modelling problems. They are generally defined as problems that are at first sight unsolvable, are authentic and are not structured in the same way as school problems, require reasoning about the necessary data for solving and evaluating them, require mathematical knowledge, and allow the development of problem-solving strategies. Fermi problems, when properly introduced into the learning process, have the potential to develop students' mathematical literacy on both cornerstones. Teachers have a key role to play in this process, as they need to acquire appropriate competencies in mathematical literacy themselves first in order to be able to organise and implement appropriate learning situations in their teaching. The aim of the empirical part was to develop a scheme for assessing the quality of Fermi problems created by prospective primary school teachers for their implementation in mathematics lessons. For this purpose, we analysed examples of Fermi problems for fifth graders designed by prospective primary school teachers The problems were evaluated according to the characteristics of the modelling process: the complexity of the mathematization, the complexity of determining the data needed for the solution, the number of cycles in modelling, and the linguistic relevance of the problem. We defined as qualitative those Fermi problems which can be identified as having a high complexity of determining the data needed for the solution, have a high level of mathematization, are linguistically appropriate, and have more than one cycle. We argue that Fermi problems with such characteristics have a great potential in the process of developing mathematical literacy in students at basically all levels. We want to encourage prospective primary school teachers, as well as other stakeholders, to create their own Fermi problems. We believe that a given scheme for evaluating the quality of Fermi problems can guide teachers in creating their own problems or in deciding which problems are appropriate for classroom implementation according to the objectives.

Keywords: mathematical literacy, problems, Fermi problems, modelling, teaching mathematics, prospective primary school teachers

A Contextual Approach to Teaching Algebra in Elementary Education

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The authors underline the importance of mastering algebra-related content, but also difficulties associated with the acquisition of such content at an early age, students' comprehension of algebra and the application of knowledge in problem-solving. Within this context, they point out the characteristics, importance and effects that can be achieved in mathematics education through the implementation of a contextual approach to teaching algebra. The aim of the study was to examine the effects that the contextual approach to teaching algebra in junior elementary school had on student achievement. We organised an experimental study (experiment with parallel groups) on a sample (N = 192) to determine whether a methodological approach based on the principles of contextual learning results in the improvement of learning and student achievement in comparison to the traditional model utilised in mathematics education, and we chose algebra for that purpose. Results of the final measurement show that students from the experimental group, who were exposed to the experimental programme, have achieved better results than students whose work was based on the traditional model. This study has shown that a contextual approach to teaching algebra has significant effects on student achievement, improves their understanding of algebra-related content, and their application of the acquired knowledge in problem-solving.

Keywords: contextual approach, early algebra, mathematics education, mathematics, student achievement.

Five Mathematicians from MacTutor History of Mathematics

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We exhibit posters describing lives and work of Jurij Vega, Franc Močnik, Josip Plemelj, Ivo Lah, and Ivan Vidav. These five Slovenian mathematicians that are covered by MacTutor History of Mathematics significantly influenced the development of mathematics in Slovenia.

Graph Theory Between The World Wars

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We focus on the development of graph theory in the interwar period. In the 1930s, there were still many mathematicians who did not regard graph theory as a serious discipline, owing partly to the fact that many results in this field were originally inspired by problems of recreational mathematics. However, the publication of the first textbook of graph theory in 1936, which presented it as a completely rigorous discipline with applications in linear algebra and set theory, helped to change this attitude. Meanwhile, some important results that are now classified as belonging to graph theory were formulated in the language of topological structures. Finally, one should not forget that numerous developments in graph theory grew out of purely practical problems, such as the construction of electricity networks, or the enumeration of chemical compounds.

Software generation of images using mathematics

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I use my software developed in the Visual Basic programming language to generate images. An algorithmic approach is used, which enables a high level of diversity of results with relatively short but mathematically supported software solutions. At each point of the image, a programming algorithm is executed, consisting of mathematical formulas that enter pieces of information, which are constant for the current cycle (genetic code) and variables generated by the process itself. The result of the calculation is the color of the point or the position where to remove the color from the color palette, which plays the role of a key variable. The image coordinate system, the Cartesian and polar coordinate system, and the logic of the complex plane are used to control the image. The whole process is supported by a time-determined random number generator, which supplies the system with random values of various parameters and thus ensures unpredictability and non-repetition of the results. From the viewpoint of mathematics, the following notions appear in the algorithms: complex algebraic expressions, trigonometric and logarithmic functions, plane geometry, iterations and recursions, determination of random numbers in areas, conversion of coordinates between different coordinate systems, control of stochastic motion of a point in plane and space, circular and spiral movements, computation with complex numbers, original formulas for various calculations, number sequences (Fibonacci) and other mathematical operations. The resulting paintings are of the abstract type, which for the most part do not show their mathematical provenance and approach fine art.

Generalizations and the history of the Butterfly theorem

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The introductory part of the talk is devoted to the relatively well-known Butterfly theorem, which is the theorem on a circle and its chords. Subsequently, several of its generalizations are discussed and various combinations of these generalizations are studied. The history of this theorem is also introduced.

Interactive teaching tools and mathematical student competency: Pedagogical experiment in higher mathematics courses teaching

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The talk is devoted to implementation of various interactive teaching tools into the up-to-date higher mathematical education. This research reports a six years long experiment which took place in Vasyl Stus Donetsk National University in Ukraine. The experiment was led by own research group of three academics (including author) of the Faculty of Mathematics and IT (now Faculty of Information and Applied Technologies). We worked with 14 students of the Faculty who specialized in Mathematics from 2013/2014 till 2018/2019 academic years. The selection of students was based on their interest and ownership of appropriate gadgets for studying (laptop, smartphone, PC, webcam, etc.). The learning was done by carrying out student activities with applying interactive tools into several mathematical courses (Analytical Geometry (2013/2014), Logic (2014/2015), Differential Geometry (2014/2015), Methodology of Teaching Mathematics (2015/2016), History of Mathematics (2016/2017), IT in Teaching Mathematics (2017/2018), Preparation of Master's Work (2018/2019). The students used educational interactive portal and distance courses at LMS Moodle developed by author. It included syllabus; tutorial video lectures; diagnostic tests; links to external webpages with mathematical resources that explore the different contents of the course; additional tasks (so-called "extraproblems"); e-lectures with links for downloading; online quizzes and surveys; online consultation module; interactive mathematical games; library of students' scientific projects; discussion forum panel. We have a long-standing experience of the applying technologies for enhancing the quality of higher mathematics learning. I started a pilot experiment with applying of educational interactive portal in freshmen course of Analytical Geometry for Mathematics students at the University in Fall 2012. This study was preceded our experiment, some results were presented in author's PhD thesis and proved the efficiency of applying interactive teaching tools in mathematical course. The test procedure of the experiment consisted of 5 parts: pre-test (Math and IT literacy), intellectual test (intellectual lability, ambition test), post-test (Math and IT literacy), several questionnaires and interviews. We also carried out some electronic surveys designed with applying Google Forms. More detail of this experiment and its results will be covered in the talk.

Cross-Curricular Integration of Knowledge in Mathematics at the Primary School Level

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Cross-curricular integration is one of the key concepts of modern orientations of education development because students who are involved in elementary school education have to gain competencies that go beyond the traditional boundaries between subjects so that they will be able to participate in the society of knowledge. It is important in the transfer of knowledge that educational institutions are also aware of the extracurricular circumstances in which students work and take these circumstances into account when teaching. Students are motivated by the opportunity to use the gained knowledge to solve more complex problems and create new solutions. In recent years, most European countries have renewed mathematics curricula (including Slovenia in 2011) by giving more emphasis to mathematical competencies, knowledge, and skills, as well as cross-curricular connections, because mathematical skills are more and more perceived as a basis for learning other school subjects. Connecting mathematics with other disciplines and solving authentic problems give mathematical learning real meaning because students feel the importance of building and upgrading mathematical concepts. At the same time, we introduce students to research and problem solving from everyday life. In this paper, we present and substantiate the cross-curricular approach to teaching mathematics which provides thinking at higher cognitive levels – the use, analysis, and generalization – while the subject approach builds and deepens the knowledge of each subject. Based on an educational experiment in which we introduced cross-curricular integration of mathematics with other school subjects, we ascertained that the students of the experimental group achieved better results at higher taxonomic levels in comparison to the students of the control group. We can conclude that cross-curricular integration in the taxonomic sense is a synthesis of knowledge of different disciplines which is reflected in a new level of integration of knowledge and understanding.

Peano- and Hilbert curve

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In this presentation, we give a historical survey on Hilbert's interpretation of the space-filling Peano-curve which Hilbert presented at the GDNÄ session in Bremen in 1890. Since Hilbert was systematically working in the field of number theory in that period, we try to explain the reason for his sudden interest and immediate reaction to Peano's surprising result from the same year which proved the possibility of a continuous surjective mapping of a line to a square. We argue that by means of his address, Hilbert was willingly trying to express his positive view on Cantor's set theory, an affinity which was also present by selecting the continuum hypothesis as the first problem on his list of the mathematical problems for the 20^{th} century and which lead his thinking also long afterwards in the 1920s while working on his so-called Hilbert's program in logic. We present also Hilbert's correspondence with H. Minkowski on this topic.

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Students' achievements in solving geometric problems using visual representations in a virtual learning environment

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The process of visualization is considered to be indispensable in mathematics learning, specifically in mathematical problem solving.

The role of visual representations is becoming increasingly important in technology-based learning. Researchers have presented a variety of frameworks for the optimal use of visual representations in complex, multimedia environments over the last two decades.

For the purpose of the study presented in this paper, we designed a model of learning geometry in a virtual learning environment with the use of different learning resources, dynamic geometry programs and applets, which fosters visualisation and the exploration of geometric concepts through the manipulation of interactive virtual representations. The instructional design incorporates Bruner's (1966) three-stage learning model, where learning activities are designed following a concrete-visual-abstract sequencing of instruction, starting with the concrete and progressing through visual to abstract representations.

We present the results of an experimental study aimed at exploring (1) whether geometry learning in a virtual learning environment (rich in various teaching material and activities, including dynamics geometry activities, and instructions) is reflected in higher student achievements in solving geometric problems and (2) whether solving geometric problems with the aid of visual representations contributes to higher student achievements.

Keywords: mathematical education, geometry, multiple representations, visualization, virtual learning environment

Minisymposia in

MATHEMATICS IN SCIENCE AND TECHNOLOGY

- Mathematics in biology and medicine (MS-35)

Minisymposium

MATHEMATICS IN BIOLOGY AND MEDICINE (MS-35)

Organized by Barbara Boldin, *University of Primorska, Slovenia*

- Solving the selection-recombination equation, *Ellen Baake*
- Phylogeny and population genetics: The mutation process on the ancestral line, *Enrico Di Gaspero*
- Lines of descent in a Moran model with frequency-dependent selection and mutation, *Luigi Esercito*
- Evolutionary escape and evolutionary suicide in host-pathogen systems, *Eva Kisdi*
- Mathematical Modelling of the impact of Quarantine and Isolation-based control interventions on the Transmission Dynamics of Lassa fever, *Chinwendu Emilian Madubueze*
- Impulsive multigroup SIS model for spread of modeling multidrug-resistant bacteriae, *Aleksandra Puchalska*
- Use of singular perturbation theory in the analysis of epidemic models, *Andrea Pugliese*
- Basic Reproduction Number for Conservation laws, *Jordi Ripoll*
- The impact of herd behavior on stochastic competition model, *Miljana Stanković*
- Stochastic Hepatitis C model - conditions for disease extinction, *Vuk Vujović*

Solving the selection-recombination equation

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The deterministic selection-recombination equation describes the evolution of the genetic type composition of a population under selection and recombination in a law of large numbers regime. So far, only the special case of three sites with selection acting on one of them has been treated, but only approximately and without an obvious path to generalisation. We use both an analytical and a probabilistic, genealogical approach for the case of an *arbitrary* number of neutral sites linked to one selected site. This leads to a recursive integral representation of the solution. Starting from a variant of the ancestral selection-recombination graph, we develop an efficient genealogical structure, which may, equivalently, be represented as a weighted partitioning process, a family of Yule processes with initiation and resetting, and a family of initiation processes. We prove them to be dual to the solution of the differential equation forward in time and thus obtain a stochastic representation of the deterministic solution, along with the Markov semigroup in closed form.

Phylogeny and population genetics: The mutation process on the ancestral line

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We consider a well-known observation at the interface of phylogeny and population genetics: mutation rates estimated via phylogenetic methods tend to be much smaller than direct estimates from pedigree studies. To understand this, we consider the Moran model with two types, mutation, and selection, and investigate the line of descent of a randomly-sampled individual from a contemporary population. We trace this ancestral line back into the distant past, far beyond the most recent common ancestor of the population (thus connecting population genetics to phylogeny) and analyze the mutation process along this line. We use a probabilistic tool, namely the pruned lookdown ancestral selection graph, which consists of the set of potential ancestors of the sampled individual at any given time. A crucial observation is that the mutation process on the ancestral line is not a Markov process by itself, but it becomes Markov when consindering a broader state space. Relative to the neutral case (that is, without selection), we obtain a general bias towards beneficial mutations. These results shed new light on previous analytical findings of Fearnhead (2002).

Lines of descent in a Moran model with frequency-dependent selection and mutation

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Dealing with the interplay of mutation and selection is one of the important challenges in population genetics. We consider two variants of the two-type Moran model with mutation and frequency-dependent selection, namely a scheme with nonlinear dominance (of the fit type) and another with what we name the *fittest-type-wins* scheme. We show the equivalence of the two variants and pursue the latter for further analysis. In particular, we trace the genealogy of a sample of individuals backward in time, via an appropriate version of the so-called *ancestral selection graph* (ASG), originally introduced by Krone and Neuhauser (1997). We use the information contained in mutation events to reduce the ASG to the parts that are informative with respect to the type distribution of the present population and their ancestors, respectively. This leads to the killed ASG and the pruned lookdown ASG in this setting, which we use to derive representations for the (factorial) moments of the type distribution and the ancestral type distribution; we do so by connecting forward and backward graphical models via duality relationships. Finally, we show how the results carry over to the diffusion limit.

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Evolutionary escape and evolutionary suicide in host-pathogen systems

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Can pathogens drive their hosts extinct? Can hosts evolutionarily escape their pathogens? The most basic models answer both questions in the negative. Here I review a number of possibilities how evolution may nevertheless lead to the extinction of the host or of the pathogen. Possible mechanisms driving extinction include Allee effects, population structure and conditional behaviour of the hosts. These models showcase examples of simple models exhibiting rich behaviour and leading to results previously thought to be unlikely.

Mathematical Modelling of the impact of Quarantine and Isolation-based control interventions on the Transmission Dynamics of Lassa fever

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Lassa fever is an acute viral hemorrhagic fever caused by the Lassa virus. It was first discovered in Nigeria in 1969 when two missionary nurses died of Lassa fever. Lassa fever infection is

endemic in West African countries such as Nigeria, Sierra Leone, Guinea, Ghana and other West Africa Countries. It infects 100,000 - 300,000 cases annually with approximately 5,000 deaths. About 15 - 20% of people hospitalized for Lassa fever died from the illness. There are few studies done on the modelling of Lassa fever transmission and control dynamics so far but none has considered the effect of quarantine and Isolation as control interventions against the transmission dynamics of Lassa fever. This paper studied the deterministic model of Lassa fever transmission dynamics with quarantine and isolation as control measures. The model is shown to be mathematically well-posed and epidemiologically meaningful. Detailed analyses (both qualitative and quantitative) were carried out to determine the equilibrium points, their stabilities, and the basic reproduction number necessary to control the Lassa fever transmission in the population. Numerical simulations were carried out to illustrate the analytical results.

Impulsive multigroup SIS model for spread of modeling multidrug-resistant bacteriae

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In the talk we present a system of SIS models coupled by impulses at fixed times in the way that it can describe the transfer of patients between healthcare facilities. The first aim for his considerations is to provide analytical background for numerical simulation of multidrug-resistant bacteriae spread based on admission/discharge data from insurance provider for Saxony and Thuringia (Germany) for years 2010 - 2016 considered in [1]. Dynamical properties developed at possibly high level of generality allow to examine statements hypothesize in [1] and can be easily applied for other settings.

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Use of singular perturbation theory in the analysis of epidemic models

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We analyse models of SIR and SIRS type [3], using singular perturbation theory exploiting the difference in time-scales between the infection period and the typical period of susceptible recruitment by new births or immunity loss. The systems are not in standard form with slow and fast equations, but it is still possible to divide the flow into slow and fast parts, and to use the entry-exit map [2] to characterize the exit from the fast flow. When applied to classical SIR and SIRS model, we recover through this method the well-known properties of convergence to equilibrium, but we also able to characterize the sequence of susceptible densities at which recurrent epidemics start. When applied to the SIRWS model, in which immunity of W (weakly immune) individuals can be boosted by encounter with infectives, we are able to prove, by joining analytical and numerical results, the existence of periodic solutions, already shown numerically in [1]. Finally, we studied an SIRS system resulting from pair approximation of a network model; although the system is 5-dimensional, the method allows to reduce the study of the system to that of a sequence of 2-dimensional maps; we are then able to prove existence of periodic solutions of the system when the degree of network nodes is sufficiently small.

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Basic Reproduction Number for Conservation laws

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The famous basic reproduction number R_0 plays a key-role in weighing birth/infection and death/recovery processes in population models. In this work we focus on continuously structured models (e.g. where individuals are characterized by age or size, spatial position, etc...) as given by conservation laws.

For constant and time periodic environments, R_0 is defined as the spectral radius of the so-called *next generation operator*, which let's say, maps a distribution of population to the distribution of population of their offspring along the whole life span of the former. Analogously in epidemic models, it maps a distribution of infected population to the distribution of their secondary cases.

On the one hand, for populations with concentrated state at birth, we have developed a systematic limit procedure to get to the basic reproduction number. Specifically, R_0 is computed as the limit of basic reproduction numbers of approximate models of populations with distributed state at birth but tending to concentrate (e.g. from an interval to a singleton). So our approach avoids the often computation of R_0 in a heuristically way. We give several examples obtaining explicit expressions.

On the other hand, for populations with distributed state at birth, the computation of the spectral radius of next generation operators poses, in general, serious obstacles to the effective and efficient determination of R_0. Either we have an explicit formula (e.g. for rank one operators) or we address this problem numerically via suitable reductions of the relevant operators to matrices, thus computing the sought quantity by solving generalized eigenvalue problems, possibly of large dimension. We also give several examples obtaining good results where in addition we proved the compactness of the corresponding next generation operator.

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The impact of herd behavior on stochastic competition model

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In this paper we extend the deterministic competition model with herd behavior and Allee effect by including environmental noise in the model. We prove that obtained stochastic model has positive, global and bounded solution. Conditions under which system is stable in some sense are established. We also apply our mathematical results to predict the time it takes for populations of certain species to reach their equilibrium state.

Stochastic Hepatitis C model - conditions for disease extinction

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Hepatitis C is an infectious liver disease caused by the Hepatitis C virus and transmitted exclusively by infected person's blood. There are many papers in the literature which consider spread of Hepatitis C. In some of them infected persons are divided in two classes: acute and chronic persons, but we construct our stochastic model on basis of the deterministic one which takes into account the isolation stage of infection, too. Hence, we obtain, five-state stochastic model by using system of stochastic differential equations of Ito type. This model better describes variability and uncertainty which may manifest through the contact between persons in population. Also stochastic model is constructed on such way, that inherits the disease free equilibrium point of deterministic model. In this presentation the conditions for coefficient of stochastic systems that provide stability in probability of disease free equilibrium state are shown. On the other words, it means, that under these conditions Hepatitis C will die out in population. Our

theoretical results show that reduction of contact rate and isolation of those with disease symptoms are the best measures for suppression of spread of infection. These theoretical results are also confirmed by numerical simulation.

Minisymposia in

NUMBER THEORY

- Number Theory (MS-63)

Minisymposium

NUMBER THEORY (MS-63)

Organized by Laura Paladino, *University of Calabria, Italy*
Coorganized by Jung Kyu Canc, *Hochschule Luzern - Technik and Architektur, Switzerland*

- Higher height paining and extensions of mixed Hodge structures, *Jose Ignacio Burgos Gil*
- Multiplicative and linear dependence in finite fields and on elliptic curves modulo primes, *Laura Capuano*
- On the variance of the nodal volume of arithmetic random waves, *Giacomo Cherubini*
- Composition series of a class of induced representations, *Igor Ciganović*
- An epsilon constant conjecture for higher dimensional representations, *Alessandro Cobbe*
- On the Galois module structure of integers of p-adic fields. The question of the minimal index, *Ilaria Del Corso*
- Uniformity for the Number of Rational Points on a Curve, *Philipp Habegger*
- Bounding the Iwasawa invariants of Selmer groups, *Sören Kleine*
- Zeros of Fekete polynomials, *Marc Munsch*
- Regulators of elliptic curves over global fields, *Fabien Pazuki*
- Kummer theory for number fields, *Antonella Perucca*
- Schinzel's Hypothesis (H) with probability 1 and random Diophantine equations, *Alexei Skorobogatov*
- GCD results on semiabelian varieties and a conjecture of Silverman, *Amos Turchet*
- Atkin-Lehner theory for Drinfeld Modular forms, *Maria Valentino*
- The field of iterates of a rational function, *Solomon Vishkautsan*

Higher height paining and extensions of mixed Hodge structures

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Coauthors: Souvik Joy, Greg Pearlstein

The height pairing between algebraic cycles over global fields is an important arithmetic invariant. It can be written as sum of local contributions, one for each place of the ground field. Following Hain, the Archimedean components of the height pairing can be interpreted in terms of biextensions of mixed Hodge structures. In this talk we will explore how to extend the Archimedean contribution of the height pairing to higher cycles in the Bloch complex and interpret it as an invariant associated to a mixed Hodge structure. This is joint work with S. Goswami and G. Pearlstein.

Multiplicative and linear dependence in finite fields and on elliptic curves modulo primes

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Coauthors: Fabrizio Barroero, Lazlo Mérai, Alina Ostafe, Min Sha

In 2008 Maurin proved that given n multiplicatively independent rational functions $\varphi_1(x), \ldots, \varphi_n(x) \in \mathbb{Q}(x)$, there are at most finitely many $\alpha \in \overline{\mathbb{Q}}$ such that $\varphi_1(\alpha), \ldots, \varphi_n(\alpha)$ satisfy two independent multiplicative relations. This statement is an instance of more general conjectures of *unlikely intersections* over tori made by Bombieri, Masser and Zannier and independently by Zilber. We consider a positive characteristic variant of this problem, proving that, for sufficiently large primes, the cardinality of the set of $\alpha \in \overline{\mathbb{F}}_p$ such that $\varphi_1(\alpha), \ldots, \varphi_n(\alpha)$ satisfy two independent multiplicative relations with exponents bounded by a certain constant K is bounded independently of K and p. We prove analogous results for products of elliptic curves and for split semiabelian varieties $E^n \times \mathbb{G}_m^k$.

On the variance of the nodal volume of arithmetic random waves

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Coauthor: Niko Laaksonen

We discuss arithmetic random waves on the d-dimensional torus $\mathbb{R}^d/\mathbb{Z}^d$. Their zero set and its volume are related to the study of linear correlations of lattice points on the sphere of radius $\sqrt{n}$ in $\mathbb{R}^d$ ($n \geq 2$). In this talk we look for bounds on the variance of the nodal volume. The problem has been solved (in the stronger form of an asymptotic with power saving) in dimension $d = 2, 3$ by using a combination of number theory and graph theory. In this seminar we will explain what is known in dimension $d \geq 4$. As the dimension increases, analysis gives the best results. The main input is a result that follows from the proof of the l^2-decoupling conjecture by Bourgain and Demeter.

Composition series of a class of induced representations

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We determine compostion series of a class of parabolically induced representation $\delta([\nu^{-b}\rho, \nu^{c}\rho]) \times \delta([\nu^{\frac{1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma$ of p-adic symplectic group in terms of Mœglin Tadić clasification. Here $\frac{1}{2} \leq a < b < c \in \mathbb{Z} + \frac{1}{2}$ are half integers, $\nu = |\det|_F$ where F is a p-adic field, ρ is a cuspidal representation of a general linear group, σ is a cuspidal representation of a p-adic symplectic group such that $\nu^{\frac{1}{2}}\rho \rtimes \sigma$ reduces and $\delta([\nu^{x}\rho, \nu^{y}\rho]) \hookrightarrow \nu^{y}\rho \times \cdots \times \nu^{x}\rho$ is a discrete series representation for $x \leq y \in \mathbb{Z} + \frac{1}{2}$.

An epsilon constant conjecture for higher dimensional representations

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Coauthor: Werner Bley

The equivariant local epsilon constant conjecture was formulated in various forms by Fontaine and Perrin-Riou, Benois and Berger, Fukaya and Kato and others. If N/K is a finite Galois extension of p-adic fields and V a p-adic representation of G_K, then the above conjecture describes the epsilon constants attached to V in terms of the Galois cohomology of T, where T is a G_K-stable $\mathbb{Z}_p$-sublattice T such that $V = T \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$.

Here we will discuss the case when N/K is at most weakly ramified (this includes the case of tame ramification) and $T = \mathbb{Z}_p^r(\chi^{\mathrm{cyc}})(\rho^{\mathrm{nr}})$, i.e. the $\mathbb{Z}_p$-module $\mathbb{Z}_p^r$ with the trivial action of G_K twisted by the cyclotomic character and by an unramified representation $\rho^{\mathrm{nr}} : G_K \to \mathrm{Gl}_r(\mathbb{Z}_p)$. The main results generalize previous work by Izychev, Venjakob, Bley and the author. This is a joint work with Werner Bley.

On the Galois module structure of integers of p-adic fields. The question of the minimal index

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Coauthors: Fabio Ferri, Davide Lombardo

Let L/K be a Galois extension with Galois group G. The Normal Basis Theorem shows that L is a free $K[G]$-module of rank 1. When L/K is a number field or a local field extension, it is natural to consider the question of determining the structure of the ring of integers $\mathcal{O}_L$ as a $\mathcal{O}_K[G]$-module. It is well-known that $\mathcal{O}_L$ contains free $\mathcal{O}_K[G]$-submodules of finite index, but, in general, it is not free.

In this talk, after a brief overview of the main classical results in this context, I will present some recent results on the minimal index of a free $\mathcal{O}_K[G]$-submodule into $\mathcal{O}_L$, in the case of p-adic fields.

Uniformity for the Number of Rational Points on a Curve

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Coauthors: Ziyang Gao, Vesselin Dimitrov

In 1983, Faltings proved the Mordell Conjecture: a smooth projective curve of genus at least 2 that is defined over a number field K has at most finitely many K-rational points. Several years later Votja gave a new proof. Neither proof provides a procedure to determine the set of rational points, they are ineffective. But the number of rational points can be bounded from above effectively with bounds given by Bombieri, David-Philippon, de Diego, Parshin, Rémond, Vojta, and others. I discuss a result where we show that the number of K-rational points is bounded from above as a function of K, the genus, and the rank of the Mordell-Weil group of the curve's Jacobian. This is joint work with Vesselin Dimitrov and Ziyang Gao and our proof is based on Vojta's approach. Thanks to earlier work by other authors mentioned above, we may reduce to bounding the number of points in a certain height range. For this we develop an inequality for the Néron-Tate height in a family of abelian varieties and use a recent functional transcendence result of Gao.

Bounding the Iwasawa invariants of Selmer groups

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Let p be a rational prime. After recalling some basic notation from Iwasawa theory, we study the growth of p-primary Selmer groups of abelian varieties with good and ordinary reduction at p in $\mathbb{Z}_p$-extensions of a fixed number field K. Proving that in many situations the knowledge of the Selmer groups in a sufficiently large number of finite layers of a $\mathbb{Z}_p$-extension of K suffices for bounding the over-all growth, we relate the Iwasawa invariants of Selmer groups in different $\mathbb{Z}_p$-extensions of K.

Zeros of Fekete polynomials

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The study of the location of zeros of polynomials with coefficients constrained in different sets has a very rich history. The case of random polynomials has been studied intensively and the asymptotic number of real zeros has been computed in various cases (Gaussian, Bernoulli etc). We investigate related questions in the deterministic family of Fekete polynomials. These are constructed with coefficients being Legendre symbols and are related to the study of zeros of real Dirichlet L- functions. We discuss previous results, conjectures and the progress we made towards the understanding of real zeros in this family of polynomials.

This is a joint work with O. Klurman and Y. Lamzouri.

Regulators of elliptic curves over global fields

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In a recent collaboration with Pascal Autissier and Marc Hindry, we prove that up to isomorphisms, there are at most finitely many elliptic curves defined over a fixed number field, with Mordell-Weil rank and regulator bounded from above, and rank at least 4. We will explain how to obtain an even stronger result in the case of elliptic curves defined over a function field of positive characteristic, in particular removing the conditions on the rank (while adding a necessary assumption on the inseparability degree). This opens up interesting questions about surfaces.

Kummer theory for number fields

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University of Luxembourg, Luxembourg

Let K be a number field and let G be a finitely generated and torsion-free subgroup of $K^\times$ of rank r. I will present various results (which are joint work with Hörmann, Perissinotto, Sgobba, and Tronto) concerning the cyclotomic-Kummer extensions $K(\zeta_N, \sqrt[n]{G})$ where $n \mid N$. For example there is an explicit finite procedure to compute a positive integer C (depending only on G and K) such that the ratio between n^r and the degree of the Kummer extension $K(\zeta_N, \sqrt[n]{G})/K(\zeta_N)$ divides C. For some families of number fields I will also present concrete strategies to compute all of the above degrees.

Schinzel's Hypothesis (H) with probability 1 and random Diophantine equations

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In a joint work with Efthymios Sofos it was proved that Schinzel's Hypothesis (H) holds for 100% of polynomials of any fixed degree. In the talk I will discuss applications of this analytic result to proving that among surfaces in specific families over $\mathbb{Q}$, a positive proportion have rational points. The main examples are diagonal conic bundles of any fixed degree over $\mathbb{P}^1_{\mathbb{Q}}$ and generalised Châtelet equations.

GCD results on semiabelian varieties and a conjecture of Silverman

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A divisibility sequence of polynomials is a sequence d_n such that whenever m divides n one has that d_m divides d_n. Results of Ailon and Rudnick, among others, have shown that pairs of divisibility sequences corresponding to subgroups of the multiplicative group have only limited common factors. Silverman conjectured that a similar behaviour should appear in (a large class of) all algebraic groups. We show, extending work of Ghioca-Hsia-Tucker and Silverman on elliptic curves, how to prove Silverman's conjecture over function fields for abelian and split semi-abelian varieties and some generalizations.

Atkin-Lehner theory for Drinfeld Modular forms

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Let $S_k(N)$ be the space of cusp forms of level $\Gamma_0(N)$ with $N \in \mathbb{Z}$. Atkin-Lehner theory deals with the notion of oldforms, namely those coming from a lower level $M|N$, and newforms, i.e. the orthogonal complement of the space of oldforms with respect to the Petersson inner product. Moreover, it is also concerned with the construction of a basis for $S_k(N)$ made up by eigenfunctions for the Hecke operators $\mathbf{T}_n$ with n prime to N. In this talk we shall present some recent advances on the analogous theory for Drinfeld modular forms, which are certain analogues over the function field $\mathbb{F}_q[T]$ of classical modular forms.

The field of iterates of a rational function

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Coauthor: Francesco Veneziano

We will discuss the field of definition of a rational function and in what ways it can change under iteration, in particular when the degree over the base field drops. We present examples of families of rational functions with the property above, and prove that in the special case of polynomials, only one of these families is possible. We also explain how this relates to Ritt's decomposition theorem on polynomials. Joint work with Francesco Veneziano (SNS Pisa).

Minisymposia in

Numerical Analysis and Scientific Computing

- EU-MATHS-IN: mathematics for industry in Europe (MS-66)
- Matrix Computations and Numerical (MS-47)
- Modeling, approximation, and analysis of partial differential equations involving singular source terms (MS-39)

Minisymposium

EU-MATHS-IN: MATHEMATICS FOR INDUSTRY IN EUROPE (MS-66)

Organized by Wil Schilders, *Eindhoven University of Technology, The Netherlands*
Coorganized by Christophe Prud'homme, *University of Strasbourg, France*

Simulation, optimal management and infrastructure planning of gas transmission networks

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Coauthors: Alfredo Bermúdez, Diego Rodríguez-Martínez, Adolfo Núñez-Fernández, Gabriel Capeáns-García, Damián Pallas-Carrillo, Mohsen Shabani

In this talk we plan to present the results of an ongoing collaboration with a Spanish company in the gas industry, for which we have developed a software, GANESO, that simulates and optimizes gas transmission networks. A gas transmission network consists basically of emission and consumption points, compressors and valves that are connected via pipes. The natural gas flows along the pipes, but the friction with the walls of the pipes decreases the pressure of the gas. At the demand points, the gas has to be delivered with a certain pressure, so it is necessary to counterbalance the pressure loss in the pipes using compressor stations. Yet, compressor stations operate consuming part of the gas that flows through the pipes, so it is important to manage the gas network in an efficient way to minimize such consumption. In order to tackle this problem in the steady-state setting, the methodology we have followed consists of implementing a slight variation of the standard sequential linear programming algorithms that can easily accommodate integer variables, combined with a control theory approach. Further, we have developed a simulator for the transient case, based on well-balanced finite volume methods for general flows and finite element methods for isothermal models. It is worth mentioning that we have also recently added two new features to GANESO in order to deal with energy coupling issues. On the one hand, GANESO was extended to be able to manage heterogeneous gas mixtures in the same network, including Hydrogen-rich mixtures. On the other hand, in order to integrate gas and electricity energy systems, a new simulation/optimization framework was developed for assessing the interdependency of both networks guaranteeing the security of supply of the whole system. In our collaboration we have also studied other related problems to gas networks, such as the allocation of gas losses, the infrastructure planning under uncertainty and the computation of tariffs for networks access according to the different methodologies proposed in EU directives. The developed software, along with the support and consulting analysis provided by the research group, has allowed the company to improve its strategic positioning in the gas sector.

Hydrodynamic load on coupled “ship” – “breast dolphin” system using a conformal mapping approach

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When a ship docks at a breast dolphin structure, such as a single pile flexible dolphin, one question remains unanswered: How do the hydrodynamic loads of the ships contribute to the deformation of the dolphins? One approach is the application of the potential flow theory and the section-by-section approximation of the ship geometry. In the literature, the method is usually referred to as "2.5D flow theory". The hydrodynamic loads of each section can now be calculated using the conformal mapping approach from circular to ship cross-section geometry. The proposed method is an improvement over Ursell’s method, which uses a circular cylinder.

We will show how a method is derived and applied to the case of a tanker berthing at the liquid jetty in the port of Koper. The results will discuss the dynamics of a coupled system.

Suboptimal scheduling of a fleet of AGVs to serve online requests

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In the talk we consider an online problem in which a fleet of vehicles must serve transportation requests arriving over time. We propose a dynamic scheduling strategy, which continuously updates the running schedule each time a new request arrives, or a vehicle completes a request. The vehicles move on the edges of a mixed graph modeling the transportation network of a workshop. Our strategy is complete in the sense that deadlock avoidence is guaranteed, and all requests get served. The primary objective is to minimize the total tardiness of serving the requests, and the secondary objective is to minimize the total distance traveled. We provide qualitative results and a comparison to another complete strategy from the literature. The main novelty of our approach is that we optimize the schedule to get better results, and we measure solution quality which is very rare in the AGV literature.

Overview of HU-MATHS-IN, the Hungarian Service Network of Mathematics for Industry and Innovation

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Inspired by the creation of EU-MATHS-IN, HU-MATHS-IN was established in 2013. Initially, the activities of the Hungarian network were based on the decades-long industrial collaborations of the national mathematical research groups. Since 2017 its activities are intensified by an EU and Hungary-supported project (EFOP 3.6.2-16-2017-00015) in which best practices of the EU-MATHS-IN national networks were analysed and some of them adapted. The main results of the project include, among others, the following ones:

- more than 50 short-term aimed basic research projects with industry,
- two collaborative projects of the HU-MATHS-IN research groups,
- national one-stop-shop for services to industry with mathematical technologies.

HU-MATHS-IN has been participating in the EU-MATHS-IN activities strongly, e.g. with succesfull research and innovation actions projects within the H2020 framework program.

In this talk an overview of the organization of the network and its activities will be given. In addition, some short-term industrial projects of HU-MATHS-IN will be overviewed from the fields of computational acoustics and model order reduction for compressible fluids, with mathematical details.

Quantitative Comparison of Deep Learning-Based Image Reconstruction Methods for Low-Dose and Sparse-Angle CT Applications

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Over the last years, deep learning methods have significantly pushed the state-of-the-art results in applications like imaging, speech recognition and time series forecasting. This development also starts to apply to the field of computed tomography (CT). In medical CT, one of the main goals lies in the reduction of the potentially harmful radiation dose a patient is exposed to during the scan. Depending on the reduction strategy, such low-dose measurements can be more noisy or starkly under-sampled. Hence, achieving high quality reconstructions with classical methods like filtered back-projection (FBP) can be challenging, which motivated the investigation of a number of deep learning approaches for this task.

With the purpose of comparing such approaches fairly, we evaluate their performance on fixed benchmark setups. In particular, two CT applications are considered, for both of which large datasets of 2D training images and corresponding simulated projection data are publicly available: a) reconstruction of human chest CT images from low-intensity data, and b) reconstruction of apple CT images from sparse-angle data.

In order to include a large variety of methods in the comparison, we organized open challenges for either task. Our current study comprises results obtained with popular deep learning approaches from various categories, like post-processing, learned iterative schemes and fully learned inversion. The test covers image quality of the reconstructions, but also aspects such as the required data or model knowledge and generalizability to other setups are considered. For reproducibility, both source code and reconstructed images are made publicly available.

MACSI and industrial mathematics in Ireland

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MACSI was established on foot of a Science Foundation Ireland grant so-called mathematics initiative grant in 2006. As a result, a fulltime business manager was employed, an industrial consultancy was established, and European study groups were brought to Ireland for the first time. Building on this, we have recently established the first Centre for Research Training in mathematics in the country (also supported by Science Foundation Ireland).

Combining advanced mathematical methods, IoT and High-Performance Computing to optimize energy in existing buildings

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At a time of mass awareness of the impacts of climate change, mainly due to the increase in carbon emissions, reducing and controlling energy consumption are major challenges for the future. Taking up this challenge will help to contain the runaway rise in climate change. This partnership between Cemosis, the platform for collaboration between mathematics and industry at University of Strasbourg, and the company Synapse-Concept will speed up the identification of sources of energy savings in existing buildings by means of clustered calculations and will enable the effectiveness of the technical solutions proposed by architects and engineers to be virtually tested before any improvement work is undertaken. In this talk, we present an overview of the mathematical and computational framework — coupling modeling simulation data assimilation, reduced order modeling and high performance computing — as well as some numerical results

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An overview of the transfer activity in Spain. Coordination with EU-MATHS-IN initiatives

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Spanish Network for Mathematics & Industry, Spain

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In this talk, the most relevant aspects of the transfer of Mathematics in Spain will be presented. In particular, the detail of the initiatives carried out to promote the transfer, some indicators on the impact of this activity in the Spanish productive landscape, as well as some relevant success stories of collaboration Academy - Industry will be introduced. Attention will also be paid to the coordination of math-in activities with those promoted by EU-MATHS-IN in order to multiply the potential of both structures facilitating transfer.

Optimizing the routes of mobile agents in a network using a multiobjective travelling salesman model

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This work is a result of the consortium formed by the company Smartgeo Solutions, Lda., in the role of leading developer, and by the University of Coimbra. The main goal of the project was the development of an innovative Geographic Information System application, which operates on a Web platform. This application should incorporate a functionality that allows to optimise, according to multiple criteria, the routes of mobile agents in a network. The problem was modelled using a multicriteria version of the travelling salesman problem to obtain the efficient routes. Several new heuristics were proposed whose performances were evaluated and compared with classical approaches. It was also proposed a new dominance criteria relation which allows to speed up the algorithms to search efficient solutions. The proposed algorithms allowed us to explore new solutions that had not been found with classic ones.

An overview of EU-MATHS-IN and its activities

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EU-MATHS-IN, the European Service Network of Mathematics for Industry and Innovation, was established in November 2013 by the European Mathematical Society (EMS) and the European Consortium of Mathematics for Industry (ECMI). It is a unique network of national networks, currently with 21 countries on board. Since its foundation, EU-MATHS-IN has been active to spread best practices, organise events and issue reports. It has set up close contacts with the EU. In 2017, an Industrial Core Committee was established which has representatives of important industries in Europe. In this talk, we will go into more detail concerning the activities of EU-MATHS-IN, as well as on future plans.

An overview of the activities of the french network for mathematics and entreprises, France

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From 2011, AMIES (French Agency for Mathematics in Interaction with Enterprises and the Society) acts to developp and make more visible the collaborations between mathematical research and companies in France. In the context of the digital transformation of companies and Industry 4.0' revolution, AMIES' provides financial support and environment to promote industrial mathematics. We have identified more than 600 French companies calling upon French mathematical research, of which nearly 300 have been supported by AMIES funding. During the talk, we will present the mechanisms put in place by AMIES and some examples of collaborations and mathematical services for companies.

Minisymposium

MATRIX COMPUTATIONS AND NUMERICAL (MS-47)

Organized by Stefano Pozza, *Charles University, Czech Republic*
Coorganized by Davide Palitta, *Max Planck Institute for Dynamics of Complex Technical Systems, Germany*

- On the convergence of the Jacobi-type method for computing orthogonal tensor decomposition, *Erna Begović*
- Inverses of k-Toeplitz matrices for resonator arrays with multiple receivers, *Jose Brox*
- A μ-mode-based integrator for solving evolution equations in Kronecker form, *Marco Caliari*
- Random multi-block ADMM: an ALM based view for the Quadratic Programming case, *Stefano Cipolla*
- A Lanczos-like algorithm for time-ordered exponentials, *Pierre Louis Giscard*
- Parallel Newton-Chebyshev Polynomial Preconditioners for the Conjugate Gradient method, *Ángeles Martínez*
- Infinite Tensor Rings, *Lana Periša*
- Core reduction: Necessary and sufficient information in linear approximation problems, *Martin Plešinger*
- On the numerical solution of certain linear multiterm matrix equations and applications, *Valeria Simoncini*
- Riemannian thresholding methods for row-sparse and low-rank matrix recovery, *André Uschmajew*

On the convergence of the Jacobi-type method for computing orthogonal tensor decomposition

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Tensor decompositions are a central problem of numerical multilinear algebra. For a general third-order tensors $\mathcal{A} \in \mathbb{R}^{n\times n\times n}$ we are looking for its SVD-like decomposition

$$\mathcal{A} = \mathcal{S} \times_1 U \times_2 V \times_3 W,$$

where U, V, W are orthogonal $n \times n$ matrices and $\mathcal{S}$ is an $n \times n \times n$ tensor such that

$$\|\text{diag}(\mathcal{S})\|^2 = \sum_{i=1}^{n} \mathcal{S}_{iii}^2 \to \max .$$

To obtain this decomposition we are using the alternating least squares approach and a Jacobi-type method. The algorithm works on $2 \times 2 \times 2$ subtensors. In each iteration the sum of the squares of two diagonal entries is maximized using Jacobi rotations. We show how the rotation angles are calculated and prove the convergence of the algorithm. Moreover, we discuss different initializations of the algorithm.

Inverses of k-Toeplitz matrices for resonator arrays with multiple receivers

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Coauthor: José Alberto

Wireless power transfer systems allow to avoid electrical contact and transfer power in rough environments with water, dust or dirt. They are used in electrical vehicle and mobile devices charging, biomedical devices powering, etc. But they have a drawback: in case of misalignment or distance from the transmitter to the receiver, the efficiency and power transmitted can drop abruptly. To overcome this inconvenience, arrays of resonators arranged in a plane are used to transfer power over longer distances through magnetic coupling, with receivers placed over the array to absorb the power transmitted. In the literature, these arrays have been examined using magnetoinductive wave theory or through the circuit analysis of the array, however considering only arrays with one receiver placed over them. Here we present the study of arrays with multiple receivers for which an arbitrary pattern of receivers is repeated over every k resonators. In this case, the impedance matrix representing the circuit is tridiagonal with equal upper and lower diagonals and periodic main diagonal of period k. We show how to invert those matrices, by computing their determinants through linear recurrence relations and then using them to compute the minors appearing in the cofactor matrix. In this way we are able to provide rational formulas for the currents, power transmission and efficiency of the system.

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A μ-mode-based integrator for solving evolution equations in Kronecker form

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In this talk, we propose a μ-mode integrator for computing the solution of stiff evolution equations. The integrator is based on a d-dimensional splitting approach and uses exact (usually precomputed) one-dimensional matrix exponentials. We show that the action of the exponentials, i.e. the corresponding matrix-vector products, can be implemented efficiently on modern computer systems. We further explain how μ-mode products can be used to compute spectral transformations efficiently even if no fast transform is available. We illustrate the performance of the new integrator by solving three-dimensional Schrödinger equations, and we show that the μ-mode integrator can significantly outperform numerical methods well established in the field. This is a joint work with Fabio Cassini, Lukas Einkemmer, Alexander Ostermann, and Franco Zivcovich.

Random multi-block ADMM: an ALM based view for the Quadratic Programming case

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Coauthor: Jacek Gondzio

Embedding randomization procedures in the Alternating Direction Method of Multipliers (ADMM) has recently attracted an increasing amount of interest as a remedy to the fact that the direct multi-block generalization of ADMM is not necessarily convergent. Even if, in practice, the introduction of such techniques could *mitigate* the diverging behaviour of the multi-block extension of ADMM, from the theoretical point of view, it can ensure just the *convergence in expectation*, which may not be a good indicator of its robustness and efficiency. In this work, analysing the strongly convex quadratic programming case, we interpret the block Gauss-Seidel sweep performed by the multi-block ADMM in the context of the inexact Augmented Lagrangian Method. Using the proposed analysis, we are able to outline an alternative technique to those present in literature which, supported from stronger theoretical guarantees, is able to ensure the convergence of the multi-block generalization of the ADMM method.

A Lanczos-like algorithm for time-ordered exponentials

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The time-ordered exponential is defined as the function that solves a system of coupled first-order linear differential equations with generally non-constant coefficients. In spite of being at the heart of much system dynamics, control theory, and model reduction problems, the time-ordered exponential function remains elusively difficult to evaluate. In this presentation we will present a Lanczos-like algorithm capable of evaluating it by producing a tridiagonalization of the original differential system. In addition, we will show that such tridiagonalization is always feasible with piecewise-smooth functions and will completely characterize algorithmic breakdowns. We will then discuss numerical implementations of the Lanczos-like procedure and present its extensions to systems of linear and non-linear partial differential equations.

Parallel Newton-Chebyshev Polynomial Preconditioners for the Conjugate Gradient method

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Discretization of PDEs modeling different processes and constrained/unconstrained optimization problems often require the repeated solution of large and sparse linear systems $A\mathbf{x} = \mathbf{b}$. The size of these system can be of order $10^6 \div 10^9$ and this calls for the use of iterative methods, equipped with ad-hoc preconditioners as accelerators running on a parallel computing environment. In most cases, the huge size of the matrices involved prevents their complete storage. In these instances only the application of the matrix to a vector is available as a routine (*matrix -free regime*). Differently from direct factorization methods, iterative methods do not need the explicit knowledge of the coefficient matrix. The issue is the construction of a preconditioner which also work in a matrix-free regime. Polynomial preconditioners, i.e. preconditioners that can be expressed as $P_k(A)$, are very attractive for several reasons i.e. their construction is only theoretical, namely only the coefficients of the polynomial are to be computed with negligible computational cost, the application of $P_k(A)$ requires a number, k, of matrix-vector products so that they can be implemented in a matrix-free regime, and the eigenvectors of the preconditioned matrix are the same as those of A.

We consider polynomial preconditioners to accelerate the Conjugate Gradient method in the solution of large symmetric positive definite linear systems in massively parallel environments. We put in connection a specialized Newton method to solve the matrix equation $X^{-1} = A$ [1] and the Chebyshev polynomials for preconditioning. We propose a simple strategy to avoid clustering of the extremal eigenvalues in order to speed-up convergence. Numerical results on very large linear systems (up to 8 billion unknowns) in a parallel environment show the efficiency of the proposed class of preconditioners.

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Infinite Tensor Rings

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While creating a flexible power method for computing the leftmost, i.e., algebraically smallest, eigenvalue of an infinite dimensional tensor eigenvalue problem, $Hx = \lambda x$, where the infinite dimensional symmetric matrix H exhibits a translational invariant structure, we study the theory of infinite Tensor Rings (iTRs). Under the assumption that the smallest eigenvalue of H is simple, representing the eigenvector as a translational invariant iTR allows the use of power iteration of e^{-H}. In order to implement this power iteration, we use a small parameter t so that the infinite matrix-vector operation e^{-Htx} can efficiently be approximated by the Lie product formula, also known as Suzuki–Trotter splitting. In this talk we further explain the motivation for defining iTRs and present their derived and used mathematical properties.

Core reduction: Necessary and sufficient information in linear approximation problems

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Technical University of Liberec, Czech Republic
Coauthors: Iveta Hnětynková, Jana Žáková

We focus on linear approximation problems $Ax \approx b$, where A is a given matrix, x an unknown vector, and the given right-hand side vector b is not in the range of A, i.e., $b \notin R(A)$. By solving of such problem we usually mean replacing it by some minimization. Typically the least squares (LS) techniques can be used. We focus on the so-called total least squares (TLS) minimization

$$\min \|[g, E]\|_F \quad \text{s.t.} \quad (b + g) \in R(A + E).$$

TLS has been studied since the early eighties. The trouble there is, contrary to the standard LS, that the minimization may not have a solution for the given (A, b).

The theory of core problem introduced in 2006 by Paige and Strakoš brings a concept of necessary and sufficient information for solving the TLS minimization. This concept allows us to distinguish cases having and not having the TLS solution. Moreover, core problem theory clearly explains why it happens. In recent years the core problems thoery has been applied on several other linear problems $A(X) = B$ where the linear mapping A as well as the right-hand side B can have some particular structure.

On the numerical solution of certain linear multiterm matrix equations and applications

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Linear matrix equations have arisen as the natural algebraic form associated with the discretization of a growing number of application problems. The case where the unknown matrix appears

in at most two additive terms in the equation has been extensively studied, and satisfactory solution strategies have been developed for various classes of problems, both in the small and large scale cases. The general multiterm setting is regarded as far more complicated, and only recently practical solution methods have been proposed. In this talk we discuss new procedures that can take advantage of certain algebraic properties of the coefficient matrices, yielding effective algorithms both in terms of computational time and memory requirements.

Part of this work is joint with Yue Hao, School of Mathematics and Statistics, Lanzhou University, (PRC).

Riemannian thresholding methods for row-sparse and low-rank matrix recovery

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The problem of recovering a jointly row-sparse and low-rank matrix from linear measurements arises for instance in sparse blind deconvolution. The ideal goal is to ensure recovery using only a minimal number of measurements with respect to the combined constraints. We present modifications of the iterative hard thresholding (IHT) method for this task. In particular a Riemannian version of IHT is considered which significantly reduces computational cost of the gradient projection in the case of rank-one measurements. We also consider a Riemannian proximal gradient method for the special case of unknown sparsity. This is joint work with H. Eisenmann, F. Krahmer and M. Pfeffer.

Minisymposium

MODELING, APPROXIMATION, AND ANALYSIS OF PARTIAL DIFFERENTIAL EQUATIONS INVOLVING SINGULAR SOURCE TERMS (MS-39)

Organized by Luca Heltai, *International School for Advanced Studies, Italy*
Coorganized by Wenyu Lei, *International School for Advanced Studies, Italy*

- Local error estimates for the discretization of elliptic problems with Dirac source term, *Silvia Bertoluzza*
- Advances on fictitious domain approach for fluid-structure interaction problems, *Daniele Boffi*
- Multiscale coupling of one-dimensional vascular models and elastic tissues, *Alfonso Caiazzo*
- Some remarks on a two phase problem in the Heisenberg group, *Fausto Ferrari*
- Finite element approximation of Stokes equations with non-smooth data, *Lucia Gastaldi*
- A priori error estimates of regularized elliptic problems, *Wenyu Lei*
- Regularizations of the Dirac delta distribution, and applications, *Nilima Nigam*
- Projection in negative norms and the regularization of rough linear functionals, *Sergio Rojas*
- Analysis and approximation of fluids under singular forcing, *Abner Salgado*
- Discontinuous Galerkin Discretisations for Problems with Dirac Delta Source, *Thomas Wihler*
- Regularity and finite element approximation for two-dimensional elliptic equations with line Dirac sources, *Peimeng Yin*
- Analysis and approximation of mixed-dimensional PDEs on 3D-1D domains coupled with Lagrange multipliers, *Paolo Zunino*

Local error estimates for the discretization of elliptic problems with Dirac source term

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It is well known that, as the solution of elliptic problems with a Dirac measure right-hand side in dimension ≥ 2 is not H^1, the convergence of the finite element solutions is suboptimal in the L^2-norm. The solution is, however, smooth whenever we are far away from the singular source term and we can therefore hope for optimal convergence rate in subregions which are disjoint from a neighbourhood of the singularity. In this work we consider problems where the right hand side is, in dimension 3, the Dirac measure along a curve and, in dimension 2, the punctual Dirac measure. We show a quasi optimal convergence in the H^s-norm, for $s \geq 1$ on subdomains which exclude a neighbourhood of the singularity; in the particular case of Lagrange finite elements, an optimal convergence in the H^1-norm is shown on a family of quasi uniform meshes. Our results are obtained using local Nitsche and Schatz-type error estimates, a weak version of Aubin-Nitsche duality lemma and a discrete inf-sup condition. This is a joint work with A. Decoene, Loïc Lacouture and Sébastien Martin.

Advances on fictitious domain approach for fluid-structure interaction problems

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We review a numerical scheme based on a fictitious domain approach for the modeling and approximation of the interaction of fluids and solids. A crucial aspect consists in the choice of the finite element spaces that need to satisfy a suitable compatibility condition. In this talk we discuss the theoretical aspects and we highlight some implementation details.

Multiscale coupling of one-dimensional vascular models and elastic tissues

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Coauthors: Luca Heltai, Lucas O. Müeller

We present a computational multiscale model for the efficient simulation of vascularized tissues, composed of an elastic three-dimensional matrix and a vascular network. The effect of blood vessel pressure on the elastic tissue is surrogated via hyper-singular forcing terms in the elasticity equations, which depend on the fluid pressure. In turn, the blood flow in vessels is treated as a one-dimensional network. The pressure and velocity of the blood in the vessels are simulated using a high-order finite volume scheme, while the elasticity equations for the tissue are solved using a finite element method.

This work addresses in particular the potential of the multiscale model for reproducing the tissue response at the effective scale (of the order of millimeters) while modeling the vasculature at the microscale. We validate the multiscale method against a full scale (three-dimensional) model and present as well simulation results obtained with the proposed approach in a realistic scenario, demonstrating that the method can robustly and efficiently handle the one-way coupling between complex fluid microstructures and the elastic matrix.

Some remarks on a two phase problem in the Heisenberg group

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We will discuss about the existence of an Alt-Caffarelli-Friedman monotonicity formula for a two phase problem associated with a Bernoulli type functional in the Heisenberg group.

Finite element approximation of Stokes equations with non-smooth data

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The lid driven cavity flow is a well known model used frequently to test the finite element approximation of the Stokes problem. Actually, this model does not meet the regularity requirements for the boundary datum which is only in L^2, so that it cannot be the trace of the velocity which belongs to H^1. In this talk, we analyze the finite element approximation of the Stokes equations with nonsmooth Dirichlet boundary data. To define the discrete solution we first approximate the boundary datum by a smooth one and then apply a standard finite element method to the regularized problem. We prove almost optimal order error estimates for two regularization procedures in the case of general data in fractional order Sobolev spaces, and for the Lagrange interpolation (with appropriate modifications at the discontinuities) for piece- wise smooth data. Our results apply in particular to the classic lid-driven cavity problem improving the existing error estimates.

Finally, we introduce and analyze an a posteriori error estimator. We prove its reliability and efficiency, and show some numerical examples which suggest that optimal order of convergence is obtained by an adaptive procedure based on our estimator.

The results reported in this talk have been obtained, in collaboration with Ricardo Duràn and Ariel Lombardi.

A priori error estimates of regularized elliptic problems

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Coauthor: Luca Heltai

Approximations of the Dirac delta distribution are commonly used to create sequences of smooth functions approximating nonsmooth (generalized) functions, via convolution. We show a-priori rates of convergence of this approximation process in standard Sobolev norms, with minimal regularity assumptions on the approximation of the Dirac delta distribution. The application of these estimates to the numerical solution of elliptic problems with singularly supported forcing terms allows us to provide sharp H^1 and L^2 error estimates for the corresponding regularized problem. As an application, we show how finite element approximations of a regularized immersed interface method result in the same rates of convergence of its non-regularized counterpart, provided that the support of the Dirac delta approximation is set to a multiple of the mesh size, at a fraction of the implementation complexity. Numerical experiments are provided to support our theories.

Regularizations of the Dirac delta distribution, and applications

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Coauthors: John Stockie, Bamdad Hosseini

Nilima Nigam Regularizations of the Dirac delta distribution, and applications.' The need to approximate singular sources is widespread in numerical analysis. In this talk, we present a historical overview of, and a framework for constructing approximations of the Dirac delta distribution. As part of this framework we study their convergence in suitable topologies. This in turn allows us to examine the consistency error incurred in their use while numerically solving PDEs. We present numerical experiments in which these ideas are illustrated. This work was inspired by notable previous works on approximation of singular source terms, and is joint with Bamdad Hosseini and John Stockie.

Projection in negative norms and the regularization of rough linear functionals

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Coauthors: Felipe Millar, Ignacio Muga, Kris van der Zee

Rough linear functionals (such as Dirac Delta distributions) often appear on the right-hand side of variational formulations of PDEs. As they live in negative Sobolev spaces, they dramatically affect adaptive finite element procedures to approximate the solution of a given PDE.

In this talk, we propose an alternative that, in a first step, computes a projection of the rough functional over piecewise polynomial spaces, up to a given desired precision in a negative norm sense. The projection (being Lp-regular) can be used as the right-hand side of a regularized problem for which adaptive Galerkin methods perform better. A complete error analysis of the proposed methodology will be shown, together with numerical experiments.

Analysis and approximation of fluids under singular forcing

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Motivated by applications, like modeling of thin structures immersed in a fluid, we develop a well posedness theory for Newtonian and some non-Newtonian fluids under singular forcing in Lipschitz domains, and in convex polytopes. The main idea, that allows us to deal with such forces, is that we study the problem in suitably weighted Sobolev spaces. We develop an

a priori approximation theory, which requires to develop the stability of the Stokes projector over weighted spaces. In the case that the forcing is a linear combination of Dirac deltas, we develop a posteriori error estimators for the stationary Stokes and Navier Stokes problems. We show that our estimators are reliable and locally efficient, and illustrate their performance within an adaptive method. We briefly comment on ongoing work regarding the Bousinessq system. Numerical experiments illustrate and complement our theory.

Discontinuous Galerkin Discretisations for Problems with Dirac Delta Source

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Coauthor: Paul Houston

We investigate the symmetric interior penalty discontinuous Galerkin (SIPG) scheme for the numerical approximation of linear second-order elliptic PDE with Dirac delta right-hand side. We outline both an a priori and a (residual-type) a posteriori error analysis on the error measured in terms of the L^2–norm. Moreover, some computational results will be presented. Finally, a brief outlook on an inf-sup theory in weighted Sobolev spaces will be given.

Regularity and finite element approximation for two-dimensional elliptic equations with line Dirac sources

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We study the elliptic equation with a line Dirac delta function as the source term subject to the Dirichlet boundary condition in a two-dimensional domain. Such a line Dirac measure causes different types of solution singularities in the neighborhood of the line fracture. We establish new regularity results for the solution in a class of weighted Sobolev spaces and propose finite element algorithms that approximate the singular solution at the optimal convergence rate. Numerical tests are presented to justify the theoretical findings.

Analysis and approximation of mixed-dimensional PDEs on 3D-1D domains coupled with Lagrange multipliers

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Coauthors: Miroslav Kuchta, Kent-Andre Mardal

Coupled partial differential equations (PDEs) defined on domains with different dimensionality are usually called mixed-dimensional PDEs. We address mixed-dimensional PDEs on three-dimensional (3D) and one-dimensional (1D) domains, which gives rise to a 3D-1D coupled problem. Such a problem poses several challenges from the standpoint of existence of solutions and numerical approximation. For the coupling conditions across dimensions, we consider the combination of essential and natural conditions, which are basically the combination of Dirich-

let and Neumann conditions. To ensure a meaningful formulation of such conditions, we use the Lagrange multiplier method suitably adapted to the mixed-dimensional case. The well-posedness of the resulting saddle-point problem is analyzed. Then, we address the numerical approximation of the problem in the framework of the finite element method. The discretization of the Lagrange multiplier space is the main challenge. Several options are proposed, analyzed, and compared, with the purpose of determining a good balance between the mathematical properties of the discrete problem and flexibility of implementation of the numerical scheme. The results are supported by evidence based on numerical experiments. The application of this technique to microcirculation and the modeling of subsurface wells will be discussed.

Minisymposia in

Optimization and Control

- Analysis, Control and Inverse Problems for Partial Differential Equations (MS-22)

Minisymposium

Analysis, Control and Inverse Problems for Partial Differential Equations (MS-22)

Organized by Francesca Bucci, *Università degli Studi di Firenze, Italy*
Coorganized by Barbara Kaltenbacher, *Alpen-Adria-Universität Klagenfurt, Austria*

- Optimal Control in Poroelasticity, *Lorena Bociu*
- The Calderón problem with corrupted data, *Pedro Caro*
- Stable determination of polygonal and polyhedral interfaces from boundary measurements, *Elisa Francini*
- Boundedness in Total Variation regularization, *Gwenael Mercier*
- Existence and regularity of weak solutions for a fluid interacting with a non-linear shell in $3D$, *Boris Muha*
- Analysis of finite-element based discretizations in nonlinear acoustics, *Vanja Nikolić*
- Asymptotic behavior of dispersive electromagnetic waves in bounded domains, *Cristina Pignotti*
- Exponential dynamical Luenberger observers for nonautonomous parabolic-like equations, *Sergio Rodrigues*
- Optimal control problem for a repulsive chemotaxis system, *María Ángeles Rodríguez-Bellido*
- Strong unique continuation at the boundary in linear elasticity and its connection with optimal stability in the determination of unknown boundaries, *Edi Rosset*
- Weak Solutions for an Implicit, Degenerate Poro-elastic Plate System, *Justin Webster*

Optimal Control in Poroelasticity

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Coauthor: Sarah Strikwerda

In this talk we address optimal control problems subject to fluid flows through deformable, porous media. In particular, we focus on quadratic poroelasticity control problems, with both distributed and boundary controls, and prove existence and uniqueness of optimal control. Furthermore, we derive the first order necessary optimality conditions. These problems have important biological and biomechanical applications. For example, optimizing the pressure of the flow and investigating the influence and control of pertinent biological parameters are relevant in ocular tissue perfusion and its relation to the development of ocular neurodegenerative diseases such as glaucoma.

The Calderón problem with corrupted data

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The inverse Calderón problem consists in determining the conductivity inside a medium by electrical measurements on its surface. Ideally, these measurements determine the Dirichlet-to-Neumann map and, therefore, one usually assumes the data to be given by such map. This situation corresponds to having access to infinite-precision measurements, which is unrealistic. In this talk, I will consider the Calderón problem assuming data to contain measurement errors and provide formulas to reconstruct the conductivity and its normal derivative on the surface (joint work with Andoni García). I will also present similar results for Maxwell's equations (joint work with Ru-Yu Lai, Yi-Hsuan Lin, Ting Zhou). When modelling errors in these two different frameworks, one realizes the existence of certain freedom that yields different reconstruction formulas. To understand the whole picture of what is going on, we will rewrite the problem in a different setting, which will bring us to analyse the observational limit of wave packets with noisy measurements (joint work with Cristóbal J. Meroño).

Stable determination of polygonal and polyhedral interfaces from boundary measurements

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I will present some results concerning Lipschitz stability in the determination of polygonal and polyhedral inclusions from the Dirichlet to Neumann map.

Boundedness in Total Variation regularization

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In this talk, we investigate boundedness and convergence of total variation regularized linear inverse problems. We present a simple proof of boundedness of the minimizer for fixed regularization parameter, and derive the existence of uniform bounds for small enough noise under a source condition and adequate a priori parameter choices. We present a few (counter)examples. This is a joint work wirth K. Bredies (Graz) and José A. Iglesias (RICAM, Linz).

Existence and regularity of weak solutions for a fluid interacting with a non-linear shell in $3D$

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Coauthor: Sebastian Schwarzacher

We study the unsteady incompressible Navier-Stokes equations in three dimensions interacting with a non-linear flexible shell of Koiter type. We study weak solutions to the corresponding fluid-structure interaction (FSI) problem. The known existence theory for weak solutions is extended to non-linear Koiter shell models. We introduce a-priori estimates that reveal higher regularity of the shell displacement beyond energy estimates. These are essential for non-linear Koiter shell models, since such shell models are non-convex (w.r.t. terms of highest order). The estimates are obtained by introducing new analytical tools that allow to exploit dissipative effects of the fluid for the (non-dissipative) solid. The regularity result depends on the geometric constitution alone and is independent of the approximation procedure; hence it holds for arbitrary weak solutions. The developed tools are further used to introduce a generalized Aubin-Lions type compactness result suitable for fluid-structure interactions.

Analysis of finite-element based discretizations in nonlinear acoustics

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Nonlinear effects can be easily observed in sound waves with sufficiently large amplitudes. The nonlinearity will be apparent even sooner in high-frequency waves because these effects accumulate over the distance measured in wavelengths. This makes high-intensity ultrasonic waves inherently nonlinear. Their many applications range from non-invasive surgery to industrial welding and motivate the mathematical investigation into nonlinear acoustics.

In this talk, we will discuss the a priori analysis of finite-element-based discretizations of nonlinear acoustic equations. In particular, we will focus on the conforming and (hybrid) discontinuous Galerkin discretizations in space for acoustic equations with nonlinearities of quadratic type, such as the Westervelt and Kuznetsov equations. These are quasilinear strongly damped wave equations that serve as classical models of sound propagation through thermoviscous fluids and gases. The general approach in the a priori error analysis combines the stability and convergence analysis of their linearizations with the Banach fixed-point theorem. Numerical experiments will illustrate the theoretical results.

The talk is based on joint research with Paola F. Antonietti, Ilario Mazzieri (Politecnico di Milano), Markus Muhr, and Barbara Wohlmuth (TU Munich).

Asymptotic behavior of dispersive electromagnetic waves in bounded domains

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Coauthor: Serge Nicaise

We analyze the stability of Maxwell equations in bounded domains taking into account electric and magnetization effects. Well-posedness of the model is obtained by means of semigroup theory. Then, a passitivity assumption guarantees the boundedness of the associated semigroup. Under suitable sufficient conditions, we prove the exponential or polynomial decay of the energy. Moreover, some illustrative examples are presented.

Exponential dynamical Luenberger observers for nonautonomous parabolic-like equations

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The estimation of the full state of a nonautonomous semilinear parabolic equation is achieved by a Luenberger type dynamical observer. The estimation is derived from an output given by a finite number of average measurements of the state on small regions. The state estimate given by the observer converges exponentially to the real state, as time increases. The result is semiglobal in the sense that the error dynamics can be made stable for an arbitrary given initial condition, provided a large enough number of measurements, depending on the norm of the initial condition, is taken. The output injection operator is explicit and involves a suitable oblique projection. The results of numerical simulations are presented showing the exponential stability of the error dynamics.

Optimal control problem for a repulsive chemotaxis system

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Coauthors: Francisco Guillén-González, Exequiel Mallea-Zepeda

The chemotaxis phenomenon can be understood as the directed movement of living organisms in response to chemical gradients. Keller and Segel [5] proposed a mathematical model that describes the chemotactic aggregation of cellular slime molds. These molds move preferentially towards relatively high concentrations of a chemical substance secreted by the amoebae themselves. Such mechanism is called *chemo-attraction* with production. However, when the regions of high chemical concentration generate a repulsive effect on the organisms, the phenomenon is called *chemo-repulsion*.

In this work, we want to study an optimal control problem for the (repulsive) Keller-Segel model and a bilinear control acting on the chemical equation in a $2D$ and $3D$ domains. The system can be written as:

$$\left\{\begin{array}{rcll} \partial_t u - \Delta u - \nabla \cdot (u\,\nabla v) & = & 0 & \text{in } \Omega \times (0,T), \\ \partial_t v - \Delta v + v & = & u + f\,v\,1_{\Omega_c} & \text{in } \Omega \times (0,T), \\ \partial_{\mathbf{n}} u = \partial_{\mathbf{n}} v & = & 0 & \text{on } \partial\Omega \times (0,T), \\ u(0,\cdot) = u_0 \geq 0, \quad v(0,\cdot) & = & v_0 \geq 0 & \text{in } \Omega, \end{array}\right. \tag{1}$$

being $f : Q_c := (0,T) \times \Omega_c \to \mathbb{R}$ (the control) with $\Omega_c \subset \Omega \subset \mathbb{R}^n$ ($n = 2,3$) the control domain, and the state $u, v : Q := (0,T) \times \Omega_c \to \mathbb{R}_+$ the celular density and chemical concentration, respectively. Here, $\mathbf{n}$ is the outward unit normal vector to $\partial\Omega$.

The existence and uniqueness of global in time weak solution (u,v) for the uncontrolled system is known (see for instance [1, 4]).

In this work we study an optimal control problem subject to a chemo-repulsion system with linear production term, and in which a bilinear control acts injecting or extracting chemical substance on a subdomain of control $\Omega_c \subset \Omega$. Existence of weak solutions are stablished (in the $3D$ case by using a regularity criterion), and, as a consequence, a global optimal solution together with first-order optimality conditions for local optimal solutions are deduced.

The results presented in this talk are based on [2, 3].

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Strong unique continuation at the boundary in linear elasticity and its connection with optimal stability in the determination of unknown boundaries

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Quantitative estimates of Strong Unique Continuation at the boundary for solutions to the isotropic Kirchhoff-Love plates subject to Dirichlet conditions, and for solutions to the Generalized plane stress problem subject to Neumann conditions are presented. These results have been applied to prove optimal stability estimates for the inverse problem of determining unknown boundaries.

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Weak Solutions for an Implicit, Degenerate Poro-elastic Plate System

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Coauthor: Elena Gurvich

We consider a recent plate model obtained as a scaled limit of the three dimensional quasi-static Biot system of poro-elasticity. The result is a “2.5" dimensional linear system that couples traditional Euler-Bernoulli plate dynamics to a pressure equation in three dimensions, where diffusion acts only transversely. Motived by application, we allow the permeability function to be time-dependent, making the problem non-autonomous and disqualifying much of the standard theory. Weak solutions are defined and the problem is framed abstractly as an implicit, degenerate evolution problem:

$$[Bp]_t + A(t)p = S.$$

Existence is obtained, and uniqueness follows under additional hypotheses on the temporal regularity of the permeability. Time permitting, we address the inertial case with constant permeability by way of semigroup theory.

Minisymposia in

Partial Differential Equations and Applications

- Analysis of PDEs on Networks (MS-26)
- CA18232: Variational Methods and Equations on Graphs (MS-40)
- Geometric-functional inequalities and related topics (MS-23)
- Harmonic Analysis and Partial Differential Equations (MS-28)
- Higher-order evolution equations (MS-43)
- Mathematical analysis: the interaction of fluids/ viscoelastic materials and solids (MS-36)
- Multiscale Modeling and Methods: Application in Engineering, Biology and Medicine (MS-80)
- Nonlocal operators and related topics (MS-55)
- Nonsmooth Variational Methods for PDEs and Applications in Mechanics (MS-8)
- PDE models in life and social sciences (MS-71)
- Partial differential equations describing far-from-equilibrium open systems (MS-51)
- Topics in sub-elliptic and elliptic PDEs (MS-31)

Minisymposium

ANALYSIS OF PDES ON NETWORKS (MS-26)

Organized by Simone Dovetta, *CNR - Centro IMATI, Italy*
Coorganized by Lorenzo Tentarelli, *Università degli Studi di Napoli Federico II, Italy*

- Towards nonlinear hybrids: the planar NLS with point interactions, *Riccardo Adami*
- Ground states of the NLSE with standard and delta nonlinearities on star graphs, *Filippo Boni*
- Bound states for nonlinear Dirac equations on metric graphs with localized nonlinearities, *William Borrelli*
- Dynamics and scattering of truncated coherent states on the star-graph in the semiclassical limit, *Claudio Cacciapuoti*
- On the nonlinear Dirac equation on noncompact metric graphs, *Raffaele Carlone*
- Macroscopic traffic flow models on road networks, *Paola Goatin*
- First order Mean Field Games on networks, *Claudio Marchi*
- Spectral minimal partitions on metric graphs, and applications, *Delio Mugnolo*
- The quintic NLS on the tadpole graph, *Diego Noja*
- On Pleijel's nodal domain theorem for quantum graphs, *Marvin Plümer*
- Discontinuous ground states for the NLSE on $\mathbb{R}$ with a *Fülöp-Tsutsui* δ interaction, *Alice Ruighi*
- Local minimizers in absence of ground states for the critical NLS energy on metric graphs, *Nicola Soave*
- Initial-boundary value problems for transport equations in one space dimension with very rough coefficients, *Laura V. Spinolo*

Towards nonlinear hybrids: the planar NLS with point interactions

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A natural, but not straightforward, generalization of networks is provided by hybrids, namely systems made by gluing together pieces of different dimension. The simplest hybrid is made of a plane connected to a halfline. Suitable matching conditions at the contact point are to be imposed in order to define a well-posed dynamics, and this procedure leads to studying propagation of waves in the presence of point interactions in the plane, subject to a possibly nonlinear dynamics. We discuss the case of the Nonlinear Schroedinger Equation, where the nonlinearity is either self-consistent, or external and concentrated at a point in the plane, morally the connection point of the plane and the halfline. This is a joint project with Filippo Boni, Raffaele Carlone, and Lorenzo Tentarelli.

Ground states of the NLSE with standard and delta nonlinearities on star graphs

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We study the existence of ground states at fixed mass of a Schrödinger equation on star graphs with two subcritical power-type nonlinear terms: a pointwise one, located at the vertex of the graph, and a standard one. We show that existence and non-existence results strongly depend on the interplay between the two nonlinearities. In particular, we see that when one nonlinearity prevails the other, existence of ground states depends both on the mass and on the number of halflines in the graph, whereas if the two nonlinearities are in a specific balance, then existence of ground states is only determined by the number of halflines of the graph. This is a joint work with R. Adami and S. Dovetta.

Bound states for nonlinear Dirac equations on metric graphs with localized nonlinearities

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Coauthors: Raffaele Carlone, Lorenzo Tentarelli

The investigation of evolution equations on metric graphs has become very popular nowadays, as they represent effective models for the dynamics of physical systems confined in branched spatial domains. In particular, the Dirac operator

$$\mathcal{D} := -\imath c\sigma_1 \frac{d}{dx} + mc^2\sigma_3, \, , \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

on metric graphs has attracted a growing interest for the description of systems where confined particles exhibit a ‘relativistic behavior’. Here $m > 0$ is the mass of the particle whose (effective) hamiltonian is given by (1) and $c > 0$ is a phenomenological parameter, playing the role of the speed of light. In this talk, I will discuss nonlinear Dirac equations with localized nonlinearity, namely

$$\mathcal{D}\psi - \chi_{\mathcal{K}}|\psi|^{p-2}\,\psi = \omega\psi\,, \qquad \psi : \mathcal{G} \to \mathbb{C}^2\,, \quad p > 2\,, \tag{2}$$

where $\mathcal{G}$ is a metric graph with finitely many edges and $\mathcal{K} \neq \emptyset$ is its *compact core*, i.e. the set of bounded edges, and $\chi_{\mathcal{K}}$ is its characteristic function. The reduction to this simplified model arises if one assumes that the nonlinearity affects only the compact core of the graph. This idea was originally exploited in the case of Schrödinger equation in and it represents a preliminary step toward the investigation of the case with the "extended" nonlinearity.

Considering the operator endowed with *Kirchoff-type* vertex conditions, we proved that (2) possesses infinitely many solutions with $\omega \in (-mc^2, mc^2)$, converging, after a suitable renormalization, in H^1-sense to solutions to the analogous Schrödinger equation $-u'' - \chi_{\mathcal{K}}|u|^{p-2}u = \lambda u$ for some $\lambda < 0$., in the *non-relativistic limit* as $c \to +\infty$.

Dynamics and scattering of truncated coherent states on the star-graph in the semiclassical limit

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Università degli Studi dell'Insubria, Italy

Coauthors: Davide Fermi, Andrea Posilicano

We consider the dynamics of a quantum particle of mass m on the star-graph constituted by n half-lines with a common origin. The generator of the dynamics is the Hamiltonian $H_K = -(2m)^{-1}\hbar^2\Delta$ with Kirchhoff conditions in the vertex, $\hbar$ is the reduced Planck constant. Our aim is to obtain the semiclassical limit of the quantum evolution, generated by H_K, of an initial state resembling a coherent state (gaussian packet) concentrated on one of the edges of the graph. Due to the Kirchhoff conditions in the vertex, the corresponding classical dynamics on the graph cannot be described by Hamilton-Jacobi equations. For this reason, we define the classical dynamics through a Liouville operator on the graph, obtained by means of the Krein's theory of singular perturbations of self-adjoint operators. For the same class of initial states, we study also the semiclassical limit of the wave and scattering operators for the couple H_K and H_D, where H_D is the free Hamiltonian with Dirichlet conditions in the vertex.

On the nonlinear Dirac equation on noncompact metric graphs

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Università Federico II Napoli, Italy

Coauthors: William Borrelli, Lorenzo Tentarelli

We discuss the Nonlinear Dirac Equation, with Kerr-type nonlinearity, on non-compact metric graphs with a finite number of edges, in the case of Kirchhoff-type vertex conditions. We will present results about the well-posedness for the associated Cauchy problem in the operator domain and, for infinite N-star graphs, and the existence of standing waves.

Macroscopic traffic flow models on road networks

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Inria, France

The talk will review the main macroscopic models of vehicular traffic flow on networks, focusing on the description of dynamics at junctions. I will then show some optimal control application, as well as some recent developments accounting for the impact of modern navigation systems.

First order Mean Field Games on networks

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Coauthors: Yves Achdou, Paola Mannucci, Nicoletta Tchou

The theory of Mean Field Games studies the asymptotic behaviour of differential games (mainly in terms of their Nash equilibria) as the number of players tends to infinity. In these games, the players are rational and indistinguishable: each player aims at choosing its trajectory so to minimize a cost which depends on the trajectory itself and on the distribution of the whole population of agents. We focus our attention on deterministic Mean Field Games with finite horizon in which the states of the players are constrained in a network (in our setting, a network is given by a finite collection of vertices connected by continuous edges which cannot self-intersect). In these games, an agent can control its dynamics and has to pay a cost formed by a running cost depending on the evolution of the distribution of all agents and a terminal cost depending on the distribution of all agents at terminal time. As in the Lagrangian approach, we introduce a relaxed notion of Mean Field Games equilibria and we shall deal with probability measures on trajectories on the network instead of probability measures on the network. This is a joint work with: Y. Achdou (Univ. of Paris), P. Mannucci (Univ. of Padova) and N. Tchou (Univ. of Rennes).

Spectral minimal partitions on metric graphs, and applications

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University of Hagen, Germany

There exist several possibilities of partitioning graphs or manifolds, including those based on Cheeger cuts or nodal domains. We present a different approach that elaborates on a theory developed in the last 15 years, among others, by Bonnaillie-Noël, Helffer, Hoffmann-Ostenhof, and Terracini. While these authors focus on domains, we are going to discuss the partitioning of metric graphs in terms of spectral quantities of the associated Laplacian.

We introduce a well-defined class of spectral partitions of metric graphs and show some of their features. While the complicated topology of metric graphs prevents us from recovering all results that hold for domains, new remarkable features also arise.

This is joint work with Matthias Hofmann, James Kennedy, Pavel Kurasov, Corentin Léna, Marvin Plümer.

The quintic NLS on the tadpole graph

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Coauthor: Dmitry Pelinovsky

The tadpole graph consists of a circle and a half-line attached at a vertex. We analyze standing waves of the nonlinear Schrödinger equation with quintic power nonlinearity and Kirchhoff boundary conditions at the vertex. The profile of a standing wave with frequency $\omega \in (-\infty, 0)$ is characterized as a global minimizer of the quadratic part of energy constrained to the unit sphere in L^6. The set of standing waves so defined strictly includes the set of ground states, i.e. the global minimizers of the energy at constant mass (L^2-norm), but it is actually wider. While ground states exist only for a certain interval of masses, the above standing waves exist for every $\omega \in (-\infty, 0)$ and correspond to a bigger interval of masses. It is proven that there exist critical frequencies ω_1 and ω_0 with $-\infty < \omega_1 < \omega_0 < 0$ such that the standing waves are the ground state for $\omega \in [\omega_0, 0)$, local constrained minima of the energy for $\omega \in (\omega_1, \omega_0)$ and saddle points of the energy at constant mass for $\omega \in (-\infty, \omega_1)$.
Joint work with D.E. Pelinovsky.

On Pleijel's nodal domain theorem for quantum graphs

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FernUniversität in Hagen, Germany

In this talk we present recent results on metric graph counterparts of Pleijel's theorem on the asymptotics of the number of nodal domains ν_n of the n-th eigenfunction(s) of a broad class of operators on compact metric graphs, including Schrödinger operators with L^1-potentials as well as the p-Laplacian with natural vertex conditions, and without any assumptions on the lengths of the edges, the topology of the graph, or the behaviour of the eigenfunctions at the vertices. We characterise the accumulation points of the sequence $(\frac{\nu_n}{n})_{n\in\mathbb{N}}$, which are shown to form a finite subset of $(0, 1]$. This extends the previously known result that $\nu_n \sim n$ *generically*, for certain realisations of the Laplacian, in several directions. In particular, we will see that the existence of accumulation smaller than 1 is strictly related to the failure of the unique continuation principle on metric graphs.

Finally we show that for (most) metric graphs – metric trees and general metric graphs with at least one Dirichlet vertex – there exists an infinite sequence of *generic* eigenfunctions – namely, eigenfunctions of multiplicity 1 that do not vanish in the graph's vertices – of the free Laplacian and infer that, in this case, 1 is always an accumulation point of $\frac{\nu_n}{n}$. In order to do so we introduce a new type of secular function.

The talk is based on joint work with Matthias Hofmann (Lisbon), James Kennedy (Lisbon), Delio Mugnolo (Hagen) and Matthias Täufer (Hagen).

Discontinuous ground states for the NLSE on $\mathbb{R}$ with a *Fülöp-Tsutsui δ* interaction

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We analyse the existence and the stability of the ground states of the one-dimensional nonlinear Schrödinger equation with a focusing power nonlinearity and a defect located at the origin. A ground state is intended as a global minimizer of the action functional on the Nehari's manifold and the defect considered is a Fülöp-Tsutsui δ type, namely a δ condition that allows discontinuities. The existence of ground states is proved by variational techniques, while the stability results follow from the Grillakis-Shatah-Strauss' theory.
This is a joint work with Riccardo Adami.

Local minimizers in absence of ground states for the critical NLS energy on metric graphs

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Coauthors: Dario Pierotti, Gianmaria Verzini

We consider the mass-critical nonlinear Schrödinger equation on non-compact metric graphs. A quite complete description of the structure of the ground states, which correspond to global minimizers of the energy functional under a mass constraint, has been recently provided by R. Adami, E. Serra and P. Tilli (*Comm. Math. Phys.* 352, no.1, 387-406, 2017). They proved that existence and properties of ground states depend in a crucial way on both the value of the mass, and the topological properties of the underlying graph. In this talk I present some results regarding cases when ground states do not exist and show that, under suitable assumptions, constrained local minimizers of the energy do exist. This result paves the way to the existence of stable solutions in the time-dependent equation in cases where the ground state energy level is not achieved.

Initial-boundary value problems for transport equations in one space dimension with very rough coefficients

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IMATI-CNR, Italy
Coauthors: Elio Marconi, Simone Dovetta

I will discuss new existence, uniqueness and regularity propagation results for solutions of transport equations defined in one-dimensional domains with boundaries. The only assumptions imposed on the coefficient are boundedness and near incompressibility, which means that the coefficient supports a nonnegative and bounded density. This analyis is motivated by applications to a source-destination model for traffic flows on road networks. The talk will be based on joint work with Simone Dovetta and Elio Marconi.

Minisymposium

CA18232: Variational Methods and Equations on Graphs (MS-40)

Organized by Aleksandra Puchalska, *University of Warsaw, Poland*
Coorganized by Delio Mugnolo, *University of Hagen, Germany*

- Telegraph systems on networks and port-Hamiltonians, *Jacek Banasiak*
- Gibbs Evolution Families, *András Bátkai*
- On transmission conditions in modeling equations on graphs., *Adam Bobrowski*
- Semigroups for flows on limits of graphs, *Christian Budde*
- Uniqueness and non-uniqueness of prescribed mass NLS ground states on metric graphs, *Simone Dovetta*
- Convergence to equilibrium of stochastic semigroups and an application to buffered networks flows, *Jochen Glück*
- Trace formulas for general Hermitian matrices: a scattering approach on their associated graphs, *Sven Gnutzmann*
- Kinetic and macroscopic diffusion models for gas mixtures in the context of respiration, *Bérénice Grec*
- Mathematical modeling of traffic flow, *Helge Holden*
- Hidden symmetries in non-self-adjoint graphs, *Amru Hussein*
- Spectral geometry of quantum graphs via surgery principles, *James Kennedy*
- Flows in infinite networks, *Marjeta Kramar Fijavž*
- Semilinear evolution problems in fractal domains, *Maria Rosaria Lancia*
- Stability and asymptotic properties of dissipative equations coupled with ordinary differential equations, *Serge Nicaise*
- Self-adjoint extensions of infinite quantum graphs, *Noema Nicolussi*
- Nonlinear models of kinetic type: On the Cauchy problem and Banach space regularity for Boltzmann flows of monatomic gas mixtures, *Milana Pavić Čolić*
- Stochastic completeness of graphs: curvature and volume growth, *Radoslaw Wojciechowski*

Telegraph systems on networks and port-Hamiltonians

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Coauthor: Adam Błoch

In this talk we consider a system of linear hyperbolic differential equations on a network coupled through general transmission conditions of Kirchhoff's type at the nodes. We discuss the reduction of such a problem to a system of 1-dimensional hyperbolic problems, also called port-Hamiltonian, for the associated Riemann invariants and provide a semigroup theoretic proof of its well-posedness in any L_p.

In the second part of the talk we consider a reverse question, that is, we derive conditions under which such a port-Hamiltonian with general linear Kirchhoff's boundary conditions can be written as a system of 2×2 hyperbolic equations on a metric graph Γ. This is achieved by interpreting the matrix of the boundary conditions as a potential map of vertex connections of Γ and then showing that, under the derived assumptions, that matrix can be used to determine the adjacency matrix of Γ.

Gibbs Evolution Families

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Pädagogische Hochschule Vorarlberg, Austria

We extend results of Zagrebnov and coauthors on Gibbs semigroups to the nonautonomous case.

Sufficient conditions will be given to evolution equations in the parabolic case so that the generated evolution family belogs to appropriate operator ideals. Applications to Schrödinger operators and the trace class ideal will be also given.

The results were achieved in part in a joint work with Bálint Farkas (Wuppertal). Support by the COST Action CA18232 - Mathematical models for interacting dynamics on networks is acknowledged.

On transmission conditions in modeling equations on graphs

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I will discuss the role of transmission conditions in modeling equations on graph-like spaces, by considering two recent examples. The first of these is a theorem on thin layer approximation, the other is a simple kinetic model with interface.

Semigroups for flows on limits of graphs

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Transport of goods is nowadays of extreme importance and indispensable considering what mankind needs for daily life. Now imagine a start-up company shipping special goods all over the world. Of course, the company starts with a small network of customers. However, assuming the company grows and retains the already existing routes and customers, the ship network grows and grows. It might come to the point in the development of the company, that one actually lost the view on all specific routes but only knows how the network works since it becomes too big. However, one still wants to know how the transport is going on the whole network.

That a network is growing, through adding vertices and edges, means that one has a sequence of graphs and each graph is a subgraph of the subsequent graph in the sequence, describing the above mentioned situation of growing networks. We will describe the transition from finite to infinite graphs by means of direct limits in a certain category. The approximation of the transport process on the direct limit graph, is done by a version of the (first) Trotter–Kato approximation theorem, which is originally due to T. Kato [2, Chapter IX, Thnm. 3.6] and H.F. Trotter [3, Thm. 5.2 & 5.3] and modified by a version by K. Ito and F. Kappel [1, Thm. 2.1]. We will also present an extension the work of Ito and Kappel which is related to the well-known second Trotter–Kato theorem.

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[1] K. Ito and F. Kappel. The Trotter-Kato theorem and approximation of PDEs. *Math. Comp.*, 67(221):21–44, 1998.

[2] T. Kato. *Perturbation theory for linear operators*. Springer-Verlag, Berlin-New York, second edition, 1976. Grundlehren der Mathematischen Wissenschaften, Band 132.

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Uniqueness and non-uniqueness of prescribed mass NLS ground states on metric graphs

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This talk addresses the problem of uniqueness of ground states of prescribed mass for the Nonlinear Schrödinger Energy with power nonlinearity on noncompact metric graphs with half–lines. We first show that, up to an at most countable set of masses, all ground states at given mass solve the same equation, that is the Lagrange multiplier appearing in the NLS equation is constant on the set of ground states of mass μ. On the one hand, we apply this result to prove uniqueness of ground states on two specific families of noncompact graphs. On the other hand, we construct a graph that admits at least two ground states with the same mass having different Lagrange multipliers. This shows that the result for Lagrange multipliers is sharp in general, in the sense that one cannot get rid of the at most countable set of masses where it may fail without further assumptions. Our proofs are based on careful variational arguments and rearrangement techniques, and hold both for the subcritical regime $p \in (2, 6)$ and in the critical case $p = 6$. This is a joint work with Enrico Serra and Paolo Tilli.

Convergence to equilibrium of stochastic semigroups and an application to buffered networks flows

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University of Passau, Germany
Coauthor: Florian Martin

The long-time behaviour of flows on finite metric graphs is known to depend heavily on the network topology. Depending on this topology, the lengths of the edges and the flow velocities, the flow might converge or behave asymptotically periodic as $t \to \infty$.

In this talk we show that the situation changes if we introduce a mass buffer in at least one of the vertices. Such a buffer has a smoothing effect on the flow and thus enforces convergence as $t \to \infty$. As we consider finite graphs only, one would even expect that the convergence is uniform (i.e., with respect to the operator norm over the L^1-space over the graph). In order to prove that this is indeed true we employ a novel characterisation of operator norm convergence to equilibrium for stochastic C_0-semigroups.

Trace formulas for general Hermitian matrices: a scattering approach on their associated graphs

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Coauthor: Uzy Smilansky

Two trace formulas for the spectra of arbitrary Hermitian matrices are presented. In either case the one associates a unitary scattering matrix to the given Hermitian matrix H such that the unitary matrix depends on the spectral parameter. In the first type the unitary matrix is obtained by exponentiation. The new feature in this case is that the spectral parameter appears in the final form as an argument of Eulerian polynomials—thus connecting the periodic orbits to combinatorial objects in a novel way. To obtain the second type, one expresses the input in terms of a unitary scattering matrix in a larger Hilbert space. One of the surprising features here is that the locations and radii of the spectral discs of Gershgorin's theorem appear naturally as the pole parameters of the scattering matrix. Both formulas are discussed and possible applications are outlined.

Kinetic and macroscopic diffusion models for gas mixtures in the context of respiration

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University of Paris, France

In this talk, I will first discuss shortly the context of respiration, and in particular the need to describe accurately the diffusion of respiratory gases in the lower part of the lung. At the macroscopic level, diffusion processes for mixtures are often modelled using cross-diffusion models. In order both to determine the regime in which such models are valid, and to compute the binary diffusion coefficients, it is of particular interest to derive these models from a description at the mesoscopic level by means of kinetic equations.

More precisely, we consider the Boltzmann equations for mixtures with general cross-sections (i.e. for any kind of molecules interactions), and obtain the so-called Maxwell-Stefan equations by performing a Hilbert asymptotic expansion at low Knudsen and Mach numbers. This allows us to compute the values of the Maxwell-Stefan diffusion coefficients with explicit formulae with respect to the cross-sections. We also justify the specific ansatz we use thanks to the so-called moment method.

This is a joint work with Laurent Boudin and Vincent Pavan.

Mathematical modeling of traffic flow

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Norwegian University of Science and Technology, Norway

Traditionally, there are two types of mathematical models for vehicular traffic, namely the Follow-the-Leader (FtL) models and the continuum models, using variants of the classical Lighthill–Whitham–Richards (LWR) models. In the FtL models individual vehicles are tracked, and this leads to a system of ordinary differential equations. On the other hands, in LWR models, traffic is represented by the density of vehicles, and the resulting equation is a first order hyperbolic conservation law. We will study the continuum limit of the FtL model when traffic becomes dense. We will also mention the problem of modeling traffic flow on networks.

In the second part of the talk, we will discuss a novel model for multi-lane traffic within the LWR framework. The talk is based on joint work with Nils Henrik Risebro (University of Oslo).

Hidden symmetries in non-self-adjoint graphs

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TU Kaiserslautern, Germany

On finite metric graphs Laplace operators subject to general non-self-adjoint matching conditions imposed at graph vertices are considered. A regularity criterium related to the Cayley transform of boundary conditions is discussed and spectral properties of such regular operators are investigated, in particular similarity transforms to self-adjoint operators and generation of C_0-semigroups. Concrete examples are discussed exhibiting that non-self-adjoint boundary conditions can yield to unexpected spectral features.

The talk is based on joint works with David Krejčiřík (Czech Technical University in Prague), Petr Siegl (Queen's University Belfast) and Delio Mugnolo (FernUniversität Hagen).

Spectral geometry of quantum graphs via surgery principles

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Coauthors: Gregory Berkolaiko, Pavel Kurasov, Delio Mugnolo

"Surgery" on a (metric) graph means making a small, generally local, change to its structure: for example, joining two vertices, lengthening an edge, or maybe removing an edge and reinserting it somewhere else.

We will introduce a number of sharp new surgery principles which allow one to control the eigenvalues of the Laplacian on a metric graph with any of the usual vertex conditions (natural, Dirichlet or delta). We will illustrate how these principles can be used to give new proofs and sharper versions of existing "isoperimetric"-type eigenvalue estimates by sketching a result which interpolates between the theorems of Nicaise and Band–Lévy for the first non-trivial eigenvalue of the Laplacian with natural vertex conditions.

This is based on joint work with Gregory Berkolaiko, Pavel Kurasov and Delio Mugnolo.

Flows in infinite networks

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Coauthor: Christian Budde

We consider linear transport processes in infinite metric graphs in the L^∞-setting. We apply the theory of bi-continuous operator semigroups to obtain well-posedness of the problem under different assumptions on the velocities and for general stochastic matrices appearing in the boundary conditions.

Semilinear evolution problems in fractal domains

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We present some results on semilinear evolutions equations, possibly non autonomous , in fractal domains.

Local existence, uniqueness and regularity results for the mild solution are proved.

Stability and asymptotic properties of dissipative equations coupled with ordinary differential equations

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In this talk, we will present some stability results of a system corresponding to the coupling between a dissipative equation (set in an infinite dimensional space) and an ordinary differential equation. Namely we consider U, P solution of the system

$$\begin{cases} U_t = \mathcal{A}U + MP, \text{ in } H, \\ P_t = BP + NU, \text{ in } X, \\ U(0) = U_0, P(0) = P_0, \end{cases} \tag{1}$$

where $\mathcal{A}$ is the generator of a C_0 semigroup in the Hilbert space H, B is a bounded operator from another Hilbert space X, and M, N are supposed to be bounded operators. Many problems from physics enter in this framework, let us mention dispersive medium models, generalized telegraph equations, Volterra integro-differential equations, and cascades of ODE-hyperbolic systems. The goal is to find sufficient (and necessary) conditions on the involved operators $\mathcal{A}$, B, M and N that garantee stability properties of system (1), i.e., strong stability, exponential stability or polynomial one. We will illustrate our general results by an example of generalized telegraph equations set on networks.

Self-adjoint extensions of infinite quantum graphs

Noema Nicolussi, noema.nicolussi@univie.ac.at
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In the last decades, quantum graphs (Laplacians on metric graphs) have become popular objects of study and the analysis of spectral properties relies on the self-adjointness of the Laplacian. Whereas on finite metric graphs the Kirchhoff Laplacian is always self-adjoint, much less is known about the self-adjointness problem for graphs having infinitely many edges and vertices. Intuitively the question is closely related to finding appropriate boundary notions for infinite graphs.

In this talk we study the connection between self-adjoint extensions and the notion of graph ends, a classical graph boundary introduced independently by Freudenthal and Halin. Our discussion includes a lower estimate on the deficiency indices of the minimal Kirchhoff Laplacian and a geometric characterization of self-ajointness of the Gaffney Laplacian.

Based on joint work with Aleksey Kostenko (Ljubljana & Vienna) and Delio Mugnolo (Hagen).

Nonlinear models of kinetic type: On the Cauchy problem and Banach space regularity for Boltzmann flows of monatomic gas mixtures

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Coauthors: Irene M. Gamba, Erica De La Canal

This talk will focus on the analysis of kinetic models for multi-component mixtures of monatomic gases with different masses. The model corresponds to a Boltzmann system for the evolution of vector valued distribution function. The collision or interaction law, as much as the modelling of the transition probability rates for pairwise interactions, are crucial components in the dynamics.

We will present some recent rigorous results for the full non-linear space homogeneous Boltzmann system of equations describing multi-component monatomic gas mixtures for binary interactions. More precisely, we will show existence and uniqueness of the vector value solution in the case of hard potentials and integrable angular scattering kernels associated to each pair of interacting species, by means of an existence theorem for ODE systems in Banach spaces. In addition, we will present several properties for such a solution, including integrability properties of the multispecies collision operator. These properties together with a control by below imply propagation of the polynomially and exponentially weighted L^p norms, $1 \leq p \leq \infty$, associated to the system solution. Additionally, for $p = 1$ we have generation of such moments.

Stochastic completeness of graphs: curvature and volume growth

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We will summarize some recent work concerning stochastic completeness of graphs. In particular, we will discuss curvature criteria, uniqueness class and volume growth results.

Minisymposium

Geometric-functional inequalities and related topics (MS-23)

Organized by Luboš Pick, *Charles University, Prague, Czech Republic*

- An eigenvalue problem in anisotropic Orlicz-Sobolev spaces, *Angela Alberico*
- Fractional Orlicz-Sobolev spaces, *Andrea Cianchi*
- A liquid-solid phase transition in a simple model for swarming, *Rupert Frank*
- Approximation and nuclear embeddings in weighted function spaces, *Dorothee Haroske*
- Attainability of the best Sobolev constant in a ball, *Noriusuke Ioku*
- Caccioppoli–type estimates and Hardy–type inequalities derived from weighted p–harmonic problems, *Agnieszka Kałamajska*
- Compactness of Sobolev embeddings with upper Ahlfors regular measures, *Zdeněk Mihula*
- Measure of noncompactness of Sobolev embeddings, *Vít Musil*
- Minimal Conditions to define BMO, *Carlos Perez Moreno*
- Blaschke–Santaló inequalities for Minkowski endomorphisms, *Franz Schuster*
- Classical multiplier theorems and their sharp variants, *Lenka Slavíková*
- Subelliptic geometric-functional inequalities, *Durvudkhan Suragan*
- An optimization problem in thermal insulation, *Cristina Trombetti*
- Characterization of Sobolev functions with zero traces via the distance function from the boundary, *Hana Turčinová*
- Reverse superposition estimates, lifting over a compact covering and extensions of traces for fractional Sobolev mappings, *Jean Van Schaftingen*

An eigenvalue problem in anisotropic Orlicz-Sobolev spaces

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Coauthors: Giuseppina di Blasio, Filomena Feo

The existence of eigenfunctions for a class of fully anisotropic elliptic equations is established. The relevant equations are associated with constrained minimization problems for integral functionals depending on the gradient of competing functions through general anisotropic Young functions. In particular, the latter need neither be radial, nor have a polynomial growth, and are not even assumed to satisfy the so called Δ_2-condition. In particular, our analysis requires the development of some new aspects of the theory of anisotropic Orlicz-Sobolev spaces.

Fractional Orlicz-Sobolev spaces

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Coauthors: Angela Alberico, Lubos Pick, Lenka Slavíková

Optimal embeddings for fractional-order Orlicz–Sobolev spaces are presented. Related Hardy type inequalities are proposed as well. Versions for fractional Orlicz-Sobolev seminorms of the Bourgain-Brezis-Mironescu theorem on the limit as the order of smoothness tends to 1 and of the Maz'ya-Shaposhnikova theorem on the limit as the order of smoothness tends to 0 are established.

A liquid-solid phase transition in a simple model for swarming

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We consider a non-local shape optimization problem, which is motivated by a simple model for swarming and other self-assembly/aggregation models, and prove the existence of different phases. In particular, we show that in the large mass regime the ground state density profile is the characteristic function of a round ball. An essential ingredient in our proof is a strict rearrangement inequality with a quantitative error estimate. The talk is based on joint work with E. Lieb.

Approximation and nuclear embeddings in weighted function spaces

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Friedrich Schiller University Jena, Germany
Coauthor: Leszek Skrzypczak

We study nuclear embeddings for weighted spaces of Besov and Triebel-Lizorkin type where the weight belongs to some Muckenhoupt class and is essentially of polynomial type. Here

we can extend our previous results on the compactness of corresponding embeddings. The concept of nuclearity goes back to Grothendieck (1955) and was the basis for many fundamental developments in functional analysis. Recently we noticed a refreshed interest to study such questions in special situations. This led us to the investigation in the weighted setting. We obtain complete characterisations for the nuclearity of the corresponding embedding. Essential tools are a discretisation in terms of wavelet bases, operator ideal techniques, as well as a very useful result of Tong (1969) about the nuclearity of diagonal operators acting in ℓ_p spaces.

Attainability of the best Sobolev constant in a ball

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Tohoku University, Japan

The best constant in the Sobolev inequality in the whole space is attained by the Aubin–Talenti function; however, this does not happen in bounded domains because of the break down of the dilation invariance. In this talk, we investigate a nonlinear scale invariant form of the Sobolev inequality in a ball and show that its best constant is attained by functions of the Aubin–Talenti type.

Caccioppoli–type estimates and Hardy–type inequalities derived from weighted p–harmonic problems

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Let $u : \mathbf{R}^n \supseteq \Omega \to \mathbf{R}$ be the nontrivial and nonnegative solution to the following anticoercive partial differential inequality of elliptic type involving weighted p–Laplacian:

$$-\Delta_{p,a} u := -\mathrm{div}(a(x)|\nabla u|^{p-2}\nabla u) \geq b(x)\Phi(u)\chi_{\{u>0\}},$$

where $\Phi : (0, \infty) \to \mathbf{R}$ is the given continuous function with certain properties. After obtaining Caccioppoli–type estimates for u, we derive from them several variants of Hardy–type inequalities in weighted Sobolev setting, some of them holding with best constants.

The talk will be based on joint work with Iwona Chlebicka and Pavel Drabek [1]. It extends earier result from [4] to the weighted setting. Both approaches are besed on the modified techniques due to Pohozhaev and Mitidieri [3] from [2].

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Compactness of Sobolev embeddings with upper Ahlfors regular measures

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A lot of different Sobolev-type embeddings (e.g., classical Sobolev embeddings in the Euclidean setting, boundary trace embeddings, trace embeddings on manifolds, some weighted Sobolev embeddings), which are often treated separately, can be viewed as special instances of Sobolev embeddings with respect to upper Ahlfors regular measures (i.e., Borel measures whose decay on balls is bounded from above by a power of their radii). The aim of this talk is to present in some sense sharp compactness results for such embeddings in the general setting of rearrangement-invariant spaces.

Measure of noncompactness of Sobolev embeddings

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Coauthors: Luboš Pick, Jan Lang, Miroslav Olšák

A bounded set can be covered by a single ball of some radius. Sometimes several balls of a smaller radius can also cover the set. A compact set can be covered by finitely many balls of arbitrary small radius. The smallest radius that allows to cover the set with finitely many balls therefore describes sets laying in between boundedness and compactness. Such quantity is called a measure of non-compactness.

Based on the property of images of the unit balls, linear mappings between Banach spaces are also classified as bounded or compact and to those staying in between, we can assign the measure of non-compactness as well.

An important instance of an operator is a Sobolev embedding. Compactness of a Sobolev embedding can constitute a crucial step in many applications in partial differential equations, probability theory, calculus of variations, mathematical physics and other disciplines. In the non-compact case, more subtle techniques have to be developed and the measure of non-compactness plays an indispensable role here.

We give a survey of some recent new results on measure of non-compactness of Sobolev embeddings and related mappings.

Minimal Conditions to define BMO

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It is well known the importance of the BMO space of functions with bounded mean oscillation especially due to the famous John-Nirenberg theorem of the early 60's of the last century. This result is the archetypical self-improving result in Analysis. In this talk we will show that there is another self-improving phenomenon attached to this class of functions which roughly gives a way of defining BMO using much weaker conditions than the usual L1 oscillation. These results improved a recent work by Logunov-Slavin-Stolyarov-Vasyunin-Zatitskiy. Our method is more flexible yielding sharp results under rougher geometries.

If there is enough time I will show some self-improving phenomenon considered for first time by B. Muckenhoupt and R. Wheeden with weights which turned out to be very useful in different situations like in the extrapolation theory.

This joint work with J. Canto and E. Rela.

Blaschke–Santaló inequalities for Minkowski endomorphisms

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In this talk we explain how each monotone Minkowski endomorphism of convex bodies gives rise to an isoperimetric inequality which directly implies the classical Urysohn inequality. Among this large family of new inequalities, the only affine invariant one – the Blaschke–Santaló inequality – turns out to be the strongest one. A further extension of these inequalities to merely weakly monotone Minkowski endomorphisms is proven to be impossible. Moreover, for functional analogues of monotone Minkowski endomorphisms, a family of analytic inequalities for log-concave functions is established which generalizes the functional Blaschke–Santaló inequality.

Classical multiplier theorems and their sharp variants

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Charles University, Czech Republic

The question of finding good sufficient conditions on a bounded function m guaranteeing the L^p-boundedness of the associated Fourier multiplier operator is a long-standing open problem in harmonic analysis. In this talk we recall the classical multiplier theorems of Hörmander and Marcinkiewicz and present their sharp variants in which the multiplier belongs to a certain fractional Lorentz-Sobolev space. The talk is based on a joint work with L. Grafakos and M. Mastyło.

Subelliptic geometric-functional inequalities

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In this talk, we discuss geometric-functional inequalities on stratified groups. In this environment, the theory of geometric-functional inequalities becomes intricately intertwined with the properties of sub-Laplacians and more general subelliptic partial differential equations. Particularly, we discuss subelliptic Hardy-Sobolev type inequalities and their applications. Moreover, we present sharp remainder terms for the higher-order Steklov inequality on stratified groups which imply short and direct proofs of the sharp (classical) higher-order Steklov inequalities. We also give representation formulae for the L^{2^m}-Friedrichs inequalities. This talk is partially based on our recent works with Tohru Ozawa and Michael Ruzhansky.

An optimization problem in thermal insulation

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We study thermal insulating of a bounded body. Under a prescribed heat source, we consider a model of heat transfer between the body and the environment determined by convection; this corresponds, before insulation, to Robin boundary conditions. We study the maximization of heat content (which measures the goodness of the insulation) among all the possible distributions of insulating material with fixed mass, and prove an optimal upper bound in terms of geometric properties. Eventually we prove a conjecture which states that the ball surrounded by a uniform distribution of insulating material maximizes the heat content.

Characterization of Sobolev functions with zero traces via the distance function from the boundary

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Let Ω be a regular domain in the Euclidean space $\mathbb{R}^n$ and let d be the distance function from the boundary of Ω. A classical result of late 1980's states that for $p \in (1, \infty)$ and $m \in \mathbb{N}$, u belongs to the Sobolev space $W_0^{m,p}(\Omega)$ if and only if $u/d^m \in L^p(\Omega)$ and $|\nabla^m u| \in L^p(\Omega)$. During the consequent decades, several authors have spent considerable effort in order to relax the characterizing condition concerning requirements on the regularity of the function u/d^m. We present a new such condition in terms of Lorentz spaces.

Reverse superposition estimates, lifting over a compact covering and extensions of traces for fractional Sobolev mappings

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Coauthor: Petru Mironescu

When $u \in W^{1,p}(\mathbb{R}^m)$, then $|u| \in W^{1,p}(\mathbb{R}^m)$ and

$$\int_{\mathbb{R}^m} |Du|^p = \int_{\mathbb{R}^m} |D|u||^p;$$

this provides an a priori control on u by $|u|$ in first-order Sobolev spaces. For fractional Sobolev spaces when $sp > 1$, we prove a reverse oscillation inequality that yields a control on u by $|u|$ in $W^{s,p}(\mathbb{R}^m)$. As another consequence of the reverse oscillation estimate, given a covering map $\pi : \widetilde{\mathcal{N}} \to \mathcal{N}$, with $\widetilde{\mathcal{N}}$ compact, we prove any $u \in W^{s,p}(\mathbb{R}^m, \mathcal{N})$ has a lifting, that is, can be written as $u = \pi \circ \widetilde{u}$, with $\widetilde{u} \in W^{s,p}(\mathbb{R}^m, \widetilde{\mathcal{N}})$. This completes the picture for lifting of fractional Sobolev maps and implies the surjectivity of the trace operator on Sobolev spaces of mappings into a manifold when the fundamental group is finite.

Minisymposium

Harmonic Analysis and Partial Differential Equations (MS-28)

Organized by Oliver Dragičević, *University of Ljubljana, Slovenia*

- Microlocal analysis of singular measures, *Valeria Banica*
- Pointwise convergence for the Schrödinger equation with orthonormal initial data, *Neal Bez*
- p-ellipticity, generalized convexity and applications, *Andrea Carbonaro*
- Weak L^1 inequalities for noncommutative singular integrals, *Jose Manuel Conde Alonso*
- Sobolev-Lorentz capacity and its regularity in the Euclidean setting, *Serban Costea*
- An optimal multiplier theorem for Grushin operators in the plane, *Gian Maria Dall'Ara*
- Multilinear singular and oscillatory integrals and applications, *Polona Durcik*
- A new proof of the weak $(1, 1)$ inequality for the dyadic square function, *Irina Holmes*
- Tales on two commuting transformations or flows, *Vjekoslav Kovač*
- Optimal Hardy weights on the Euclidean lattice, *Marius Lemm*
- Spectral analysis of a confinement model in relativistic quantum mechanics, *Albert Mas*
- Fractional Integrals with Measure in Grand Lebesgue and Morrey spaces, *Alexander Meskhi*
- Pointwise ergodic theorems for bilinear polynomial averages, *Mariusz Mirek*
- Inequalities for noncommutative martingales with applications to quantum harmonic analysis, *Adam Osekowski*
- Fractional Degenerate Poincaré-Sobolev inequalities, *Carlos Perez Moreno*
- Regularity of solutions of complex coefficient elliptic systems: the p-ellipticity condition, *Jill Pipher*
- L^p estimates for wave equations with specific Lipschitz coefficients, *Pierre Portal*
- Improved bounds for the Kakeya maximal conjecture using semialgebraic geometry, *Keith Rogers*
- Hardy–Littlewood–Sobolev inequality for $p = 1$, *Dmitriy Stolyarov*
- On a control of the maximal truncated Riesz transform by the Riesz transform; dimension-free estimates, *Błażej Wróbel*
- Multiplicative inequalities on BMO, *Pavel Zatitskii*
- Boundary unique continuation of Dini domains, *Zihui Zhao*

Microlocal analysis of singular measures

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In this talk I shall present a study of scalar and vectorial measures from a microlocal point of view, by introducing a notion of L^1-regularity wave front set. I shall give several results including a full L^1-elliptic regularity result, properties of the polarisation of vectorial measures constrained by a PDE, as well as a propagation of singularities result. This is a joint work with Nicolas Burq.

Pointwise convergence for the Schrödinger equation with orthonormal initial data

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For the one-dimensional Schrödinger equation, we will explain how to obtain maximal-in-time estimates for systems of orthonormal initial data and, as a result, certain pointwise convergence results associated with systems of infinitely many fermions. Our argument proceeds by establishing a maximal-in-space estimate for the fractional Schrödinger equation which, at the same time, addresses an endpoint problem raised by Rupert Frank and Julien Sabin. The talk is based on joint work with Sanghyuk Lee and Shohei Nakamura.

p-ellipticity, generalized convexity and applications

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I will review some recent applications of the notions of p-ellipticity and generalized convexity introduced by O. Dragičević and myself [3]. M. Dindoš and J. Pipher [7], simultaneously and independently of us, found that p-ellipticity is a critical tool in different elliptic regularity problems they were studying. A condition weaker than p-ellipticity appeared in a different formulation in the 2005 work by A. Cialdea and V. Maz'ya [6].

The applications I will discuss include: (i) optimal holomorphic functional calculus for generators of symmetric contraction semigroups [1] and for non-symmetric Ornstein–Uhlenbeck operators [2] (ii) L^p-contractivity of the semigroups generated by divergence-form operators with complex coefficients [3,4,8,6] and (iii) maximal parabolic regularity of the generators subject to mixed boundary conditions on generic open subsets of $\mathbb{R}^d$ [4] (iv) trilinear estimates and Kato-Ponce-type inequalities [5].

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Weak L^1 inequalities for noncommutative singular integrals

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The classical Calderón-Zygmund decomposition is a fundamental tool that helps one study endpoint estimates near L^1. In this talk, we shall study an extension of the decomposition to a particular operator valued setting where noncommutativity makes its appearance, allowing us to get rid of the (usually necessary) UMD property of the Banach space where functions take values. The noncommutative extension entails a number of applications. One that we shall discuss concerns weak L^1 estimates for Fourier multipliers on groups. Based on joint work with L. Cadilhac and J. Parcet.

Sobolev-Lorentz capacity and its regularity in the Euclidean setting

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We study the Sobolev-Lorentz capacity and its regularity in the Euclidean setting whenever $n \geq 1$ is an integer. We extend here our previous results on the Sobolev-Lorentz capacity obtained for $n > 1$ integer. Moreover, for $n > 1$ integer we obtain a few new results concerning the $n, 1$ relative and global capacities. Specifically, we obtain sharp estimates for the $n, 1$ relative capacity of the concentric condensers $(B(0, r), B(0, 1))$ for all r in $[0, 1)$. As a consequence we obtain the exact value of the $n, 1$ capacity of a point relative to all its bounded open neighborhoods from $\mathbf{R}^n$ when $n > 1$ is an integer. We also show that this aforementioned constant is the value of the $n, 1$ global capacity of any point from $\mathbf{R}^n$, where $n > 1$ is an integer. Finally, we prove that whenever $n > 1$ is an integer, the relative and the global $p, 1$ capacities are Choquet whenever p is finite and greater than n.

An optimal multiplier theorem for Grushin operators in the plane

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Let $\mathcal{L} = -\partial_x^2 - V(x)\partial_y^2$ be the Grushin operator on $\mathbb{R}^2$ with coefficient $V : \mathbb{R} \to [0, \infty)$. Under the sole assumptions that $V(-x) \simeq V(x) \simeq xV'(x)$ and $x^2|V''(x)| \lesssim V(x)$, we prove a spectral multiplier theorem of Mihlin–Hörmander type for $\mathcal{L}$, whose smoothness requirement is optimal and independent of V. The proof hinges on the spectral analysis of one-dimensional Schödinger operators, including universal estimates of eigenvalue gaps and matrix coefficients of the potential.

Multilinear singular and oscillatory integrals and applications

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Coauthors: Michael Christ, Joris Roos, Vjekoslav Kovač

We give an overview of some recent results in the area of multilinear singular and oscillatory integrals. We discuss their connection with certain questions about point configurations in subsets of the Euclidean space and convergence of some ergodic averages. The talk is based on joint works with Michael Christ, Vjekoslav Kovač, and Joris Roos.

A new proof of the weak $(1, 1)$ inequality for the dyadic square function

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This project (joint with Paata Ivanisvili and Sasha Volberg) is concerned with finding the (strange) sharp constant in the weak $(1, 1)$ inequality for the dyadic square function, using the Bellman function method. This constant was conjectured by Bollobas in the 1980's and proved first by Osekowski using Brownian motion methods. The interesting aspect of our new proof is that it required the invention of a new way to work with Bellman functions – a way which we hope can be implemented in other problems.

Tales on two commuting transformations or flows

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We will give an overview of conjectures, results, and techniques related to long-term dynamics of two commuting measure-preserving actions of $\mathbb{Z}$ or $\mathbb{R}$ on a probability space. All problems will be approached via the Calderón transference principle, reducing the questions on convergence of various ergodic averages to boundedness of integral operators on the Euclidean space. This scheme might be an overkill when merely establishing convergence, but it is usually very

quantitative and appealing to harmonic analysts. The talk will be based on several papers coauthored with M. Christ, P. Durcik, J. Roos, K. A. Škreb, and C. Thiele.

Optimal Hardy weights on the Euclidean lattice

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Coauthor: Matthias Keller

We investigate the large-distance asymptotics of optimal Hardy weights on $\mathbb{Z}^d$, $d \geq 3$, via the super solution construction. For the free discrete Laplacian, the Hardy weight asymptotic is the familiar $\frac{(d-2)^2}{4}|x|^{-2}$ as $|x| \to \infty$. We prove that the inverse-square behavior of the optimal Hardy weight is robust for general elliptic coefficients on $\mathbb{Z}^d$ in various senses. The proofs leverage Green's function estimates rooted in homogenization theory.

Spectral analysis of a confinement model in relativistic quantum mechanics

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In this talk we will focus on the Dirac operator on domains of $\mathbb{R}^3$ with confining boundary conditions of scalar and electrostatic type. This operator is a generalization of the MIT-bag operator, which is used as a simplified model for the confinement of quarks in hadrons that has interested many scientists in the last decades. It is conjectured that, under a volume constraint, the ball is the domain which has the smallest first positive eigenvalue of the MIT-bag operator.

I will describe our results —in collaboration with N. Arrizabalaga (U. País Vasco), T. Sanz-Perela (U. Edinburgh and BCAM), and L. Vega (U. País Vasco and BCAM)— on the spectral analysis of the generalized operator. I will discuss on the parameterization of the eigenvalues, their symmetry and monotonicity properties, the optimality of the ball for large values of the parameter, and the connection to boundary Hardy spaces.

Fractional Integrals with Measure in Grand Lebesgue and Morrey spaces

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Coauthor: Vakhtang Kokilashvili

In the last two decades, the theory of grand Lebesgue spaces $L^{p)}$ introduced by T. Iwaniec and C. Sbordone [2] is one of the intensively developing directions in modern analysis. The necessity to investigate these spaces emerged from their rather essential role in various fields, in particular, in the integrability problem of Jacobian under minimal hypotheses. It turns out that in the theory of PDEs, the generalized grand Lebesgue spaces $L^{p),\theta}$ introduced by Greco, Iwaniec, and Sbordone [1] are appropriate for treating the existence and uniqueness, as well as the regularity problems for various non-linear differential equations.

The aim of our talk is to give a complete characterization of a class of measures μ governing the boundedness of fractional integral operators I^γ defined on a quasi-metric measure space (X, d, μ) (nonhomogeneous space) from one grand Lebesgue spaces $L^{p),\theta_1}_\mu(X)$ into another one $L^{q),\theta_2}_\mu(X)$. As a corollary, we have a generalization of the Sobolev inequality for potentials with measure. D. Adams trace inequality (i.e., $L^{p),\theta_1}_\mu(X) \mapsto L^{q),\theta_2}_\nu(X)$ boundedness) is also derived for these operators in grand Lebesgue spaces. Appropriate problems for grand Morrey spaces are also studied. In the case of Morrey spaces, we assume that the underlying sets of spaces might be of infinite measure. Under some additional conditions on a measure, we investigate the sharpness of the second parameter θ_2 in the target space.

Acknowledgement: The work was supported by the Shota Rustaveli National Foundation grant of Georgia (Project No. DI-18-118).

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Pointwise ergodic theorems for bilinear polynomial averages

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We shall discuss the proof of pointwise almost everywhere convergence for the non-conventional (in the sense of Furstenberg and Weiss) bilinear polynomial ergodic averages. This is joint work with Ben Krause and Terry Tao: arXiv:2008.00857. We will also talk about recent progress in this area.

Inequalities for noncommutative martingales with applications to quantum harmonic analysis

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In the recent twenty years, the theory of noncommutative (or quantum) martingales has gained a lot interest in the literature, and most of the classical results has been successfully transferred to this new, operator context. The purpose of the talk is to survey some recent progress in this field and discuss several applications related to boundedness of Fourier multipliers on some group von Neumann algebras

Fractional Degenerate Poincaré-Sobolev inequalities

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In this lecture we will discuss some recent results concerning fractional Poincaré and Poincaré-Sobolev inequalities with weights. These results improve some celebrated results by Bourgain-Brezis-Minorescu, Maz'ya-Shaponiskova unified by M. Milman. Our approach is based on methods from Harmonic Analysis. We will first consider the usual context of cubes but also we will discuss some new results in the multiparameter setting improving some results by Shi-Torchinsky and Lu-Wheeden from the 90's.

Regularity of solutions of complex coefficient elliptic systems: the p-ellipticity condition

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Solving boundary value problems for divergence form real elliptic equations has been an active and productive area of research ever since the foundational work of De Giorgi - Nash - Moser established Hölder continuity of solutions when the operator coefficients are merely bounded and measurable. The solutions to such real-valued equations share some important properties with harmonic functions: maximum principles, Harnack principles, and estimates up to the boundary that enable one to solve Dirichlet problems in the classical sense of nontangential convergence. Weak solutions of complex elliptic equations and elliptic systems do not necessarily share these good properties of continuity or maximum principles.

In joint work with M. Dindoš, we introduced in 2017 a structural condition (p-ellipticity) on divergence form complex elliptic equations that was inspired by a condition related to L^p contractivity due to Cialdea and Maz'ya. The p-ellipticity condition was simultaneously discovered by Carbonaro and Dragičević to prove a bilinear embedding result. Subsequently, the condition has proven useful in the study of well-posedness of a degenerate elliptic operator associated with domains with lower-dimensional boundary.

In this talk we discuss p-ellipticity for complex divergence form equations, and then describe recent work, joint with J. Li and M. Dindoš, extending this condition to elliptic systems. In particular, we discuss applications to the Dirichlet problem for the Lamé systems.

L^p estimates for wave equations with specific Lipschitz coefficients

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Coauthor: Dorothee Frey

For the standard linear wave equation $\partial_t^2 u = \Delta u$, the solution at time t belongs to $L^p(\mathbb{R}^d)$ for initial data $u(0,.) \in W^{(d-1)|\frac{1}{p}-\frac{1}{2}|,p}$, $\partial_t u(0,.) = 0$. This is a classical result of Peral/Myachi from the 1980's, which motivated the development of Fourier Integral Operator theory. It is optimal in terms of the order of the Sobolev space of initial data. In this talk, we discuss an extension of this result for certain wave equations, such as $\partial_t^2 u = \sum_{j=1}^{d} \partial_j a_j \partial_j u$, where a_j are $C^{0,1}$ functions bounded from above and below and depending only on the j-th variable. The fact that such a result holds is somewhat surprising, given that other space-time (Strichartz) estimates typically fail for coefficients rougher than $C^{1,1}$. The proof is based on an approach to FIO theory via phase space Hardy spaces (recently developed by Hassell, P., Rozendaal) combined with operator theoretic and harmonic analytic methods. The talk presents the scheme of the proof, focusing on the ideas behind each of the key steps.

Improved bounds for the Kakeya maximal conjecture using semialgebraic geometry

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Instituto de Ciencias Matemáticas, Spain

We consider tubes, each pointing in a different direction, and the degree to which they can be compressed by positioning them strategically. The Kakeya problem consists of bounding the measure of the union of the tubes (placed in any position). On the one hand, we will show that the measure of any semialgebraic set that contains the tubes must satisfy the expected lower bound, confirming a conjecture of Guth and Zahl. The proof employs tools from real algebraic geometry including Gromov's algebraic lemma and Tarski's projection theorem. On the other hand, we will use polynomial partitioning to prove that the expected bound holds if there is no algebraic structure at all. Balancing between the two cases yields improved bounds for the Kakeya maximal conjecture in higher dimensions. This is joint work with Hickman, Katz and Zhang.

Hardy–Littlewood–Sobolev inequality for $p = 1$

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The Hardy–Littlewood–Sobolev inequality $\|I_\alpha f\|_{L_q(\mathbb{R}^d)} \lesssim \|f\|_{L_p(\mathbb{R}^d)}$, where I_α is the Riesz potential of order α and $1/p - 1/q = \alpha/d$, fails at the endpoint $p = 1$. I will show two ways to make the inequality true in this case. One way is to impose further restrictions on f (like f is a divergence free vector field), this way is related to the so-called Bourgain–Brezis inequalities. Another way is to replace the L_q-norm on the left hand side by an expression that has the same homogeneity as the L_q-norm, but possesses additional cancellations. The phenomenon has an analog in the world of martingale transforms, whose consideration suggests the right way to install induction on scales that proves those HLS inequalities for $p = 1$.

On a control of the maximal truncated Riesz transform by the Riesz transform; dimension-free estimates

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University of Wrocław, Poland

W prove a dimension-free estimate for the $L^2(\mathbb{R}^d)$ norm of the maximal truncated Riesz transform in terms of the $L^2(\mathbb{R}^d)$ norm of the Riesz transform. Consequently, the vector of maximal truncated Riesz transforms has a dimension-free estimate on $L^2(\mathbb{R}^d)$. We also show that the maximal function of the vector of truncated Riesz transforms has a dimension-free estimate on all $L^p(\mathbb{R}^d)$ spaces, $1 < p < \infty$. The talk is based on joint work with Maciej Kucharski.

Multiplicative inequalities on BMO

Pavel Zatitskii, pavelz@pdmi.ras.ru
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We will talk about the so-called multiplicative inequality for BMO functions:

$$\|\varphi\|_{L^r}^r \le C_{p,r}\|\varphi\|_{L^p}^p\|\varphi\|_{BMO}^{r-p},$$

where $1 < p < r < \infty$. We will discuss how to find sharp constants in this inequality for the case of quadratic norm on BMO space based on a segment, circle or a real line. Talking about cases of segment and circle we assume the average of φ to be equal to zero. Also, we prove this inequality with dimension-free constant for the Garsia-type norm on BMO. The talk is based on joint work with D. Stolyarov, V. Vasyunin and I. Zlotnikov.

Boundary unique continuation of Dini domains

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Coauthor: Carlos Kenig

Let u be a harmonic function in $\Omega \subset \mathbb{R}^d$. It is known that in the interior, the singular set $\mathcal{S}(u) = \{u = |\nabla u| = 0\}$ is $(d-2)$-dimensional, and moreover $\mathcal{S}(u)$ is $(d-2)$-rectifiable and its Minkowski content is bounded (depending on the frequency of u). We prove the analogue near the boundary for C^1-Dini domains: If the harmonic function u vanishes on an open subset E of the boundary, then near E the singular set $\mathcal{S}(u) \cap \overline{\Omega}$ is $(d-2)$-rectifiable and has bounded Minkowski content. Dini domain is the optimal domain for which ∇u is continuous towards the boundary, and in particular every $C^{1,\alpha}$ domain is Dini. The main difficulty is the lack of monotonicity formula near the boundary of a Dini domain. This is joint work with Carlos Kenig.

Minisymposium

HIGHER-ORDER EVOLUTION EQUATIONS (MS-43)

Organized by Mario Bukal, *University of Zagreb, Croatia*

- Sixth-order thin-film equations as reduced models for fluid-structure interaction problems, *Mario Bukal*
- A Lagrangian scheme for the solution of nonlinear diffusion equations, *Bertram Düring*
- Discretely selfsimilar solutions for fourth order PDEs in lubrication models, *Marco Fontelos*
- Weak solutions to the stochastic thin-film equation with nonlinear noise in divergence form, *Manuel Gnann*
- Gradient flow structure of a sixth order parabolic equation, *Daniel Matthes*
- High order PDEs arising in immiscible multilayer flows, *Demetrios Papageorgiou*
- Thin-film problems with dynamic contact angle, *Dirk Peschka*
- Higher-Order Total Directional Variation, *Carola Bibiane Schönlieb*

Sixth-order thin-film equations as reduced models for fluid-structure interaction problems

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University of Zagreb, Croatia
Coauthor: Boris Muha

Starting from a nonlinear fluid-structure interaction (FSI) problem between a thin layer of a viscous fluid and a thin elastic structure we derive a reduced model given by a nonlinear sixth-order evolution equation describing the out-of-plane displacement of the structure. The derivation is carried out based on suitable energy inequalities and a uniform no-contact result for the FSI system. Both estimates are quantified in terms of the relative fluid thickness, which is a small parameter in the FSI system. The reduced model is justified in terms of weak convergence results in the sense that weak limits of solutions to the original FSI problem are identified in a relation with solutions of the sixth-order thin-film equation. The talk is based on a joint work with B. Muha (University of Zagreb).

A Lagrangian scheme for the solution of nonlinear diffusion equations

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University of Warwick, United Kingdom

Many nonlinear diffusion equations can be interpreted as gradient flows whose dynamics are driven by internal energies and given external potentials. Examples include the heat equation, the porous medium equation, and the fourth-order Derrida-Lebowitz-Speer-Spohn equation. When solving these equations numerically, schemes that respect the equations' special structure are of particular interest. I this talk we present a Lagrangian scheme for nonlinear diffusion equations. For discretisation of the Lagrangian map, we use a finite subspace of linear maps in space and a variational form of the implicit Euler method in time. We present numerical experiments for the porous medium equation in two space dimensions.

Discretely selfsimilar solutions for fourth order PDEs in lubrication models

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Instituto de Ciencias Matemáticas, Spain

We use a generalized version of the equation of motion for a thin film of liquid on a solid, horizontal substrate as a model system to study the formation of singularities in space dimensions greater than one. Varying both the exponent controlling long-ranged forces, as well as the exponent of the nonlinear mobility, we predict the structure of the singularity as the film thickness goes to zero. The spatial structure of rupture may be either 'pointlike' (approaching axisymmetry) or 'quasi-one-dimensional', in which case a one-dimensional singularity is unfolded into two or higher space dimensions. The scaling of the profile with time may be either strictly self-similar (the 'regular' case) or discretely self-similar and perhaps chaotic (the 'irregular' case).

Weak solutions to the stochastic thin-film equation with nonlinear noise in divergence form

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Delft University of Technology, Netherlands
Coauthors: Konstantinos Dareiotis, Benjamin Gess, Günther Grün

We investigate a degenerate-parabolic fourth-order stochastic partial differential equation modelling the spreading of thin liquid droplets under the influence of thermal noise. Using a combination of entropy and energy estimates, we are able to control the formation of shocks caused by the nonlinear noise in divergence form. In conjunction with a tailor-made approximation and regularization of the equation, we are thus able to prove existence of weak (martingale) solutions through a sequence of compactness arguments.

Gradient flow structure of a sixth order parabolic equation

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Technische Universität München, Germany

The second order linear heat equation and the fourth order nonlinear DLSS equation are gradient flows in the L^2-Wasserstein metric, for the entropy $H(\rho) = \int \rho \log \rho$ and the Fisher information $F(\rho) = \int \rho |\nabla \log \rho|^2$, respectively. Whereas H is geodesically convex, the functional F is very non-convex, but the DLSS equation shares the self-similar asymptotics of the heat equation, thanks to the intimate relation between H and F. This talk is about a sixth order nonlinear PDE that is a gradient flow for the second-order functional $E(\rho) = \int \rho \|\nabla^2 \log \rho\|^2$. We prove existence of weak solutions, and then study their self-similar asymptotics using a structural relation connecting E to both H and F.

High order PDEs arising in immiscible multilayer flows

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When two or more immiscible viscous fluids are set into motion by a driving pressure gradient, for instance, the underlying shear can induce instabilities and interfacial waves. The problem is a moving boundary one and is coupled to the bulk flow (typically the Navier-Stokes equations) via nonlinear interfacial boundary conditions that are statements of velocity and stresses continuity. The problems are high order due to the presence of surface tension and other physical effects like electric or magnetic fields. This is a rich and complex problem and this presentation will attempt to (i) provide the essentials of the mathematical modelling, (ii) describe asymptotic analysis techniques that produce nonlinear evolution PDEs, (iii) present results of analysis and computations of the resulting PDEs including certain local and global existence results and computer-assisted proofs, (iv) provide an outlook of open problems in the area. Throughout the presentation computations will be used to motivate analysis and vice versa.

Thin-film problems with dynamic contact angle

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Weierstraß-Institut für Angewandte Analysis und Stochastik, Germany

The evolution of a thin viscous fluid over a solid surface is often described using the thin-film equations

$$\partial_t h - \nabla \cdot \left(m|h|^\alpha \nabla \frac{\delta E}{\delta h} \right) = 0, \qquad E = \int_\omega \frac{1}{2}|\nabla h|^2 + W(x,h)\,\mathrm{d}x,$$

which are fourth-order degenerate parabolic equations for the height h of the fluid layer with the support $\omega(t) = \{x : h(t,x) > 0\}$. The motion of the liquid layer is driven by an energy E, which in addition to a surface energy contains other sources of internal energy in W. By complementing this PDE with suitable boundary conditions on $\partial\omega(t)$, this becomes a free boundary problem with a moving contact line.

In this talk I will introduce the gradient structure underlying the thin-film problem, detail the variational structure of the bulk-interface coupling that leads to dynamic contact angles, and investigate different limiting cases of low and high viscosities, i.e., $m \to 0$ and $m \to \infty$ for $0 < \alpha < 3$.

Higher-Order Total Directional Variation

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Coauthors: Simone Parisotto, Simon Masnou, Jan Lellmann

In this talk we discuss a new higher-order and anisotropic total variation model for image processing. This new model combines higher-order total variation regularisation with possibly inhomogeneous, smooth elliptic anisotropies.

We prove some properties of this total variation model and of the associated spaces of tensors with finite variations. We show the existence of solutions to a related regularity-fidelity optimization problem and prove a decomposition formula which we will use to develop a primal-dual hybrid gradient approach for its numerical approximation.

This choice of total variation regularisation allows to preserve and enhance intrinsic anisotropic features in images. We illustrate this on various examples from different imaging applications: image denoising, wavelet-based image zooming, and reconstruction of surfaces from scattered height measurements.

This talk is based on the two papers:

References

[1] Parisotto, Simone; Lellmann, Jan; Masnou, Simon; Schönlieb, Carola-Bibiane: Higher-order total directional variation: Imaging Applications. In: SIAM Journal on Imaging Sciences, 13 (4), pp. 2063-2104, 2020.

[2] Parisotto, Simone; Masnou, Simon; Schönlieb, Carola-Bibiane: Higher-order Total Directional Variation: Analysis. In: SIAM Journal on Imaging Sciences, 13 (1), pp. 474-496, 2020.

Minisymposium

Mathematical analysis: the interaction of fluids/ viscoelastic materials and solids (MS-36)

Organized by Sarka Necasova, *Institute of Mathematics, Academy of Sciences, Czech Republic*

Coorganized by

Anja Schlömerkemper, *University of Würzburg, Germany*

Justin Webster, *Univ. Maryland, Baltimore, United States*

- Qualitative Analysis of a Lamé-Wave-Stokes/Navier-Stokes System, *George Avalos*
- Fluid-poroelastic structure interaction motivated by the design of a bioartificial pancreas, *Sunčica Čanić*
- Dynamics of Hibler's sea-ice model, *Karoline Disser*
- Regularity for the 3D evolution Navier-Stokes equations under Navier boundary conditions in some Lipschitz domains, *Alessio Falocchi*
- Regularity of a weak solution to a linear fluid-composite structure interaction problem, *Marija Galić*
- On the Exponential Stability of A Compressible FSI PDE System, *Pelin Guven Geredeli*
- Description of contacts in fluid-beam systems, *Matthieu Hillairet*
- Motion of a Rigid body in a Compressible Fluid, *Arnab Roy*
- Optimal boundary control for steady motions of a self-propelled body in a Navier-Stokes liquid, *Ana Leonor Silvestre*
- An energetic variational approach for wormlike micelle solutions: Coarse graining and dynamic stability, *Yiwei Wang*
- Flow of heat conducting fluid on domain changing in time, *Aneta Wróblewska Kamińska*

Qualitative Analysis of a Lamé-Wave-Stokes/Navier-Stokes System

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In this talk, we will discuss an appropriate Babuška-Brezzi variational formulation, and subsequent spectral and stability analysis for a multilayered structure-fluid interaction (FSI) which arises in the mathematical modeling of vascular blood flow. The coupled PDE system which we will consider mathematically accounts for the fact that mammalian veins and arteries will typically be composed of various layers of tissues: each layer will generally manifest its own intrinsic material properties, and will moreover be separated from the other layers by thin elastic laminae. Consequently, the resulting modeling FSI system will manifest an additional PDE, which evolves on the boundary interface, so as to account for the thin elastic layer. (This is in contrast to the FSI PDE's which appear in the literature, wherein elastic dynamics are largely absent on the boundary interface.) As such,the PDE system will constitute a coupling of 3D fluid-2D wave-3D elastic dynamics. This is joint work with Pelin Güven Geredeli.

Fluid-poroelastic structure interaction motivated by the design of a bioartificial pancreas

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In this talk we present a recent well-posedness result in the area of fluid-poroelastic structure interaction, motivated by the design of a first implantable bioartificial pancreas without the need for immunosuppressant therapy. We show global existence of a weak solution to a fluid-structure interaction (FSI) problem between the flow of an incompressible, viscous fluid, modeled by the time-dependent Stokes equations, and a multi-layered poroelastic medium consisting of a thin poroelastic plate and a thick poroelastic medium modeled by a Biot model. This is the first global (weak) solution existence result in the context of poroelastic FSI. Numerical simulations of the underlying problem showing optimal design of a bioartificial pancreas, will be presented. If time permits, I will also preview a well-posedness results for a stochastically perturbed FSI problem. This is a joint work with bioengineer Shuvo Roy (UCSF), and mathematicians Yifan Wang (UCI), Lorena Bociu (NCSU), Boris Muha (University of Zagreb), and Justin Webster (University of Maryland, Baltimore County). The stochastically perturbed FSI result was obtained with PhD student Jeffrey Kuan (UC Berkeley).

Dynamics of Hibler's sea-ice model

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Coauthors: Felix Brandt, Robert Haller-Dintelmann, Matthias Hieber

Sea ice is a material with a complex mechanical and thermodynamical behaviour. Freezing sea water forms a composite of pure ice, liquid brine, air pockets and solid salt. The details of this formation depend on the laminar or turbulent environmental conditions. The governing equations of large-scale sea ice dynamics that form the basis of many sea ice models in climate science were suggested in a seminal paper by Hibler in 1979. We show that Hibler's sea ice model that couples a 2D-velocity and two parameters for thickness and compactness of sea ice based on viscous-plastic rheology is locally strongly well-posed and globally strongly well-posed for initial data close to constant equilibria.

Regularity for the 3D evolution Navier-Stokes equations under Navier boundary conditions in some Lipschitz domains

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Coauthor: Filippo Gazzola

For the evolution Navier-Stokes equations in bounded 3D domains, it is well-known that the uniqueness of a solution is related to the existence of a regular solution. They may be obtained under suitable assumptions on the data and smoothness assumptions on the domain (at least C^2). With a symmetrization technique, we prove these results in the case of Navier boundary conditions in a wide class of merely *Lipschitz domains* of physical interest, that we call *sectors*.

Regularity of a weak solution to a linear fluid-composite structure interaction problem

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We deal with the regularity of a weak solution to the fluid-composite structure interaction problem. The problem describes a linear fluid-structure interaction between an incompressible, viscous fluid flow, and an elastic structure composed of a cylindrical shell supported by a mesh-like elastic structure. The fluid and the mesh-supported structure are coupled via the kinematic and dynamic boundary coupling conditions describing continuity of velocity and balance of contact forces at the fluid-structure interface. It has been shown that there exists a weak solution to the described problem. By using the standard techniques from the analysis of partial differential equations, we prove that such weak solution possesses an additional regularity in both time and space variables.

On the Exponential Stability of A Compressible FSI PDE System

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Iowa State University, United States

We consider a linearized compressible flow structure interaction (FSI) PDE model with a view of analyzing the stability properties of both the compressible flow and plate solution components. We operate in the time domain by way of obtaining the necessary energy estimates which culminate in an alternative proof for the uniform stability of finite energy compressible flow-structure solutions. Unlike the frequency domain approach followed in the earlier papers, we give the proof of our stability result in time domain via an approach which involves a gradient type multiplier to obtain the necessary energy estimates.

Description of contacts in fluid-beam systems

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Universite Montpellier, France

In this talk we consider a family of systems describing the interactions between a film of fluid deposited on a horizontal substrate and a beam delimiting the upper boundary of the film. The fluid motion is prescribed by solving the incompressible Navier Stokes equations. The beam is assumed to move vertically only, several models are proposed depending on whether damping/viscosity terms are included or not. The coupling between the fluid and the beam is imposed via the continuity of velocity-fields and normal stress.

Such systems have been thorouhgly studied in the recent years especially to develop a Cauchy for strong/weak solutions up to the possible first time of contact between the moving beam and the substrate. In this talk, we will focus on the decription of beam/substrate contact. We will first discuss finite-time occurence and then provide a Cauchy theory that handle contacts. This talk is based on collaborations with C. Grandmont (INRIA Paris), J. Lequeurre (Univ. Lorraine) and J-J Casanova (Univ. Paris Dauphine)

Motion of a Rigid body in a Compressible Fluid

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Institute of Mathematics of the Czech Academy of Sciences, Czech Republic

In this talk, we discuss the motion of a rigid body in a bounded domain which is filled with a compressible isentropic fluid. We consider the Navier-slip boundary condition at the interface as well as at the boundary of the domain. We prove existence of a weak solution of the fluid-structure system up to collision.

Optimal boundary control for steady motions of a self-propelled body in a Navier-Stokes liquid

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Instituto Superior Técnico, Universidade de Lisboa, Portugal

Consider a rigid body $\mathcal{S} \subset \mathbb{R}^3$ immersed in a Navier-Stokes liquid and the motion of the body-fluid interaction system described from a reference frame attached to $\mathcal{S}$. We are interested in steady motions of this coupled system, where the region occupied by the fluid is the exterior domain $\Omega = \mathbb{R}^3 \setminus \mathcal{S}$. An important question that arises in this context is: How can a self-propelled motion of $\mathcal{S}$ with a target velocity $V(x) := \xi + \omega \times x$ be generated in such a way that the drag about $\mathcal{S}$ is minimized?

We solve this problem using boundary controls v_*, acting on the whole $\partial\Omega$ or just on a portion Γ of $\partial\Omega$. Firstly, an appropriate drag functional is derived from the energy equation of the fluid and the problem is formulated as an optimal control problem.

The drag minimization problem is solved for localized controls, such that supp $v_* \subset \Gamma$, and for tangential controls, i.e, $v_* \cdot n|_{\partial\Omega} = 0$, where n is the outward unit normal to $\partial\Omega$. Under smallness restrictions on the objectives $|\xi|$ and $|\omega|$ and on the boundary controls, we prove the existence of optimal solutions, justify the Gâteaux derivative of the control-to-state map, establish the well-posedness of the corresponding adjoint equations and, finally, derive the first order optimality conditions.

This is joint work with Toshiaki Hishida (Nagoya University, Japan) and Takéo Takahashi (Université de Lorraine, CNRS, Inria, IECL, Nancy, France).

An energetic variational approach for wormlike micelle solutions: Coarse graining and dynamic stability

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Coauthors: Chun Liu, Teng-Fei Zhang

Wormlike micelles are self-assemblies of polymer chains that can break and recombine reversibly. In this talk, we present a thermodynamically consistent two-species micro-macro model of wormlike micellar solutions by employing an energetic variational approach. The model incorporates a breakage and combination process of polymer chains into the classical micro-macro dumbbell model of polymeric fluids in a unified variational framework. The modeling approach can be applied to other reactive or active complex fluids. Different maximum entropy closure approximations to the new model will be discussed. By imposing a proper dissipation in the coarse-grained level, the closure model, obtained by "closure-then-variation", preserves the thermodynamical structure of both mechanical and chemical parts of the original system. The resulting model is an Oldroyd-B type system coupled with a chemical reaction. We'll also present the dynamic stability analysis on the micro-macro model. In particular, we show the global existence of classical solutions near the global equilibrium, which indicates the consistency between the detailed balance conditions in a chemical reaction and the global equilibrium state of each species. The is joint work with Prof. Chun Liu (IIT) and Prof. Teng-Fei Zhang (CUG).

Flow of heat conducting fluid on domain changing in time

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Institute of Mathematics, Polish Academy of Sciences, Poland

We consider a flow of heat conducting fluid inside a moving domain whose shape in time is prescribed by a given velocity field. The flow in this case is governed by the compressible Navier-Stokes-Fourier system consisting of equation of continuity, momentum balance, entropy balance and energy equality. The velocity is supposed to fulfil the full-slip boundary condition and we assume that the fluid is thermally isolated. In the presented article we show the existence of a variational solution. To this end we construct proper penalising approximation. This result is a joint work with O. Kreml, V. Macha, and S. Necasova.

Minisymposium

Multiscale Modeling and Methods: Application in Engineering, Biology and Medicine (MS-80)

Organized by Grigory Panassenko, *University Jean Monnet, France,* and *Vilnius University, Lithuania*
Coorganized by Konstantinas Pileckas, *Vilnius University, Lithuania*

- Mathematical Modeling of Inflammatory Processes of Atherosclerosis, *Ghada Abi Younes*
- Stokes Equations In An Infinite Strip With a Hole And transmission Conditions, *Olivier Bodart*
- On weakly singular kernels arising in equations set on a graph, modelling a flow in a networtk of thin tubes, *Éric Canon*
- Homogenization for elliptic operators in a strip perforated along a curve, *Giuseppe Cardone*
- Numerical solution of the viscous flows in a network of thin tubes: equations on the graph, *Frédéric Chardard*
- Robust parameter estimation in fluid flow models from velocity measurements, *Jeremias Garay*
- Time periodic Navier-Stokes equations in a thin tube structures motivated by hemodynamic, *Rita Juodagalvytė*
- Time-periodic Poiseuille-type solution with minimally regular flow-rate, *Kristina Kaulakytė*
- Blood velocity computation inside of a human heart left atrium, *Nikolajus Kozulinas*
- Asymptotically based simulation of the Stokes flow in a layer through periodic flexural plates made of beams, *Maxime Krier*
- Steady state non-Newtonian flow with strain rate dependent viscosity in thin tube structure with no slip boundary condition, *Grigory Panasenko*
- Unsteady micropolar fluid flow in a thick domain with multiscale oscillating roughness and a subdifferential boundary condition, *Laetitia Paoli*
- Modeling the evolution of COVID-19 in Lithuania, *Olga Štikonienė*
- ADI scheme for partially dimension reduced heat conduction models, *Vytenis Šumskas*
- FSI and reduced models for 3D hemodynamic simulations in time-dependent domains, *Yuri Vassilevski*

Mathematical Modeling of Inflammatory Processes of Atherosclerosis

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Coauthors: Nader El Khatib, Vitaly Volpert

Atherosclerosis is a chronic disease which involves the build up of cholesterol and fatty deposits within the inner lining of the artery. It is associated with a progressive thickening and hardening of the arterial wall that result in narrowing of the vessel lumen and restriction of blood flow to vital organs. These events may cause heart attack or stroke, the commonest causes of death worldwide. We study the early stages of atherosclerosis via a mathematical model of partial differential equations of reaction-diffusion type. The model includes several key species and identifies endothelial hyperpermeability, believed to be a precursor on the onset of atherosclerosis. For simplicity, we reduce the system to a monotone system and provide a biological interpretation for the stability analysis according to endothelial functionality. The existence of solutions of traveling waves type are as well investigated along with numerical simulations. The results obtained are in good agreement with current biological knowledge. Likewise, they confirm and generalize results of mathematical models previously performed in literature. Then, we study the non monotone reduced model and prove the existence of perturbed solutions and perturbed waves, particularly in the bistable case. Finally, we extend the study by considering the complete model proposed initially, perform numerical simulations and provide more specific results. We examine the consistency between the reduced and complete model analysis for a certain range of parameters, we elaborate bifurcation diagrams showing the evolution of inflammation upon endothelial permeability and LDL accumulation and we consider the effect of anti-inflammatory process on the system behavior. The study of the model shows that the regulation of atherosclerosis progression is mediated by anti-inflammatory responses that, up to certain extent, lead to plaque regression.

Stokes Equations In An Infinite Strip With a Hole And transmission Conditions

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Let $0 < a_i < b_i < l_i$, $i = 1,2$ and $S = (0,l_1)\times(0,l_2)$, $\widetilde{S} = (a_1,b_1)\times(a_2,b_2)$. Let also $Y \subset S\times]-1,1[$ be a convex open set with smooth boundary ∂Y. Let Λ be the infinite vertical domain in $\mathbf{R}^3$ defined by

$$\Lambda = \big(S\times]-\infty,+\infty[\big)\setminus\overline{Y}.$$

We define the following subsets in Λ:

$$\begin{aligned}
\Lambda_- &= \big\{y=(y',y_3)\in\mathbf{R}^3\ ;\ y'\in S\ ,\ y_3<-1\big\}\,,\\
\Omega &= \big(S\times]-\infty,+\infty[\big)\setminus\overline{Y},\\
\Lambda_+ &= \Big\{y=(y',y_3)\in\mathbf{R}^3\ ;\ y'\in\widetilde{S}\ ,\ y_3>1\Big\}\,,\\
\Gamma_- &= \big\{y=(y',y_3)\in\mathbf{R}^3\ ;\ y'\in S\ ,\ y_3=-1\big\}\,,\\
\Gamma_- &= \big\{y=(y',y_3)\in\mathbf{R}^3\ ;\ y'\in S\ ,\ y_3=1\big\}\,,
\end{aligned}$$

where we denoted $y' = (y_1,y_2)$. Then we can decompose Λ as follows:

$$\Lambda = \Lambda_-\cup\overline{\Omega}\cup\Lambda_+.$$

We seek a couple (u, p) defined in Λ as

$$u(x) = \begin{cases} u_-(x), & x \in \Lambda_- \\ u_0(x), & x \in \Omega \\ u_+(x), & x \in \Lambda_+ \end{cases} \qquad \text{and} \qquad p(x) = \begin{cases} p_-(x), & x \in \Lambda_- \\ p_0(x), & x \in \Omega \\ p_+(x), & x \in \Lambda_+ \end{cases}$$

where the pairs (u_-, p_-), (u_0, p_0) and (u_+, p_+) satisfy the following system

$$(\mathrm{S}) \begin{cases} -\nu\Delta u_\pm + \nabla p_\pm &= 0 & \text{in } \Lambda_\pm, \\ -\nu\Delta u_0 + \nabla p_0 &= 0 & \text{in } \Omega, \\ \nabla \cdot u_\pm &= 0 & \text{in } \Lambda_\pm, \\ \nabla \cdot u_0 &= 0 & \text{in } \Omega, \\ u_0 &= 0 & \text{on } \partial Y \\ \sigma(u_-, p_-) \cdot \mathrm{n} &= \sigma(u_0, p_0) \cdot \mathrm{n} + \nu\, g & \text{on } \Gamma_-, \\ \sigma(u_0, p_0) \cdot \mathrm{n} &= \sigma(u_+, p_+) \cdot \mathrm{n} + \nu\, h & \text{on } \Gamma_+, \end{cases}$$

with (u_-, p_-) and (u_+, p_+) are periodic with respect to y_1 and y_2, with periods l_1 and l_2. Here $\nu > 0$ is the viscosity parameter and n is the unit normal vector on Γ_- (resp. Γ_+) external to Λ_- (resp. Ω), i.e. $\mathrm{n} = (0, 0, 1)$. The vector functions $g = (g', 0)$ and $h = (h', 0)$ are supposed to be given in suitable function spaces.

We study the existence and uniqueness of a solution (u, p) to the system (S) which decays exponentially fast, as well as all its derivatives, as $y_3 \to \pm\infty$.

The main result of this work is the following:

Theorem. Suppose that

$$g \in (H^{-1/2}_{per}(\Gamma_-))^3, \quad g_3 = 0, \quad h \in (H^{-1/2}_{per}(\Gamma_+))^3, \quad h_3 = 0.$$

There exists a unique solution of the system (S) (up to an additive constant for the pressures) satisfying

$$u \in \left(H^1_{per,loc}(\Lambda)\right)^3, \ \nabla u_- \in \left(L^2(\Lambda)\right)^9, \ p_- \in L^2_{per,loc}(\Lambda).$$

Moreover, let $\delta > 1$ and let $\beta_\pm$ be the mean of the velocity over cross sections of $\Lambda_\pm$, i.e.

$$\beta_- = \frac{1}{|S|} \int_S u_-(y', -\delta)\, dy', \quad \beta_+ = \frac{1}{|S|} \int_S u_+(y', \delta)\, dy'.$$

The following decay estimates hold:

- for any $\alpha \in \mathbf{N}^3$, $y' \in S$, $y_3 \le -\delta$,

$$|\partial^\alpha (u - \beta_-)(y', y_3)| + |\partial^\alpha p(y', y_3)| \le C(\delta, \alpha)\, \|g\|_{\left(H^{-1/2}(\Gamma_-)\right)^3} \exp{(c\, y_3)};$$

- for any $\alpha \in \mathbf{N}^3$, $y' \in \widetilde{S}$, $y_3 \ge \delta$,

$$|\partial^\alpha (u - \beta_+)| + |\partial^\alpha p(y', y_3)| \le C(\delta, \alpha)\, \|h\|_{\left(H^{-1/2}(\Gamma_+)\right)^3} \exp{(-c\, y_3)},$$

where $c > 0$ is a constant independent of the data and $C(\delta, \alpha)$ is a constant depending only on δ and α. The subscript “per” denotes periodic Sobolev Spaces.

This work answers a question addressed to the author by G. Panasenko. It will be be used in a forthcoming work to build boundary layers correctors in an homogenization framework.

On weakly singular kernels arising in equations set on a graph, modelling a flow in a networtk of thin tubes

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This talk follows the talk by Frédéric Chardard entitled: Numerical solution of the viscous flows in a network of thin tubes: equations on the graph. These equations are set on a 1D graph and were obtained by letting the diameters of the tubes tend to zero in some asymptotic process. They are characterized by a convolution in time in the diffusion operator, with a weakly singular kernel in time that are computed from the solution of local heat equations (with Dirichlet Conditions) in 2D domains that represent the cross sections of the tubes of the initial network. This talk is more about these kernels: theoretical results and numerical computations of the kernels. We obtain in particular asymptotic expansions for small times in different ways: for smooth cross section inspired by techniques developped by Gie, Hamouda, Jung and Temam (Singular Perturbations and Boundary Layers, Springer), or for specific cross sections (rectangles, disks, equilateral triangles) with specific techniques.

Homogenization for elliptic operators in a strip perforated along a curve

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We consider an elliptic operator in a planar infinite strip perforated by small holes along a curve: we impose mixed classical boundary conditions (Dirichlet, Neumann and Robin) on the holes, assuming that the perforation is non-periodic and satisfies rather weak assumptions.

We describe the homogenized operators, establish the norm resolvent convergence of the perturbed resolvents to the homogenized one, prove the estimates for the rate of convergence and study the convergence of the spectrum.

Based on a joint work with D. Borisov and T. Durante

Numerical solution of the viscous flows in a network of thin tubes: equations on the graph

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A non-stationary flow in a network of thin tubes is considered. Its one-dimensional approximation was proposed in a paper by G.Panasenko and K.Pileckas [1]. It consists of a set of equations with weakly singular kernels, on a graph, for the macroscopic pressure. A new difference scheme for this problem is proposed. Several variants are discussed. Stability and convergence are studied theoretically and numerically. Numerical results are compared to the direct numerical solution of the full dimension Navier-Stokes equations.

More details about the kernels will be discussed in the talk by Éric Canon: On weakly singular kernels arising in equations set on a graph, modelling a flow in a networtk of thin tubes.

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Robust parameter estimation in fluid flow models from velocity measurements

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Parameter estimation in blood flow models from measured velocity data is a key step for patient-specific hemodynamic analysis. However, the quality of the estimation can be compromised if the measurements suffer from high noise levels and aliasing artifacts, which usually affect this type of acquisitions. Moreover, due to the change in the parameter values during the parameter optimization process, efficient and robust forward solvers – unconditionally stable – are also needed.

The aim of this work is hence twofold. Firstly, we propose an easy-to-implement, unconditionally stable fractional step solver for a multiscale 3D-0D coupled flow problem, with which we can formulate non-linear Kalman filtering strategy. Secondly, we propose as adaptation a new inverse problem being able to tackle aliased and noisy velocity measurements.

Time periodic Navier-Stokes equations in a thin tube structures motivated by hemodynamic

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The time-periodic Navier-Stokes equations are considered in thin tube structures in three and two-dimensional settings with Dirichlet boundary conditions. A thin tube structure is defined as finite union of thin cylinders which are characterized by a small parameter ε which is the ratio of the height and the diameter of the cylinders. We consider the case of the finite or big coefficient before the time derivative. This setting is motivated by hemodynamic (small vessels). Theorems of existence and uniqueness of a solution are proved. Complete asymptotic expansion of a solution is constructed and justified. The method of asymptotic partial decomposition of the domain is justified for the time-periodic problem. The conductivity problem on the graph is solving using numerical methods. The numerical results are obtained in collaboration with Frédéric Chardard.

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Time-periodic Poiseuille-type solution with minimally regular flow-rate

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The nonstationary Navier–Stokes equations are studied in the infinite cylinder $\Pi = \{x = (x', x_n) \in \mathbb{R}^n : x' \in \sigma \subset \mathbb{R}^{n-1},\ -\infty < x_n < \infty, n = 2, 3\}$ under the additional condition of the prescribed time-periodic flow-rate (flux) $F(t)$. It is assumed that the flow-rate F belongs to the space $L^2(0, 2\pi)$ only. The existence and uniqueness of a time-periodic Poiseuille-type solution to this problem is proved. The time-periodic Poiseuille solution has the form $\mathbf{u}(x,t) = \big(0, \dots, 0, U(x', t)\big),\ p(x,t) = -q(t)x_n + p_0(t)$, where $(U(x', t), q(t))$ is a time-periodic solution of an inverse problem for the heat equation with a specific over-determination condition.

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Blood velocity computation inside of a human heart left atrium

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In this presentation a model of the blood flow for a geometry of LAA (left atrial appendage) will be described. The goal is to improve early prediction of thrombus formation during atrial fibrillation which can be considered as one of the triggers for stroke. There are four of the most well-known shapes of LAA: chicken wing, windsock, cauliflower and cactus. In the first two cases there is no high blood stagnation inside of LAA, therefore possibility of thrombi formation is not high enough. But the latter two oppositely due to a very low blood velocity, during atrial fibrillation convey a higher possibility for a blood clot formation. By implementing of computer tomography, we obtain the views of a human heart and construct computational mesh. Afterwards, in accordance with numerical methods for the Navier-Stokes equations we calculate blood velocity magnitude distribution inside of LAA.

This research is partially funded from European Social Fund (project No 09.3.3-LMT-K-712-01-0012) under grant agreement with the Research Council of Lithuania (LMTLT).

Asymptotically based simulation of the Stokes flow in a layer through periodic flexural plates made of beams

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Stokes fluid is flowing through a spacer fabric, a porous layer between two parallel hyperplanes with periodically distributed parallel beam latices, which are orthogonal to the hyperplanes. The flow direction is parallel to the hyperplanes and orthogonal to the latices. The top and the bottom of the spacer fabric is insulated for the in- and outflow. The thickness of the spacer fabric is assumed to be one, while the thickness of the latices or porous layers is a small parameter ε. The fluid viscosity is assumed to be $\varepsilon^3 E$, where E is the Young's modulus of the beams. Fluid-solid interaction is considered in the structure. A dimension reduction as $\varepsilon \to 0$ was considered in [1] and the latice-layer is replaced in the paper by its mean surface with a condition: the pressure jump through the surface is proportional to the biharmonic operator in the surface applied to the velocity trace at this surface. The normal component of the limit macroscopic velocity field is an H^2-function of the lattice mean-plane variable and the limit problem is non-local in time. This corresponds to the non-stationarity of the initial problem.

For the numerical computations, a further dimension reduction is performed (see [2], [3], [4]), reducing the elasticity problem to beam equations on one-dimensional lattices and further to a linear algebraic system with 6 unknown degrees of freedom in the nodes of the lattice. Using the equivalence of finite dimensional interpolated norms on segments of the lattice and in the 3D-domain spanned between periodic latices (hexaedral mesh), [5], the nodal solution on the lattices is extended by Q_1 interpolation into the fluid part and the limit problem is solved staying with just 6 degrees of freedom in the lattice nodes. The convergence estimates from the corresponding analysis will be used to estimate the numerical accuracy of the reduced dimension algorithm. Finally, local stresses in the beams and the fluid pressure will be reconstructed as in [2] with the help of interpolated and extended piece-wise polynomial function sequences which are strongly convergent to the solution.

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Steady state non-Newtonian flow with strain rate dependent viscosity in thin tube structure with no slip boundary condition

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Thin tube structures are finite unions of thin cylinders depending on the small parameter, ratio of the diameter of the cross section to the length of the cylinder. Flows in such domains model blood flow in a network of vessels. The asymptotic expansion of the solution of the steady Stokes and Navier-Stokes equations in these domains with no slip boundary condition was constructed in the papers [1], [2] and the book [3]. However, the blood exhibits a non-Newtonian rheology, when the viscosity depends on the strain rate. In the present talk we consider such rheology. Applying the Banach fixed point theorem we prove the existence and uniqueness of a solution and its regularity. An asymptotic approximation is constructed and justified by an error estimate. The first and the second authors are supported by the European Social Fund (project No 09.3.3-LMT-K-712-01-0012) under grant agreement with the Research Council of Lithuania (LMTLT).

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Unsteady micropolar fluid flow in a thick domain with multiscale oscillating roughness and a subdifferential boundary condition

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Motivated by lubrication problems involving complex fluids we consider an unsteady micropolar fluid flow in a two dimentional thick domain Ω^ε. Following [2] the problem is thus described by a non-linear coupled variational system for the fluid velocity v^ε, the pressure p^ε and the angular micro-rotation field Z^ε. We assume moreover a fluid-solid interface law of friction type modelled by a subdifferential condition (see [3]).

Existence, uniqueness and uniform estimates for $(v^\varepsilon, p^\varepsilon, Z^\varepsilon)$ are derived. Then we assume that the thickness and roughness of Ω^ε are described by multiple separated scales of periodic oscillations i.e.

$$\Omega^\varepsilon = \left\{(z_1, z_2) :\ 0 < z_1 < L,\ 0 < z_2 < \varepsilon^m h^\varepsilon(z_1)\right\}$$

with $h^\varepsilon(z_1) = h(z_1, \frac{z_1}{\varepsilon}, \frac{z_1}{\varepsilon^2}, \cdots, \frac{z_1}{\varepsilon^m})$, $0 < \varepsilon << 1$ and $m \geq 2$.

In order to study the asymptotic behaviour of the flow as ε tends to zero we apply the multiple scale convergence technique for reiterated homogenization problems ([1]). The assumption $m \geq 2$ raises several technical difficulties and leads to a new type of divergence free conditions for the limit velocity which play a crucial role in the derivation of the limit problem.

Finally we prove that the limit velocity and pressure (v^0, p^0) and angular micro-rotation field Z^0 solve a totally decoupled system of elliptic variational inequality on one hand and elliptic partial differential equation on the other hand, where the time variable appears as a parameter. Furthermore v^0, p^0 and Z^0 are uniquely determined through auxiliary well-posed problems.

[joint work with M.Boukrouche and F.Ziane]

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Modeling the evolution of COVID-19 in Lithuania

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Models based on data analysis are being developed for the spread of the COVID-19 epidemic. A mathematically based methodology was developed for the analysis and assessment of the spread of potential epidemics and their consequences. It has the potential of becoming one of the tools of the state to manage crisis situations caused by epidemics, especially in the early phase of epidemic spread. We present the model for the spread of COVID-19 in Lithuania for the period March—December, 2020. Our approach is based on the generalized SEIR model which is derived from a set of ordinary differential equations that incorporates the transition rates at which population moves from one compartment to another. It is important to note that in the initial phase of the virus spread, when the data were very limited, it was necessary to rely on the experience of other countries and adapt the used epidemiological models to Lithuania. As the modeling results show, in the early phase the generalized SEIR model showed rather accurate forecasts. Later it became possible to construct more accurate and flexible models due to a wide range of data. As with many mathematical-statistical models, the accuracy of prediction relies heavily on the quality of the available data and the level of model abstraction.

ADI scheme for partially dimension reduced heat conduction models

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In this talk, an alternating direction implicit (ADI) type finite volume numerical scheme is proposed to solve a non-classical non-stationary heat conduction problem set in a 3D tube with radial symmetry. The original 3D model is reduced to a hybrid dimension model in a large part of the domain. Special junction conditions are defined between 3D and 1D parts. The finite volume method is applied to approximate spatial differential operators and ADI splitting is used for time integration. The ADI scheme is unconditionally stable and under a mix of Dirichlet and Neumann boundary conditions the approximation error is of second order in space and time. An efficient factorization algorithm is presented to solve the obtained systems of equations. Results of computational experiments confirm the theoretical error analysis. Visual representations and computational times are compared for various sizes of reduced dimension zones, thus contributing to a conclusion that hybrid mathematical models can be used to simulate heat models for a quite broad set of domains and coefficients.

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FSI and reduced models for 3D hemodynamic simulations in time-dependent domains

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In the talk we briefly present our approach to fluid-structure interaction (FSI) simulations [6, 4] and consider three biomedical problems for flow in time-dependent domains which can be solved by simpler formulations than the FSI formulation: blood flow in the human ventricles [5, 3, 1], blood flow in the aortic bifurcation [2], coaptation characteristics of the aortic valve [7]. The numerical schemes are summarized in the book [8].

This is the joint work with M.Olshanskii, A.Lozovskiy, A.Danilov, T.Dobroserdova, G.Panassenko, V.Salamatova, A.Lyogkii.

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Minisymposium

Nonlocal operators and related topics (MS-55)

Organized by María del Mar González Nogueras, *Universidad Autónoma de Madrid, Spain*
Coorganized by
Luz Roncal, *BCAM Basc Centre for Applied Mathematics, Spain*
Juan Luis Vazquez, *Universidad Autónoma de Madrid and Real Academia de Ciencias, Spain*

- Boundary value problems for the loaded equation with integro-differential operator, *Umida Baltaeva*
- The Bernstein technique for integro-differential equations, *Xavier Cabré*
- Blow-up phenomena in nonlocal eigenvalue problems: when theories of L^1 and L^2 meet, *Hardy Chan*
- Fractional dissipations in fluid dynamics: the surface quasigeostrophic equation, *Maria Colombo*
- Local asymptotics and unique continuation from boundary points for fractional equations, *Veronica Felli*
- Three balls inequalities for discrete Schrödinger operators, *Aingeru Fernández Bertolin*
- Non-local ODEs in conformal geometry, *María Del Mar González*
- Uniqueness issues for nonlinear diffusions, *Gabriele Grillo*
- Blow-up analysis of conformal metrics with prescribed curvatures on the disk, *María Medina*
- Uniqueness of very weak solutions for a fractional nonlinear diffusion, *Matteo Muratori*
- Should I stay or should I go? Zero-size jumps in random walks for Lévy flights, *Gianni Pagnini*
- A heat equation with memory: large-time behavior, *Fernando Quirós*
- On the dynamics of Ginzburg-Landau vortices on a Riemannian Manifold, *Antonio Segatti*
- Global Harnack principle for a class of fast diffusion equations, *Nikita Simonov*
- Nonlocal minimal graphs in the plane are generically sticky, *Enrico Valdinoci*
- Nonlinear fractional parabolic equations, *Juan Luis Vazquez*
- On boundary decay of harmonic functions, Green kernels and heat kernels for some non-local operators, *Zoran Vondraček*

Boundary value problems for the loaded equation with integro-differential operator

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It is well known that fractional derivatives (e.g., Riemann-Liouville and Caputo) were widely used to model the complex phenomenon in science and engineering practice. In this connection, linear fractional partial differential equations models are commonly encountered in applied mathematics and engineering.

On the other hand, in recent years it has become increasingly important to investigate a new class of equations, known as loaded equations, as a direct result of issues with the optimal control of the agro economical system, long-term forecasting and regulating the level of ground water and soil moisture. However, we would like to note that boundary value problems for the loaded equations of a mixed types with the integro-differential operator are not well studied. Hence, the main aim of the work is to establish unique solvability boundary value problems for the loaded integro-differential equations associated with non-local problems, for the classical partial differential equations.

The Bernstein technique for integro-differential equations

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In this talk I will present a joint work with S. Dipierro and E. Valdinoci in which we extend the classical Bernstein technique to the setting of integro-differential operators. As a consequence, we provide first and one-sided second derivative estimates for solutions to fractional equations, including some convex fully nonlinear equations of order smaller than two, for which we prove uniform estimates as their order approaches two. Our method is new even in the linear integro-differential case. We will also raise some intriguing open questions, one of them concerning the "pure" linear fractional Laplacian.

Blow-up phenomena in nonlocal eigenvalue problems: when theories of L^1 and L^2 meet

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We develop a linear theory of very weak solutions for nonlocal eigenvalue problems $\mathcal{L}u = \lambda u + f$ involving integro-differential operators posed in bounded domains with homogeneous Dirichlet exterior condition, with and without singular boundary data. We consider mild hypotheses on the Green's function and the standard eigenbasis of the operator. The main examples in mind are the fractional Laplacian operators.

Without singular boundary datum and when λ is not an eigenvalue of the operator, we construct an L^2-projected theory of solutions, which we extend to the optimal space of data for the operator $\mathcal{L}$. We present a Fredholm alternative as λ tends to the eigenspace and characterise the possible blow-up limit. The main new ingredient is the transfer of orthogonality to the test function.

We then extend the results to singular boundary data and study the so-called large solutions, which blow up at the boundary. For that problem we show that, for any regular value λ, there exist "large eigenfunctions" that are singular on the boundary and regular inside. We are also able to present a Fredholm alternative in this setting, as λ approaches the values of the spectrum.

We also obtain a maximum principle for weighted L^1 solutions when the operator is L^2-positive. It yields a global blow-up phenomenon as the first eigenvalue is approached from below.

Finally, we recover the classical Dirichlet problem as the fractional exponent approaches one under mild assumptions on the Green's functions. Thus "large eigenfunctions" represent a purely nonlocal phenomenon.

Fractional dissipations in fluid dynamics: the surface quasigeostrophic equation

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The surface quasigeostrophic equation (SGQ) is a 2d physical model equation which emerges in meteorology and shares many of the essential difficulties of 3d fluid dynamics. In the supercritical regime for instance, where dissipation is modelled by a fractional Laplacian of order less than 1/2, it is not known whether or not smooth solutions blow-up in finite time.

The goal of the talk is to show that every L^2 initial datum admits an a.e. smooth solution of the dissipative surface quasigeostrophic equation (SGQ); more precisely, we prove that those solutions are smooth outside a compact set (away from t=0) of quantifiable Hausdorff dimension. We draw analogies between SQG and other PDEs in fluid dynamics in several aspects, including the partial regularity results, and underline some extra structure that SQG enjoys.

This is a joint work with Silja Haffter (EPFL).

Local asymptotics and unique continuation from boundary points for fractional equations

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Coauthors: Alessandra De Luca, Stefano Vita

In this talk I will present some results in collaboration with A. De Luca and S. Vita on unique continuation and local asymptotics of solutions to fractional elliptic equations at boundary points, under some outer homogeneous Dirichlet boundary conditions. I will describe a blow-up procedure which involves an Almgren type monotonicity formula and provides a classification of all possible homogeneity degrees of limiting entire profiles. The Caffarelli-Silvestre extension provides an equivalent formulation of the fractional equation as a local degenerate or singular problem in one dimension more, with mixed Dirichlet and Neumann boundary conditions. In the development of a monotonicity argument, the mixed boundary condition raises delicate regularity issues, which turn out to be quite difficult in dimension $N \geq 2$ due to the positive dimension of the junction set and some role played by the geometry of the domain. Such difficulties are overcome by a double approximation procedure: by approximating the potential with functions vanishing near the boundary and the Dirichlet N-dimensional region with smooth $(N+1)$-dimensional sets with straight vertical boundary, it is possible to construct a sequence of approximating solutions which enjoy enough regularity to derive Pohozaev type identities, needed to obtain Almgren type monotonicity formulas and consequently to perform blow-up analysis.

Three balls inequalities for discrete Schrödinger operators

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Coauthors: Luz Roncal, Angkana Rüland, Diana Stan

In this talk we will study the so-called three balls inequalities in the discrete setting, which in the continuous setting would be stated as inequalities of the form

$$\|u\|_{L^2(B_1)} \leq C\|u\|^{\alpha}_{L^2(B_{1/2})}\|u\|^{1-\alpha}_{L^2(B_2)},$$

for some $\alpha \in (0,1)$ and functions u such that $Pu = 0$ for a given operator P. Such an inequality is used then to study propagation of smallness and unique continuation.

In the discrete setting it is known that these properties cannot be true in general, even in the simplest case of discrete harmonic functions. However, one can prove a similar inequality in the lattice with a correction term that tends to zero as the step size of the lattice tends to zero.

In this talk we will see how we can extend this discrete inequality for the discrete Laplacian to a large variety of discrete magnetic Schrödinger operators, by means of a discrete Carleman estimate for the discrete Laplace operator.

Non-local ODEs in conformal geometry

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When one looks for radial solutions of an equation with fractional Laplacian, it is not generally possible to use usual ODE methods. If such equation has some conformal invariances, then one may rewrite it in Emden-Fowler (cylindrical) coordinates and to use the properties of the conformal fractional Laplacian on the cylinder. After giving the necessary background, we will briefly consider two particular applications of this technique: 1. Symmetry breaking, non-degeneracy and uniqueness for the fractional Caffarelli-Kohn-Nirenberg inequality (joint work with W. Ao and A. DelaTorre). 2. Existence and regularity for fractional Laplacian equations with drift and a critical Hardy potential (joint with H. Chan, M. Fontelos and J. Wei).

Uniqueness issues for nonlinear diffusions

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I discuss uniqueness, and nonuniqueness, of solutions to classes of nonlinear diffusions in different contexts, that include slow and fast diffusions on general manifolds and the fractional porous medium equation on the hyperbolic space.

Blow-up analysis of conformal metrics with prescribed curvatures on the disk

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We will establish necessary conditions on the blow-up points of conformal metrics of the disk with prescribed Gaussian and geodesic curvatures, where a non local restriction will appear. Conversely, given a point satisfying these conditions, we will construct an explicit family of approximating solutions that explode at such a point. These results are contained in several works in collaboration with A. Jevnikar, R. López-Soriano and D. Ruiz, and with L. Battaglia and A. Pistoia.

Uniqueness of very weak solutions for a fractional nonlinear diffusion

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In this talk I will present some recent results, obtained in collaboration with G. Grillo and F. Punzo, dealing with existence and uniqueness of (distributional) bounded solutions to a fractional parabolic equation of porous medium type, posed in the whole Euclidean space. Existence is established by means of a classical approximation scheme, while uniqueness is much more involved and requires a careful analysis of a suitable dual problem.

Should I stay or should I go? Zero-size jumps in random walks for Lévy flights

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Motivated by the fact that, in the literature dedicated to random walks for anomalous diffusion, it is disregarded if the walker does not move in the majority of the iterations because the most frequent jump-size is zero (i.e., the jump-size distribution is unimodal with mode located in zero) or, in opposition, if the walker always moves because the jumps with zero-size never occur (i.e., the jump-size distribution is bi-modal and equal to zero in zero), we provide an example in which indeed the shape of the jump-distribution plays a role. In particular, we show that the convergence of Markovian continuous-time random walk (CTRW) models for Lévy flights to a density function that solves the fractional diffusion equation is not guaranteed when the jumps follow a bi-modal power-law distribution equal to zero in zero, but, as a matter of fact, the resulting diffusive process converges to a density function that solves a double-order fractional diffusion equation. Within this framework, self-similarity is lost. The consequence of this loss of self-similarity is the emergence of a time-scale for realizing the large-time limit. Such time-scale results to span from zero to infinity accordingly to the power-law displayed by the tails of the walker's density function. Hence, the large-time limit could not be reached in real systems. The significance of this result is two-fold: i) with regard to the probabilistic derivation of the fractional diffusion equation and also ii) with regard to recurrence and the related concept of site fidelity in the framework of Lévy-like motion for wild animals.

A heat equation with memory: large-time behavior

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We study the large-time behavior in all L^p norms and in different space-time scales of solutions to the Cauchy problem for a heat equation with a Caputo α-time derivative. The initial data are assumed to be integrable, and, when required, to be also in L^p. A main difficulty in the analysis comes from the singularity in space at the origin of the fundamental solution of the equation when $N > 1$.

In the characteristic scale $|x| \asymp t^{\alpha/2}$, dictated by the scaling invariance of the equation, solutions behave, when properly scaled to kill their decay, like M times the fundamental solution, where M is the integral of the initial datum. In compact sets they converge to the newtonian potential of the initial datum if $N \geq 3$, one of the main novelties of the paper, and to a constant if $N = 1, 2$, with a logarithmic correction in the decay rate for the critical dimension $N = 2$. In intermediate scales, going to infinity more slowly than the characteristic one, solutions approach a multiple of the fundamental solution of the laplacian if $N \geq 3$, and a constant in low dimensions, again with logarithmic corrections for the critical dimension.

The asymptotic behavior in scales that go to infinity faster than the characteristic one depends strongly on the behavior of the initial datum at infinity. We give results for certain initial data with specific decays.

Joint work with C. Cortázar and N. Wolanski

On the dynamics of Ginzburg-Landau vortices on a Riemannian Manifold

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We consider a Ginzburg-Landau energy defined for vector fields u on a 2 dimensional closed Riemannian manifold. The Ginzburg-Landau energy depends on a small parameter $\varepsilon > 0$ that favors configurations with $|u| = 1$. It is has been recently proved by R. Jerrard & R. Ignat that under appropriate hypothesis, as $\varepsilon \to 0$, a finite number of point vortices emerges. The number and the topological charges of the vortices are related to the topology of the manifold. Moreover, the positions of the vortices is governed by the so called renormalized energy. The goal of the talk is to show that the vortices move, as in the two dimensional euclidean case, according to the gradient flow of the renormalized energy. More precisely, we prove that the gradient flow of the renormalized energy emerges as the Γ-*convergence of gradient flows* (in the sense of E. Sandier & S. Serfaty) limit of the gradient flow of the Ginzburg-Landau energy. This is a joint work with G. Canevari (Verona).

Global Harnack principle for a class of fast diffusion equations

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Coauthors: Matteo Bonforte, Diana Stan

We study global properties of non-negative, integrable solutions to the Cauchy problem of the weighted fast diffusion equation $u_t = |x|^\gamma \mathrm{div}(|x|^{-\beta}\nabla u^m)$ with $(d-2-\beta)/(d-\gamma) < m < 1$. The weights $|x|^\gamma$ and $|x|^{-\beta}$, with $\gamma < d$ and $\gamma - 2 < \beta \le \gamma(d-2)/d$ can be both degenerate and singular and need not belong to the class $\mathcal{A}_2$, this range of parameters is optimal for the validity of a class of Caffarelli-Kohn-Nirenberg inequalities.

We characterize the largest class of data for which the so-called Global Harnack Principle (GHP) holds (a global lower and upper bound in terms of suitable Barenblatt solutions). As a consequence of the GHP, we prove convergence of the *uniform relative error*, namely $|(u - \mathfrak{B})/\mathfrak{B}| \to 0$ as $t \to \infty$ uniformly in R^d, where $\mathfrak{B}$ is a suitable Barenblatt solution. In the case with no weights ($\gamma = \beta = 0$) and for a special class of data, we give (almost) sharp rates of convergence to the Barenblatt profile in the L^1 and the L^∞ topologies, in the radial case we give sharp rates.

We extend some of the results to non-negative, integrable solutions to the Cauchy problem of the p-Laplace evolution equation $u_t = \Delta_p u$, where $\Delta_p w := \mathrm{div}(|\nabla w|^{p-2}\nabla w)$, with $2d/(d+1) < p < 2$.

The above results were obtained in collaboration with Prof. M. Bonforte and D. Stan.

Nonlocal minimal graphs in the plane are generically sticky

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University of Western Australia, Australia

We discuss some recent boundary regularity results for nonlocal minimal surfaces in the plane. In particular, we show that nonlocal minimal graphs in the plane exhibit generically stickiness effects and boundary discontinuities. More precisely, if a nonlocal minimal graph in a slab is continuous up to the boundary, then arbitrarily small perturbations of the far-away data necessarily produce boundary discontinuities. Hence, either a nonlocal minimal graph is discontinuous at the boundary, or a small perturbation of the prescribed conditions produces boundary discontinuities. The proof relies on a sliding method combined with a fine boundary regularity analysis, based on a discontinuity/smoothness alternative. Namely, we establish that nonlocal minimal graphs are either discontinuous at the boundary or their derivative is Hölder continuous up to the boundary. In this spirit, we prove that the boundary regularity of nonlocal minimal graphs in the plane "jumps" from discontinuous to differentiable, with no intermediate possibilities allowed. In particular, we deduce that the nonlocal curvature equation is always satisfied up to the boundary. As a byproduct of our analysis, one describes the "switch" between the regime of continuous (and hence differentiable) nonlocal minimal graphs to that of discontinuous (and hence with differentiable inverse) ones. These results have been obtained in collaboration with Serena Dipierro and Ovidiu Savin.

Nonlinear fractional parabolic equations

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We report on the progress done on the theory of evolution equations that combine the strongly nonlinear parabolic character with the presence of fractional operators representing long-range interaction effects. The results deal with the topics of optimal existence, regularity, self-similarity, and asymptotics.

On boundary decay of harmonic functions, Green kernels and heat kernels for some non-local operators

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In this talk, I will discuss non-local operators in open subsets of Euclidean space with critical potentials and kernels admitting a decay at the boundary. The focus will be on the boundary decay of non-negative harmonic functions, Green kernels, and heat kernels. I will explain how decay depends on the critical potential and the possible decay of the kernel at the boundary

Minisymposium

NONSMOOTH VARIATIONAL METHODS FOR PDES AND APPLICATIONS IN MECHANICS (MS-8)

Organized by Victor A. Kovtunenko, *University of Graz, Austria*
Coorganized by
Hiromichi Itou, *Tokyo University of Science, Department of Mathematics, Japan*
Alexandr M. Khludnev, *Novosibirsk State University, Department of Mathematics and Mechanics, and Lavrentyev Institute of Hydrodynamics SB RAS, Russian Federation*
Evgeny M. Rudoy, *Novosibirsk State University, Department of Mathematics and Mechanics, and Lavrentyev Institute of Hydrodynamics SB RAS, Russian Federation*

- On hysteresis reaction-diffusion systems and application in population dynamics, *Klemens Fellner*
- Lagrange multipliers and nonconstant gradient constrained problem, *Sofia Giuffrè*
- On an inverse crack problem in a linearized elasticity by the enclosure method, *Hiromichi Itou*
- Analysis and Numerical Experiments of a Variance Reduction Technique for Effective Energies of Random Atomic Lattices, *Michael Kniely*
- Shape differentiability of semilinear equilibrium-constrained optimization, *Victor A. Kovtunenko*
- Singular perturbation approximation for the Kuramoto-Sakaguchi integro-differential model, *Mikhail Lavrentiev*

On hysteresis reaction-diffusion systems and application in population dynamics

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University of Graz, Austria

We consider a general class of PDE-ODE reaction-diffusion systems, which exhibits a singular fast-reaction limit towards a reaction-diffusion equation coupled to a scalar hysteresis operator.

As applicational motivation, we present a PDE model for the growth of a population according to a given food supply coupled to an ODE for the turnover of a food stock. Under realistic conditions the stock turnover is much faster than the population growth yielding an intrinsic scaling parameter. We emphasise that the structural assumptions on the considered PDE-ODE models are quite general and that analogue systems might describe e.g. cell-biological buffer mechanisms, where proteins are stored and used at the same time.

Finally, we present a new kind of hysteresis-diffusion driven instability caused by the non-linear coupling between a reaction-diffusion equation and a scalar generalised play operator. We discuss in detail how this coupling with a generalised play operator can lead to spatially inhomogeneous large-time behaviour or equilibration to a homogeneous state.

Lagrange multipliers and nonconstant gradient constrained problem

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The talk is aimed at studying a gradient constrained problem associated to a linear operator. This classical problem was subject to an intense study a few decades ago (see [1, 2, 4, 6, 8], but some very important issues were left open. In particular, we are able to prove two kinds of results (see [5]): first, we prove the equivalence of a non-constant gradient constraint problem to a suitable obstacle problem, where the obstacles solve a Hamilton-Jacobi equation in the viscosity sense (see [7]) and, second, we obtain the existence of Lagrange multipliers associated to the problem. The Lagrange multipliers exist as a Radon measure in the case that the free term of the equation $f \in L^p$, $p > 1$, whereas, if f is a positive constant, it is possible to regularize the result, namely to prove that they belong to L^2. These results have been obtained, using a new theory of infinite dimensional duality contained in [3]. The classical strong duality theory does not work in an infinite dimensional setting, when the interior of the ordering cone of the sign constraints is empty and this new theory overcomes this difficulty.

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On an inverse crack problem in a linearized elasticity by the enclosure method

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In this talk, we discuss a reconstruction problem for several linear cracks located on a line between two linearized elastic plates from measured data which are a loading surface traction and the resulted displacement field on the boundary of the joined plates. This is a typical problem from the nondestructive testing of materials. For this problem, we introduce an extraction formula of the cracks from a single set of the data by means of the enclosure method. In the case of a *single* linear crack, the extraction formula of the location and shape of an unknown crack is established by using the enclosure method [2]. However, this result cannot be extended to *several* cracks case directly because the original enclosure method can give an extraction formula of the convex hull of cracks. As one of ways to overcome the difficulty, we apply the Kelvin transform to the indicator function of the classical enclosure method. In [1, 3], by virtue of this transform we derived extraction procedure of information about the location of tips of several cracks located on a line between two electric conductive plates from a single set of an electric current density and the corresponding voltage potential on the boundary of the material formed by the plates. In the present talk, I will consider further extension of the result [1] to the linearized elastic case.

This research is based on a joint work with Masaru Ikehata (Hiroshima University) and is partially supported by Grant-in-Aid for Scientific Research (C)(No. 18K03380) and (B)(No. 17H02857) of Japan Society for the Promotion of Science and JSPS and RFBR under the Japan - Russia Research Cooperative Program (project No. J19-721).

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Analysis and Numerical Experiments of a Variance Reduction Technique for Effective Energies of Random Atomic Lattices

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Coauthor: Julian Fischer

We discuss the calculation of effective energies of random materials modeled by the Thomas–Fermi–von Weizsäcker (TFW) equations in the framework of the method of representative volume elements (RVEs). The TFW equations constitute a coupled system of nonlinear elliptic equations and describe the distribution of electrons in the presence of a prescribed nuclear charge density. The representative volume approximation is subject to a systematic error (due to the restriction to finite material samples) and a random error (due to material differences in different RVEs). Our emphasis lies on the reduction of the variance of the energy when evaluated for the RVE, as the systematic error decreases exponentially as a function of the diameter of the RVE. This variance reduction can be achieved by selecting the RVE in such a way that it represents the statistical properties of the underlying material particularly well, an approach proposed by Le Bris, Legoll, and Minvielle in the numerical homogenization of linear elliptic equations. A rigorous analysis of this strategy has been carried out recently by Fischer for linear elliptic PDEs.

For establishing the variance reduction in the case of the nonlinear TFW equations, we need a locality result which ensures that perturbations of the nuclear density inside a bounded region result in a change of the electronic density decaying exponentially away from this region. We prove the required locality by extending a recent result by Nazar and Ortner for smeared nuclear charges to the case of point nuclei represented by Dirac measures. We finally illustrate the performance of the proposed selection method for RVEs compared to the standard RVE approach by calculating the energy per atom of random AlTi lattices on RVEs of different size.

Shape differentiability of semilinear equilibrium-constrained optimization

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Coauthor: Karl Kunisch

A class of semilinear optimization problems linked to variational inequalities is studied with respect to its shape differentiability. One typical example stemming from quasi-brittle fracture describes an elastic body with a Barenblatt cohesive crack under the inequality condition of non-penetration at the crack faces. The other conceptual model is described by a generalized Stokes-Brinkman-Forchheimer's equation under divergence-free and mixed boundary conditions. Based on the Lagrange multiplier approach and using suitable regularization, an analytical formula for the shape derivative is derived from the Delfour-Zolesio theorem. The explicit expression contains both primal and adjoint states and is useful for finding descent direction of a gradient algorithm to identify an optimal shape, e.g., from boundary measurement data.

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Singular perturbation approximation for the Kuramoto-Sakaguchi integro-differential model

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The Kuramoto-Sakaguchi (or Kuramoto) model was generalized in 1996 so to take into account *inertial* effects of the nonlinearly coupled random oscillators. The ensuing nonlinear integro-differential partial differential equation is of the Fokker-Planck type, possesses several peculiarities, and was studied in the following years under the restrictive hypothesis that the oscillators frequency distribuiton had a *bounded* support. In this paper, such an assumption is relaxed, and existence, uniqueness, and regularity of solution are established.

Minisymposium

PDE models in life and social sciences (MS-71)

Organized by Lisa Maria Kreusser, *University of Cambridge, United Kingdom*
Coorganized by Jan Haskovec, *KAUST, Saudi Arabia*

- Pedestrian Models with Congestion Effects, *Rafael Bailo*
- Phase separation in active Brownian particles, *Maria Bruna*
- Linking graph Allen–Cahn and MBO with fidelity, towards applications in classification and imaging, *Jeremy Budd*
- Particle methods for local mean-field games, *Diogo Gomes*
- Asymptotic consensus in the Hegselmann-Krause model with finite speed of information propagation, *Jan Haskovec*
- Mean-field optimal control for biological pattern formation, *Lisa Maria Kreusser*
- A Hamilton-Jacobi formalism for the study of propagation in reaction-subdiffusion systems, *Álvaro Mateos González*
- Classification and stability analysis of polarising and depolarising travelling wave solutions for a model of collective cell migration., *Dietmar Oelz*
- On Generalised Gradient Flows, *Oliver Tse*
- Asymptotic behaviour of the run and tumble equation for bacterial chemotaxis, *Havva Yoldaş*
- Kinetic and macroscopic models for epidemic dynamics, *Mattia Zanella*

Pedestrian Models with Congestion Effects

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Coauthors: Pedro Aceves-Sanchez, Pierre Degond, Zoé Mercier

We propose a macroscopic model for pedestrian dynamics with a congestion effect. Said effect is achieved through the introduction of a singular pseudo-pressure term in the hydrodynamic formulation of the model, resulting in an asymptotic transition between two different dynamics. To handle the numerical simulation of the model, we adapt an implicit, asymptotic-preserving scheme that has proven successful in dealing with Euler system with capacity constraints. We present numerical examples and discuss their validity with respect to the microscopic setting.

Phase separation in active Brownian particles

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In this talk, I will discuss models for active matter systems consisting of many self-propelled particles. These can be used to describe biological systems such as bird flocks, fish schools, and bacterial suspensions. In contrast to passive particles, these systems can undergo phase separation without any attractive interactions, a mechanism known as motility-induced phase separation. Starting with a microscopic model for active Brownian particles with repulsive interactions, I will discuss four possible macroscopic PDEs (ranging from a nonlocal model to a local cross-diffusion system). I will discuss work in progress concerning the stability and analysis of such models.

Linking graph Allen–Cahn and MBO with fidelity, towards applications in classification and imaging

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Coauthor: Yves van Gennip

An emerging technique in clustering, segmentation and classification problems is to consider the dynamics of flows defined on finite graphs. In particular Bertozzi and co-authors considered dynamics related to Allen–Cahn flow (Bertozzi, Flenner, 2012) and the MBO algorithm (Merkurjev, Kostic, Bertozzi, 2013) for this purpose. In (Budd, Van Gennip, 2019, arxiv preprint) the authors showed that the MBO algorithm is a special case of a "semi-discrete" scheme for Allen–Cahn flow.

In this talk, we extend this semi-discrete link to the case with fidelity forcing. Furthermore, this semi-discrete scheme yields a family of MBO-like schemes for classification applications, which we shall explore as alternatives to the original MBO scheme.

Particle methods for local mean-field games

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We study a particle approximation for one-dimensional first-order mean-field games (MFGs) with local interactions with planning conditions. Our problem comprises a system of a Hamilton-Jacobi equation coupled with a transport equation. As we are dealing with the planning problem, we prescribe initial and terminal distributions for the transport equation. The particle approximation builds on a semi-discrete variational problem. First, we address the existence and uniqueness of the semi-discrete variational problem. Next, we show that our discretization preserves some conserved quantities. Finally, we prove that the approximation by particle systems preserves displacement convexity. We use this last property to establish uniform estimates for the discrete problem. All results for the discrete problem are illustrated with numerical examples.

Asymptotic consensus in the Hegselmann-Krause model with finite speed of information propagation

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We introduce a variant of the Hegselmann-Krause model of consensus formation where information between agents propagates with a finite speed $\mathfrak{c} > 0$. This leads to a system of ordinary differential equations (ODE) with state-dependent delay. Observing that the classical well-posedness theory for ODE systems does not apply, we provide a proof of global existence and uniqueness of solutions of the model. We prove that asymptotic consensus is always reached in the spatially one-dimensional setting of the model, as long as agents travel slower than $\mathfrak{c}$. We also provide sufficient conditions for asymptotic consensus in the spatially multi-dimensional setting. Finally, we discuss the mean-field limit of the model, showing that it does not facilitate a description in terms of a Fokker-Planck equation.

Mean-field optimal control for biological pattern formation

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University of Cambridge, United Kingdom

In this talk I will discuss a class of interacting particle models with anisotropic repulsive-attractive interaction forces. These models are motivated by the simulation of fingerprint databases, which are required in forensic science and biometric applications. In existing models, the forces are isotropic and particle models lead to non-local aggregation PDEs with radially symmetric potentials. The central novelty in the models I consider is an anisotropy which can describe complex patterns accurately. I will discuss the role of anisotropic interaction in the forward models. Then, I will propose a mean-field optimal control problem for the parameter identification of a given pattern which is an inverse problem. The cost functional is based on the Wasserstein distance between the probability measures of the modeled and the desired patterns. The first-order optimality conditions corresponding to the optimal control problem are

derived using a Lagrangian approach on the mean-field level. Based on these conditions we propose a gradient descent method to identify relevant parameters such as angle of rotation and force scaling which may be spatially inhomogeneous. We discretize the first-order optimality conditions in order to employ the algorithm on the particle level. Moreover, we prove a rate for the convergence of the controls as the number of particles used for the discretization tends to infinity. Numerical results for the spatially homogeneous case demonstrate the feasibility of the approach. This is joint work with M. Burger (FAU Erlangen-Nürnberg) and C. Totzeck (Wuppertal).

A Hamilton-Jacobi formalism for the study of propagation in reaction-subdiffusion systems

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Coauthors: Vincent Calvez, Pierre Gabriel

Certain intracellular protein exhibit random motion that deviates from standard diffusion due to trapping phenomena. These systems may be described by a probability density function $n(t, x, a)$ depending on time t, space x, and also on a structural memory or 'age' variable a that allows to account for the trapping. Roughly speaking, n is governed by a renewal equation in (t, a) (with a heavy-tailed waiting times distribution) coupled with spatial relocation at renewal.

I will motivate and give an overview of certain results obtained at the end of my PhD thesis on the hyperbolic space-time asymptotics of those equations, how they tend to a limiting Hamilton-Jacobi equation, and what this means. The interesting features of our work lie in how we dealt with complications in the limit procedure due to the memory effects being 'non-Markovian' in a certain sense.

Classification and stability analysis of polarising and depolarising travelling wave solutions for a model of collective cell migration.

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University of Queensland, Australia

We study travelling wave solutions of a 1D continuum model for collective cell migration in which cells are characterised by position and polarity. Four different types of travelling wave solutions are identified which represent polarisation and depolarisation waves resulting from either colliding or departing cell sheets as observed in model wound experiments. We study the linear stability of the travelling wave solutions numerically and using spectral theory. This involves the computation of the Evans function most of which we are able to carry out explicitly, with one final step left to numerical simulation.

On Generalised Gradient Flows

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Evolution equations in spaces of measures describe a wide variety of natural phenomena. The theory for such evolutions has seen tremendous growth in the last decades, of which resulted in general metric space theories for analysing variational evolutions—evolutions driven by one or more energies/entropies. On the other hand, physics and large-deviation theory suggest the study of *generalised* gradient flows—gradient flows with non-homogeneous dissipation potentials—which are not covered in metric space theories. In this talk, we introduce a framework for these generalisations and provide examples of why and how they play a central role in a large class of variational evolutions.

Asymptotic behaviour of the run and tumble equation for bacterial chemotaxis

Havva Yoldaş, yoldas@math.univ-lyon1.fr
Claude Bernard University of Lyon 1, France
Coauthor: Josephine Evans

In this talk, I will present a recent work (arXiv:2103.16524) on the long-time behaviour of the run and tumble equation which is a kinetic-transport equation modelling the bacteria movement under the effect of chemical stimulus. The movement of bacteria is a combination of a transport with a constant velocity, run, and a random change in the direction of the movement, tumble. We show the exponential convergence to unique stationary state for the linear run and tumble equation. This result is an improvement of a recent work by Mischler and Weng (Kinet. Relat. Models, 10 (3), 799-822 (2017)). We also consider a weakly nonlinear equation with a nonlocal coupling on the chemoattractant concentration. We construct a unique stationary solution for the weakly non-linear equation and show the exponential convergence towards it. I will also mention how this result give insights of tackling the higher nonlinearities with more physically relevant couplings. The last part is a subject of an ongoing work. This talk is based on joint works with Josephine Evans (University of Warwick).

Kinetic and macroscopic models for epidemic dynamics

Mattia Zanella, mattia.zanella@unipv.it
University of Pavia, Italy

We introduce a mathematical description of the impact of sociality in the spread of infectious diseases by integrating epidemiological dynamics with a kinetic modeling of population-based contacts. The kinetic description leads to study the evolution over time of Boltzmann-type equations describing the number densities of social contacts of compartmental models in epidemiology. Explicit calculations show that the spread of the disease is closely related to moments of the contact distribution. Motivated by the COVID-19 pandemic, part of the talk will be dedicated to the calibration of the proposed model based on data of the Province of Pavia thanks to an ongoing collaboration with the regional health agency. We conduct numerical experiments which confirm the ability of the model to describe different phenomena characteristic of the rapid spread of an epidemic.

Minisymposium

Partial differential equations describing far-from-equilibrium open systems (MS-51)

Organized by Miroslav Bulíček, *Charles University, Faculty of Mathematics and Physics, Czech Republic*
Coorganized by Dalibor Pražák, *Charles University, Czech Republic*

- On the stability of generalized viscous heat-conducting incompressible fluids with non-homogeneous boundary temperature, *Anna Abbatiello*
- Bifurcations, pattern formation and synchronization in a few RD systems and networks of RD systems, *Benjamin Ambrosio*
- A quantitative approach to the Navier-Stokes equations, *Tobias Barker*
- Polynomial decay of solutions to integrodifferential equations, *Tomas Barta*
- Far-from-equilibrium open systems: problems and tasks, *Miroslav Bulíček*
- Nonlinear inviscid damping and shear-buoyancy instability in the two-dimensional Boussinesq equations, *Michele Coti Zelati*
- Stability of trivial defect solutions, *Paige Davis*
- Non-isothermal viscoelastic flows with conservation laws and relaxation, *Mark Dostalík*
- Modelling and analysis for multicomponent incompressible fluids, *Pierre-Etienne Druet*
- Ergodic theory for energetically open fluid systems, *Eduard Feireisl*
- A semismooth Newton method for implicitly constituted flow, *Pablo Alexei Gazca Orozco*
- Uniqueness and regularity of flows of non-Newtonian fluids below critical power-law growth, *Petr Kaplicky*
- On planar flows of viscoelastic fluids of the Burgers type, *Tomas Los*
- A simple thermodynamic framework for heat-conducting flows of mixtures of two mechanically interacting fluids, *Josef Málek*
- Global existence analysis of fractional cross-diffusion systems, *Erika Maringová*
- Thermodynamics of viscoelastic rate-type fluids and its implications for stability analysis, *Vít Průša*
- The Maxwell-Stefan system, its gradient flow structure, and the problem of uniqueness of weak solutions, *Athanasios Tzavaras*
- Hydrodynamic stability for the dynamic slip, *Michael Zelina*

On the stability of generalized viscous heat-conducting incompressible fluids with non-homogeneous boundary temperature

Anna Abbatiello, anna.abbatiello@uniroma1.it
"Sapienza" Università di Roma, Italy

The motions of a generalized viscous heat-conducting incompressible fluid are governed by the non-standard Navier-Stokes-Fourier system where the non-linear viscosity depends on the shear-rate and the temperature. Assuming the fluid occupies a mechanically isolated container with a spatially non-homogeneous temperature boundary condition, the issue of stability concerns the investigation of the long-time behaviour of the fluid, which is expected to reach a steady state. The steady state is the state where the velocity field vanishes and the steady temperature field satisfies the steady heat equation with non-homogeneous boundary temperature. The aim of our study is to develop a rigorous stability analysis in the setting of weak solutions satisfying the equation for the entropy production. This is a joint work with Miroslav Bulíček and Petr Kaplický.

Bifurcations, pattern formation and synchronization in a few RD systems and networks of RD systems

Benjamin Ambrosio, benjamin.ambrosio@univ-lehavre.fr
Le Havre Normandie University, France

In this talk, I will provide theoretical and numerical insights in bifurcation phenomena and pattern formation occurring in some nonlinear reaction diffusion systems. I will also discuss the synchronization phenomenon in networks of reaction-diffusion systems. This includes the synchronization of patterns.

A quantitative approach to the Navier-Stokes equations

Tobias Barker, tobiasbarker5@gmail.com
University of Warwick, United Kingdom
Coauthor: Christophe Prange

It remains open as to whether or not the 3D Navier-Stokes equations lose smoothness ('blow-up') in finite time. Very recently, Terence Tao used a new quantitative approach to infer that certain 'slightly supercritical' quantities for the Navier-Stokes equations must become unbounded near a potential blow-up time. In this talk I'll discuss a new strategy for proving quantitative bounds for the Navier-Stokes equations, as well as applications to behaviours of potentially singular solutions.

Polynomial decay of solutions to integrodifferential equations

Tomas Barta, barta@karlin.mff.cuni.cz
Charles University, Prague, Czech Republic

We study convergence to equilibrium for second order integrodifferential equations. The speed of convergence depends on the decay of the convolution kernel, the right-hand side and the Lojasiewicz exponent. The result is valid provided one a-priori knows that the solution has a precompact trajectory.

Far-from-equilibrium open systems: problems and tasks

Miroslav Bulíček, mbul8060@karlin.mff.cuni.cz
Charles University, Czech Republic

Most of the processes that are of interest in continuum thermodynamics take place far from equilibrium. The existing mathematical approaches applicable for the description of the dissipative structures however face principal difficulties. Either the physically relevant description is achieved at the price of sacrificing the mathematical rigour, or the existing approaches are fully rigorous, but they are of no physical relevance. We discuss certain possibilities how to overcome such problem and provide also several ideas what are the possible directions of further research.

Nonlinear inviscid damping and shear-buoyancy instability in the two-dimensional Boussinesq equations

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Imperial College London, United Kingdom

We investigate the long-time properties of the two-dimensional inviscid Boussinesq equations near a stably stratified Couette flow. We prove that the system experiences a shear-buoyancy instability: the density variation and velocity undergo inviscid damping while the vorticity and density gradient grow. The result holds at least until a natural, nonlinear timescale. The proof relies on several ingredients: (A) a suitable symmetrization that makes the linear terms amenable to energy methods and takes into account the classical Miles-Howard spectral stability condition; (B) a variation of the Fourier time-dependent energy method introduced for the inviscid, homogeneous Couette flow problem developed on a toy model adapted to the Boussinesq equations.

Stability of trivial defect solutions

Paige Davis, davis@karlin.mff.cuni.cz
Charles University, Czech Republic

Heterogeneous defects have been shown to have a profound impact on the existence and stability of localised solutions, potentially pinning, rebounding or annihilating travelling wave solutions. We examine pinned stationary solutions located near a jump-type defect for which substantial existence analysis exists showing there are three main types of solution, trivial defect solutions, local defect solutions and global defect solutions. The jump-type defect is simple enough to make analysis tenable but provides insight for future analysis of far more complicated heterogeneities. Through the use of an Evans function we locate the leading order spectrum of the general trivial defect solution which agrees with that of the associated homogeneous problem and derive the first order correction term, thus providing conditions for spectral stability to first order. The examination of the stability of the trivial defect solution is a crucial first step towards the analysis of the stability of local defect solutions.

Non-isothermal viscoelastic flows with conservation laws and relaxation

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Charles University, Czech Republic
Coauthor: Sébastien Boyaval

We propose a system of balance laws for non-isothermal viscoelastic flows of Maxwell fluids. The system is an extension of the polyconvex elastodynamics of hyperelastic bodies using additional structure variables. We establish a strictly convex mathematical entropy to show that the proposed system is symmetrizable and, as a consequence, we obtain the short-time existence and uniqueness of smooth solutions which define genuinely causal viscoelastic flows with waves propagating at finite speed. To model heat-conductors the system is complemented by the hyperbolic Maxwell–Cattaneo heat conduction law.

Modelling and analysis for multicomponent incompressible fluids

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Weierstrass Institute Berlin, Germany

In this talk, we consider fluids consisting N chemical substances at small compressibility. Starting from a thermodynamically consistent constitutive model for the free energy, we analyze the incompressible limit, where the molar volume becomes independent of pressure. We show that, as a consequence of thermodynamic consistency, the molar volume must be independent on temperature as well, and moreover that it is given as a linear constitutive function of the composition variable (concentration fractions). Compared to our knowledge of thermal expansion in incompressible fluids, and to well-known nonlinear volume effects during the mixing of substances, these two properties seem surprising. We will discuss the two problems, in particular putting them into the light of the low Mach-number limit in PDEs describing the dynamics of

multicomponent fluids in non–equilibrium. If time allows, we shall also discuss recent well–posedness issues concerning the PDEs for multicomponent incompressible fluids. The talk relies on joined works with D. Bothe (TU Darmstadt) and W. Dreyer (WIAS Berlin).

Ergodic theory for energetically open fluid systems

Eduard Feireisl, feireisl@math.cas.cz
Czech Academy of Sciences, Czech Republic

We consider global in time (weak) solutions for the complete Navier-Stokes-Fourier system describing the motion of a compressible, viscous and heat conducting fluid driven by inhomogeneous boundary conditions. We show that any globally bounded trajectory generates a stationary statistical solution. Then the Birkhoff-Khinchin theorem can be applied to show the validity of the (weak) ergodic hypothesis on the associated omega-limit set.

A semismooth Newton method for implicitly constituted flow

Pablo Alexei Gazca Orozco, alexei.gazca@math.fau.de
FAU Erlangen-Nürnberg, Germany

We propose a semismooth Newton method for non-Newtonian models of incompressible flow where the constitutive relation between the shear stress and the symmetric velocity gradient is given implicitly; as a motivating example, we consider the Bingham model for viscoplastic flow. The proposed method avoids the use of variational inequalities and is based on a particularly simple regularisation introduced recently by Bulíček et al., for which the (weak) convergence of the approximate stresses is known to hold. The system is analysed at the function space level and results in mesh-independent behaviour of the nonlinear iterations.

Uniqueness and regularity of flows of non-Newtonian fluids below critical power-law growth

Petr Kaplicky, kaplicky@karlin.mff.cuni.cz
Charles University, Czech Republic

We deal with flows of non-Newtonian fluids in three-dimensional setting subjected to the homogeneous Dirichlet boundary condition. We assume the natural monotonicity, coercivity and growth condition on the Cauchy stress tensor expressed by a power index p.

First we present the results for $p \geq 11/5$, namely regularity properties of a solution with respect to time variable and uniqueness of solutions for nice data.

Second we concentrate on estimates for $p < 11/5$. We show regularity and uniqueness for small data and show that the restriction on data explodes if p goes to 11/5.

On planar flows of viscoelastic fluids of the Burgers type

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Coauthors: Miroslav Bulíček, Yong Lu, Josef Málek

Viscoelastic rate-type fluid models involving the stress and its observer-invariant time derivatives of higher order are used to describe a large class of viscoelastic mixtures - geomaterials like asphalt, biomaterials such as vitreous in the eye, synthetic rubbers such as SBR. A standard model that belongs to the category of viscoelastic rate-type fluid models of the second order is the model due to Burgers, which can be viewed as a mixture of two Oldroyd-B models of the first order. This viewpoint allows one to develop the whole hierarchy of generalized models of a Burgers type. We study one such generalization. Carrying on the study by Masmoudi (2011), who briefly proved the weak sequential stability of weak solutions to the Giesekus model, we prove long time and large data existence of weak solutions to a mixture of two Giesekus models in two spatial dimensions.

A simple thermodynamic framework for heat-conducting flows of mixtures of two mechanically interacting fluids

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Charles University, Faculty of Mathematics and Physics, Czech Republic

Within the theory of interacting continua, we develop a model for a heat conducting mixture of two mechanically interacting fluids described in the terms of the densities and the velocities for each fluid and the temperature field for the whole mixture. We use a general thermodynamic framework that determines the response of the material from the knowledge of two pieces of information, namely how the material stores the energy and how the energy of material is dissipated. This information is expressed in the form of the constitutive equations for two scalars: the Helmholtz free energy and the entropy production. Additionally, we follow the goal to determine the response of a mixture from a small (minimal) set of material parameters (bulk and shear viscosity, heat conductivity, the drag coefficient) that can be associated with the mixture as the whole. The same thermodynamic approach is used to obtain the model when the whole mixture responses as an incompressible material. For both the compressible and incompressible variants, we investigate two variants stemming from different definitions of the velocity associated with the mixture as a whole.

Global existence analysis of fractional cross-diffusion systems

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TBA

Thermodynamics of viscoelastic rate-type fluids and its implications for stability analysis

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Charles University, Czech Republic

We introduce a thermodynamic basis for some non-Newtonian fluids, namely we explicitly characterise energy storage and entropy production mechanisms that lead to the frequently used viscoelastic rate-type models such as the Oldroyd-B model, the Giesekus model, the Phan-Thien–Tanner model, the Johnson–Segalman model, the Bautista–Manero–Puig model and their diffusive variants. Knowing the thermodynamical basis of the models, we show how this knowledge can be used in nonlinear (finite amplitude) stability analysis of steady flows of these fluids.

The Maxwell-Stefan system, its gradient flow structure, and the problem of uniqueness of weak solutions

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We consider the Maxwell-Stefan system and discuss the following items: (i) How it emerges in the high-friction limit of multi-component Euler flows. (ii) The question of uniqueness of weak solutions for the Maxwell-Stefan model. (iii) The construction of numerical schemes for the Maxwell-Stefan system associated with the minimization of frictional dissipation.
(based on joint works with Xiaokai Huo, Ansgar Jüngel, Hailiang Liu and Shuaikun Wang)

Hydrodynamic stability for the dynamic slip

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Charles University, Czech Republic

We consider the incompressible Navier-Stokes equation with the dynamic slip boundary condition. Our first goal is to prove the so-called linearization principle in the class of weak solutions satisfying the energy inequality. By this we mean that if the spectrum of certain operator has only positive real parts, then the stationary solution u^* of the Navier-Stokes equation is stable with respect to sufficiently small initial perturbations. We deal further with two explicit geometries, namely with either two infinite parallel planes or two concentric cylinders, where the solution u^* corresponds either to Couette/Poiseuille or Taylor-Couette flow. We eventually compare our results with well-known analogue results in the case of Dirchlet boundary condition.

Minisymposium

TOPICS IN SUB-ELLIPTIC AND ELLIPTIC PDEs (MS-31)

Organized by Francesca Da Lio, *ETH, Zurich, Switzerland*
Coorganized by Annamaria Montanari, *Department of Mathematics, University of Bologna, Italy*

- Exponentially Subelliptic Harmonic Maps, *Yuan Jen Chiang*
- On the mean value formula for harmonic functions, *Giovanni Cupini*
- The Dirichlet problem for fully nonlinear degenerate elliptic equations with a singular nonlinearity, *Giulio Galise*
- Time-dependent focusing Mean Field Games with strong aggregation, *Daria Ghilli*
- Epsilon-regularity for p-harmonic maps at a free boundary on a sphere, *Katarzyna Mazowiecka*
- Monotone sets in Carnot groups: an interesting class of "convex" sets, *Daniele Morbidelli*
- A Resolution of the Poisson Problem for Elastic Plates, *Francesco Palmurella*
- Conformal fractional powers and heat kernels in Heisenberg-type groups, *Giulio Tralli*

Exponentially Subelliptic Harmonic Maps

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Coauthor: Sorin Dragomir

Exponentially harmonic maps were first introduced by Eells and Lemaire [5] in 1990. Exponential wave maps are exponentially harmonic maps on Minkowski spaces, which were first studied by Chiang and Yang [1, 4] since 2007. Firstly, we deal with the critical points of maps $\phi : \mathbb{H}_n \to S^m$ from the Heiserberg group into a sphere with energy $E_1(\phi) = \int_\Omega \exp(\frac{1}{2}||\nabla^H\phi||^2_\theta)\theta \wedge (d\theta)^n$ for domains $\Omega \subset\subset \mathbb{H}_n$ and a contact structure θ on $\mathbb{H}_n$. They are solutions to the 2nd order quasi-linear subelliptic PDE system

$$-\triangle_b\phi^j + 2e_b(\phi)\phi^j + G_\theta(\nabla^H e_b(\phi),\, \nabla^H\phi^j) = 0, 1 \le j \le m+1,$$

and arise through Fefferman's construction, i.e. as base maps $\phi : \mathbb{H}_n \to S^m$ associated to S^1 invariant exponential wave maps $\Phi : C(\mathbb{H}^n) \to S^m$ from the total space of the canonical circle bundle $S^1 \to C(\mathbb{H}^n) \to S^m$ endowed with the Fefferman's metric F_θ. We establish Caccioppoli type estimates

$$\int_{B_{r(x)}} \exp\Big(\frac{Q}{2}||\nabla^H\phi||^2_\theta\Big)||\nabla^H u||^Q_\theta\,\theta \wedge (d\theta)^n \le Cr^\beta \;\; (0 < \beta < 1)$$

with $Q = 2n + 2$ (the homogeneous dimension of $\mathbb{H}^n$), and show that any weak solution $\phi \in \bigcap_{p \ge Q} W^{1,p}_H(\Omega, S^n)$ of finite p-energy $E_p(\phi) < \infty$ for some $p \ge 2Q$ is locally Hölder continuous, i.e. $\phi^j \in S^{0,\alpha}_{loc}(\Omega)$ (Hölder like spaces) for $0 < \alpha \le 1$, built in terms of the Carnot-Carathéodory metric ρ_θ. The main theorems and results are based on [2]. Secondly, we study exponentially subelliptic harmonic (e.s.h.) maps from a compact pseudo hermitian manifold $(M,\,\theta)$ into a Riemannian manifold $(N,\,h)$, i.e. C^2 solutions of $\phi : M \to N$ to nonlinear PDE system $\tau_b(\phi) + \phi_*\nabla^H e_b(\phi) = 0$ which are the Euler-Legrange equation of $\delta E_b(\phi) = 0$ with $E_b(\phi) = \int_M \exp(e_b(\phi))\theta \wedge d\theta^n$, where e.s.h. maps arise in a similar way as the first setting. We study the second variation formula and stability of exponentially subelliptic harmonic maps based on [3].

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On the mean value formula for harmonic functions

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The mean integral of harmonic functions on a ball is equal to the value of the functions at the center of the ball. This is the well known Gauss mean value formula for harmonic functions. This formula is "stable" and provides a harmonic characterizion of balls. In the talk I will discuss these and related results, obtained in collaboration with E. Lanconelli and with N. Fusco, E. Lanconelli, X. Zhong.

The Dirichlet problem for fully nonlinear degenerate elliptic equations with a singular nonlinearity

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Sapienza Università di Roma, Italy

Coauthor: Isabeau Birindelli

We consider the homogeneous Dirichlet problem, in uniformly convex domains, for a large class of degenerate elliptic equations with singular zero order term. In particular we establish sharp existence and uniqueness results of positive viscosity solutions.

Time-dependent focusing Mean Field Games with strong aggregation

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University of Padua, Italy

Mean Field Games (MFG) theory models the behavior of an infinite number of indistinguishable rational agents aiming at minimising a common cost. A large part of MFG literature is devoted to the study of MFG systems with increasing coupling ("non focusing"case). Heuristically, this assumption means that agents prefer sparsely populated areas (indeed concentration costs), and it is well-suited to model competitive cases. Moreover, the increasing monotonicity of the coupling ensures existence and regularity of solutions in many circumstances. We are interested in the "focusing"case, that is, where the coupling is monotone decreasing and it is a local function of the distribution, so that no regularising effect can be expected. These systems describe Nash equilibria of games with a large number of agents aiming at aggregation. In this talk, we will introduce the model in the focusing case and we will show that there is a threshold for the growth of the coupling, after which the solutions to the MFG system may not exist. This is coherent with the focusing character of the MFG, which induces solutions to concentrate and develop singularities.

Epsilon-regularity for p-harmonic maps at a free boundary on a sphere

Katarzyna Mazowiecka, katarzyna.mazowiecka@uclouvain.be
Université catholique de Louvain, Belgium

We prove an epsilon-regularity theorem for vector-valued p-harmonic maps, which are critical with respect to a partially free boundary condition, namely that they map the boundary into a round sphere. As a consequence we obtain partial regularity of stationary p-harmonic maps up to the boundary away from a set of $(n-p)$-dimensional Hausdorff measure. Joint work with R. Rodiac and A. Schikorra.

Monotone sets in Carnot groups: an interesting class of "convex" sets

Daniele Morbidelli, daniele.morbidelli@unibo.it
Università di Bologna, Italy

In this talk we discuss some properties of horizontal convexity in Carnot groups. Namely, in the setting of a step-2 Carnot group G of rank at most 3, we show a classification of all sets $E \subset G$ which are horizontally convex and whose complement is convex too. Horizontal convexity is relevant in the theory of second-order subelliptic pde and the monotonicity notion we discuss here plays also a role in bilipschitz embeddability properties for subRiemannian spaces. This is a joint paper with Séverine Rigot, from Nice.

A Resolution of the Poisson Problem for Elastic Plates

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Coauthors: Francesca Da Lio, Tristan Rivière

The Poisson problem consists in finding a surface immersed in the Euclidean space minimising Germain's elastic energy (known as Willmore energy in geometry) with assigned boundary, boundary Gauss map and area; it constitutes a non-linear model for the equilibrium state of thin, clamped elastic plates. We present a solution, and discuss its partial boundary regularity, to a variationally equivalent version of this problem when the boundary curve is simple and closed, as in the most classical version of the Plateau problem. This is a Joint work with F. Da Lio and T. Rivière.

Conformal fractional powers and heat kernels in Heisenberg-type groups

Giulio Tralli, giulio.tralli@unipd.it
University of Padova, Italy

In this talk we discuss the conformal fractional powers of the horizontal Laplacian in groups of Heisenberg type. We present a new approach, based on the heat equation and on extension operators, to the derivation of the fundamental kernels for these nonlocal operators and to the construction of explicit solutions to fractional CR-Yamabe type problems. The talk is based on joint works with N. Garofalo.

Minisymposia in

PROBABILITY

- A game theory and its applications (MS-56)
- Modeling roughness and long-range dependence with fractional processes (MS-18)
- Stochastic Evolution Equations (MS-68)

Minisymposium

A GAME THEORY AND ITS APPLICATIONS (MS-56)

Organized by Krzysztof Szajowski, *Wrocław University of Science and Technology, Poland*

Coorganized by Vladimir V. Mazakov, *KRC RAS, Petrozavodsk, Russian Federation*

- Potential discrete-time dynamic games, *Alejandra Fonseca Morales*
- How to beat the $1/e$-strategy of best choice (the random arrivals problem), *Alexander Gnedin*
- Schumpeterian Evolution of Consumers' Optima – A Game Theory Insight, *Marta Kornafel*
- Nash Equilibria in certain two-choice multi-player games played on the ladder graph, *Victoria Sánchez Muñoz*
- Expected duration in multilateral selection problems, *Krzysztof Szajowski*

Potential discrete-time dynamic games

Alejandra Fonseca Morales, alejandra.fonseca@unison.mx
UNISON, Mexico

A potential dynamic game is a dynamic game for which one can find an optimal control problem whose optimal solutions are also Nash equilibria for the original dynamic game. We cover in this talk team games and action independent transition games in order to identify Markov-Nash equilibria via the potential approach.

How to beat the $1/e$-strategy of best choice (the random arrivals problem)

Alexander Gnedin, a.gnedin@qmul.ac.uk
Queen Mary, University of London, United Kingdom

Elimination of dominated strategies is the fundamental technique used to reduce the size of a finite zero-sum game. For infinite games, however, the dominance phenomenon may occur within the set of minimax strategies. See [1] and references therein for the Blackwell-Hill-Cover paradox of this kind. In this talk we consider a more involved optimal stopping game.

In the best choice problem with random arrivals, an unknown number n of rankable items arrive at times sampled from the uniform distribution. As is well known, a real-time player can ensure stopping at the overall best item with probability at least $1/e$ by means of the strategy τ^* that waits until time $1/e$ then selects the first relatively best item to appear (if any). The number $1/e$ is also the value of the game against an adversary in charge of the variable n.

We show that the adversary has no minimax strategy and
(i) for every u there exist stopping strategies that strictly improve upon τ^* simultaneously for all $n \leq u$,
(ii) there exists a simple strategy outperforming τ^* simultaneously for all $n > 1$ (strictly for $n > 2$),
(iii) there exist more complex strategies strictly outperforming τ^* simultaneously for all $n > 2$,
(iv) for every $\ell \geq 1$ there exist still more complex strategies that guarantee the winning probability at least $1/e$ for all n, and are outperforming τ^* simultaneously for all $n > \ell$.
(v) in the other direction (as the $\ell = \infty$ case of (iv)), there exist stopping strategies that guarantee the winning chance $1/e$, but are strictly dominated by τ^*.

The stopping strategies we employ are defined in terms of multiple time cutoffs, and rely decisions on both the arrival time of relatively best item in question and the number of arrivals seen so far.

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Schumpeterian Evolution of Consumers' Optima – A Game Theory Insight

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Cracow University of Economics, Poland

We will consider the behaviour of consumers' optimal allocations in the result of Schumpeter evolution of economy. The goal of consumer in general equilibrium model is to choose an optimal allocation that maximises his preferences over the budget set, given the price and initial allocation. The crucial point is to justify – taking into account the changing preferences – that the consumers choosing the optimal allocation at every stage of evolutionary process will end up at an optimal state of the final economy. In the talk we will provide the conditions, under which the positive answer is possible.

Nash Equilibria in certain two-choice multi-player games played on the ladder graph

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Coauthor: Michael Mc Gettrick

We compute analytically the number of Nash Equilibria (NE) for a two-choice game played on a (circular) ladder graph with $2n$ players. We consider a set of games with generic payoff parameters, with the only requirement that a NE occurs if the players choose opposite strategies (anti-coordination game). The results show that for both, the ladder and circular ladder, the number of NE grows exponentially with (half) the number of players n, as $N_{NE}(2n) \sim C(\varphi)^n$, where $\varphi = 1.618...$ is the golden ratio and $C_{circ} > C_{ladder}$. In addition, the value of the scaling factor C_{ladder} depends on the value of the payoff parameters. However, that is no longer true for the circular ladder (3-degree graph), that is C_{circ} is constant, which might suggest that the topology of the graph indeed plays an important role for setting the number of NE.

Expected duration in multilateral selection problems

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This paper treats the decision problem related to the observation of a Markov process by decision makers. The information delivered to the players is based on the aggregation of the high-frequency data by some functions. Admissible strategies are stopping moments related to the available information. The payments are defined by the state at the time of stopping. The players' decision to stop has various effects which depend on the decision makers' type (v. [3]). The knowledge about the type of the players is not public and in this way, the payers have also different information. The details of the description allow to formulate the problem as a Bayesian game with sets of strategies based on the stopping times. It is an extension of the Dynkin's game related to the observation of a Markov process with the random assignment mechanism of states to the players. The main question considered now is the expected duration of each DM in the game (v. [1]). Some examples related to the best choice problem (BCP) are analyzed (cf. [4] and [2]).

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Minisymposium

Modeling roughness and long-range dependence with fractional processes (MS-18)

Organized by Yuliya Mishura, *Taras Shevchenko National University of Kyiv, Ukraine*
Coorganized by
Mark Podolskij, *Aarhus University, Denmark*
Nikolaj Leonenko, *Cardiff University, Wales, United Kingdom*
Giulia Di Nunno, *University of Oslo, Norway*

- Time-Changed Fractional Ornstein-Uhlenbeck Process, *Giacomo Ascione*
- Persistence probabilities of fractional processes, *Frank Aurzada*
- Mild solutions of partial differential equations driven by general stochastic measures, *Iryna Bodnarchuk*
- Recent developments in stochastic analysis of Rosenblatt processes, *Petr Čoupek*
- Rough volatility: SDE driven by Hölder continuous noise and unbounded drift, *Giulia Di Nunno*
- Exponential moments of hitting times for time-inhomogeneous atomic Markov chains, *Vitaliy Golomoziy*
- Exact spectral asymptotics of fractional processes and its applications, *Marina Kleptsyna*
- Log-periodically disturbed fractional calculus, *Svenja Lage*
- The multifaceted behaviour of supOU processes: intermittency, multiscaling in limit theorems, *Nikolai Leonenko*
- Self-stabilizing processes, *Jacques Levy Vehel*
- Fractional integrals, derivatives and integral equations with weighted Takagi-Landsberg functions, *Vitalii Makogin*
- Financial markets with a memory, *Yuliya Mishura*
- On some fractional queues, *Enrica Pirozzi*
- On recent advancement in limit theory for fractional type processes, *Mark Podolskij*
- Prabhakar fractional operators and some related stochastic processes, *Federico Polito*
- On simulation of rough Volterra stochastic volatility models, *Jan Pospíšil*
- Hypotheses testing of the drift parameter sign for the fractional Ornstein–Uhlenbeck process, *Kostiantyn Ralchenko*
- Integration-by-Parts Characterizations of Gaussian Processes, *Tommi Sottinen*
- On mixed fractional SDEs with discontinuous drift coefficient, *Ercan Sönmez*
- Prediction of missing functional data with memory, *Lauri Viitasaari*
- Decomposition formula for rough Volterra stochastic volatility models, *Josep Vives*
- Approximating expected value of an option with non-Lipschitz payoff in fractional Heston-type model, *Anton Yurchenko Tytarenko*

Time-Changed Fractional Ornstein-Uhlenbeck Process

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Coauthors: Enrica Pirozzi, Yuliya Mishura

In this talk I will focus on the description of some characteristics of a time-changed fractional Ornstein-Uhlenbeck process, i. e. a process obtained by considering a fractional Ornstein-Uhlenbeck process as introduced in [“Fractional Ornstein-Uhlenbeck Process" by Cheridito, Kawaguchi and Maejima] and we substitute the time by the inverse of an independent driftless subordinator. I will focus first on the existence and the asymptotics of the even-order moments. Moreover, I will discuss some generalized Fokker-Planck equations that arise from the Gaussian nature of the fractional Ornstein-Uhlenbeck process. Finally, I will focus on the stable case, for which more explicit results can be achieved. Such things are the result of a joint work with Yuliya Mishura from University of Kiev and Enrica Pirozzi from University of Naples and are all contained in [“Time-changed fractional Ornstein- Uhlenbeck process", to appear].

Persistence probabilities of fractional processes

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The talk will introduce the area of persistence probabilities. The questions studied in this area are as follows: We are given a real-valued stochastic process, e.g. a fractional process such as fractional Brownian motion. What is the probability that the process has a long excursion, e.g. that is stays positive for a long time? What does the process look like if one conditions on having a long excursion? These questions are classical for Brownian motion, random walks and Lévy processes, with many applications in applied probability, such as queueing, finance, insurance, etc. Contrary, for fractional processes, research on this type of problems has just begun. We will survey the recent progess in this area.

Mild solutions of partial differential equations driven by general stochastic measures

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Let $\mathsf{L}_0(\Omega, \mathcal{F}, \mathsf{P})$ be the set of all real-valued random variables defined on complete probability space $(\Omega, \mathcal{F}, \mathsf{P})$, X be an arbitrary set and $\mathcal{B}(\mathsf{X})$ be a σ-algebra of Borel subsets of X. Let μ be a general stochastic measure, i.e., a σ-additive mapping $\mu : \mathcal{B}(\mathsf{X}) \to \mathsf{L}_0(\Omega, \mathcal{F}, \mathsf{P})$.

We investigate Cauchy problems of a wave and heat equations driven by general stochastic measures. The existence and uniqueness of the mild solutions are proved. Hölder regularity of the paths in time and spatial variables is obtained. Asymptotic behavior of the mild solutions is established.

Recent developments in stochastic analysis of Rosenblatt processes

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The talk will be devoted to some recent developments in stochastic analysis of Rosenblatt and related fractional processes.

Rosenblatt processes arise naturally as limits of suitably normalized sums of long-range dependent random variables in non-central limit theorems. They share many properties with regular fractional Brownian motions; in particular, they have stationary increments and they exhibit self-similarity and long-range dependence. However, unlike fractional Brownian motions, Rosenblatt process are not Gaussian which makes their analysis somewhat intriguing and which is also the reason why they have received considerable attention in recent years.

In the talk, some recent results regarding Rosenblatt processes will be given and an approach to stochastic integration for Rosenblatt processes that is based on the Malliavin calculus will be discussed. In particular, an Itô-type stochastic chain rule for processes with a second-order fractional stochastic differential will be presented.

Rough volatility: SDE driven by Hölder continuous noise and unbounded drift

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Coauthors: Yuliya Mishura, Anton Yurchenko-Tytarenko

Having in mind possible stochastic volatility models in finance, we consider an SDE driven by a general Hölder continuous noise. The drift b is exploding with a control from below:

$$b(t,y) > C_T(y-\varphi(t))^{-\gamma} \qquad (\gamma > 0),$$

where φ is a continuous function and C_T, γ are constants. We study the solution of such SDE and its properties. In particular, we prove that it has a unique solution which is bound preserving.

Furthermore, modifying the controls on the drift, we obtain an SDE sandwiched between two given bounds φ and ψ, with $\psi(x) > \varphi(x)$. This sandwich solution turns out to be the most useful for applications.

Among the properties presented, we show that the solution admits *all* moments. This remarkable result paves the way for efficient numerical methods.

Exponential moments of hitting times for time-inhomogeneous atomic Markov chains

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The result we present is devoted to studying exponential moments of hitting times for time-inhomogeneous Markov chains. It is well-known that a necessary and sufficient condition for the existence of such a moment for a homogeneous Markov chain is a drift condition of the form $PV \leq \lambda V + b\mathbb{I}_C$, $\lambda < 1$. We generalized this result to the time-inhomogeneous case and proved that it is sufficient to have a similar drift condition with different λ_t at different time steps t. We showed that homogeneous condition $\lambda < 1$ could be relaxed in the time inhomogeneous case.

The second result of the presentation is related to studying the simultaneous hitting time for an atom α by two time-inhomogeneous Markov chains. We established conditions for the existence of the exponential moment for the hitting time and found computable bounds using the drift condition described above.

Exact spectral asymptotics of fractional processes and its applications

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Coauthors: Pavel Chigansky, Dmytro Marushkevych

Many results in the theory of Gaussian processes rely on the eigenstructure of the covariance operator. However, eigenproblems are notoriously hard to solve explicitly and closed form solutions are known only in a limited number of cases. In this talk we set up a framework for the spectral analysis of the fractional type covariance operators, corresponding to an important family of processes, which includes the fractional Brownian motion, its noise, the fractional Ornstein–Uhlenbeck process and the integrated fractional Brownian motion. We obtain accurate asymptotic approximations for the eigenvalues and the eigenfunctions. Our results provide a key to several problems, whose solution is long known in the standard Brownian case, but was missing in the more general fractional setting. This includes computation of the exact limits of L^2-small ball probabilities and asymptotic analysis of singularly perturbed integral equations, arising in mathematical physics and applied probability.

Log-periodically disturbed fractional calculus

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It is well-known that stable distributions solve particular fractional diffusion equations. In this talk, we develop a similar connection between semistable densities and diffusion equations involving log-periodically disturbed fractional derivatives. Starting from this connection, we discuss the properties of these operators, which allow us to model log-periodically disturbed long-range dependencies. Furthermore, we solve corresponding diffusion equations and apply our theory to real-world applications.

The multifaceted behaviour of supOU processes: intermittency, multiscaling in limit theorems

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Superpositions of Ornstein-Uhlenbeck type (supOU) processes provide a rich class of stationary stochastic processes for which the marginal distribution and the dependence structure may be modeled independently [1]. In this paper we investigate the limiting behavior of integrated supOU processes with finite variance [2,3] (see also [5] for multifaceted behavior of supOU processes in the case the infinite variance).

We show that after suitable normalization four different limiting processes may arise. The type of limit depends on the decay of the correlation function as well as on the characteristic triplet of the marginal distribution. supOU processes, moreover, may exhibit intermittency, a phenomenon affecting the rate of growth of moments [2,3,4]. We establish this rate for each of the four limiting scenarios. The rate changes at some point indicating that there is a change-point in the asymptotic behavior of absolute moments. For such a behavior to be possible, the moments in the limit theorem do not converge beyond some critical point. We show that this point is related to the dependence structure of the supOU process. The intermittency phenomenon appears also in some other models, for example, in the subclass of ambit processes known as trawl processes [3]. We also discuss the contrasts between convergence in distributions and almost sure convergence using intermittency and multiscaling.

Joint work with D. Grahovac (Osijek University) and M. Taqqu (Boston University).

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Self-stabilizing processes

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A self-stabilizing processes $\{Z(t), t \in [t_0, t_1)\}$ is a random process which when localized, that is scaled to a fine limit near a given $t \in [t_0, t_1)$, has the distribution of an $\alpha(Z(t))$-stable process, where $\alpha : \mathbb{R} \to (0, 2)$ is a given continuous function. Thus the stability index near t depends on the value of the process at t. In the case where $\alpha : \mathbb{R} \to (0, 1)$, we first construct deterministic functions which satisfy a kind of autoregressive property involving sums over a plane point set Π. Taking Π to be a Poisson point process then defines a random pure jump process, which we show has the desired localized distributions.

When α may take values greater than 1, convergence of the considered sums may no longer be absolute. We generalize the construction in two stages, firstly by setting up a process based on a fixed point set but taking random signs of the summands, and then randomizing the point set to get a process with the desired local properties.

Fractional integrals, derivatives and integral equations with weighted Takagi-Landsberg functions

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In the talk, we find fractional Riemann-Liouville derivatives for the Takagi-Landsberg functions. Moreover, we introduce their generalizations called weighted Takagi-Landsberg functions. Namely, for constants $c_{m,k} \in [-L, L]$, $k, m \in \mathbb{N}_0$, we define a *weighted Takagi-Landsberg function* as $y_{c,H} : [0, 1] \to \mathbb{R}$ via

$$y_{c,H}(t) = \sum_{m=0}^{\infty} 2^{m\left(\frac{1}{2}-H\right)} \sum_{k=0}^{2^m-1} c_{m,k} e_{m,k}(t), t \in [0, 1],$$

where $H > 0$, $\{e_{m,k}, m \in \mathbb{N}_0, k = 0, \ldots, 2^m - 1\}$ are the Faber-Schauder functions on [0,1]. The class of the weighted Takagi-Landsberg functions of order $H > 0$ on $[0, 1]$ coincides with the H-Hölder continuous functions on $[0, 1]$. Based on computed fractional integrals and derivatives of the Haar and Schauder functions, we get a new series representation of the fractional derivatives of a Hölder continuous function. This result allows to get the new formula of a Riemann-Stieltjes integral. The application of such series representation is the new method of numerical solution of the Volterra and linear integral equations driven by a Hölder continuous function.

Financial markets with a memory

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We present general conditions for the weak convergence of a discrete-time additive scheme to a stochastic process with memory in the space $D[0,T]$. Then we investigate the convergence of the related multiplicative scheme to a process that can be interpreted as an asset price with memory. As an example, we study an additive scheme that converges to fractional Brownian motion, which is based on the Cholesky decomposition of its covariance matrix. The second example is a scheme converging to the Riemann–Liouville fractional Brownian motion. The multiplicative counterparts for these two schemes are also considered. As an auxiliary result of independent interest, we obtain sufficient conditions for monotonicity along diagonals in the Cholesky decomposition of the covariance matrix of a stationary Gaussian process.

On some fractional queues

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Coauthors: Giacomo Ascione, Nikolai Leonenko

We show some recents advances in the study of fractional queueing models such as fractional M/M/1 queues and fractional Erlang queues $M/E_k/1$. We also focus on a fractional M/M/1 queue with catastrophes. Starting from fractional M/M/1 queues, we study the transient behaviour, in which the time-change plays a key role. An alternative expression for the transient distribution of the fractional M/M/1 model is provided. The state probabilities for the fractional queue with catastrophes, the distributions of the busy period for fractional queues without and with catastrophes and the distribution of the time of the first occurrence of a catastrophe are also obtained.

Furthermore, we introduce a fractional generalization of the Erlang Queues $M/E_k/1$. Such process is obtained through a time-change via inverse stable subordinator of the classical queue process. The fractional Kolmogorov forward equation for such process is considered, then we use such equation to obtain an interpretation of this process in the queuing theory context. We give some results such as the transient state probabilities and some features of this fractional queue model, the mean queue length, the distribution of the busy periods and some conditional distributions of the waiting times.

Finally, we also show some results of the study of a fractional $M/M/\infty$ queueing system constructed as a suitable time–changed birth–death process.

On recent advancement in limit theory for fractional type processes

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In this talk we review some recent results on limit theorems for fractional type Levy driven processes. In particular, we will discuss central and non-central limit theorems for statistics of

Levy moving average models and present techniques from Malliavin calculus on Poisson spaces to get quantitative versions associated with normal limits.

Prabhakar fractional operators and some related stochastic processes

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The aim of this talk is to present properties and relevant definitions of some stochastic processes related to generalized Mittag-Leffler functions, also known as Prabhakar functions. We first describe a generalization of classical fractional calculus in which integral and differential operators involve generalized Mittag-Leffler functions as kernels, highlighting especially how differential operators should be regularized in order to turn them into Caputo-like operators. Then, we analyze two renewal processes both based on generalized Mittag-Leffler functions in two different manners. Lastly, we end the talk by characterizing time-changed Lévy processes whose governing equations present generalized Mittag-Leffler fractional operators.

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On simulation of rough Volterra stochastic volatility models

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Coauthor: Jan Matas

Rough Volterra volatility models are a progressive and promising field of research in derivative pricing. Although rough fractional stochastic volatility models already proved to be superior in real market data fitting, techniques used in simulation of these models are still inefficient in terms of speed and accuracy. This talk aims to present the accurate tools and techniques that could be used also in nowadays largely emerging pricing methods based on machine learning. In particular we compare three widely used methods: the Cholesky method, Hybrid scheme and the rDonsker scheme for simulation of the rBergomi model and for a more general αRFSV model. We also comment on implemention of variance reduction techniques, especially we show the obstacles of the so called turbocharging technique, whose importance is sometimes overestimated in the literature. To overcome these obstacles we suggest several modifications.

Hypotheses testing of the drift parameter sign for the fractional Ornstein–Uhlenbeck process

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We study the inference problem for the *fractional Ornstein–Uhlenbeck process* $X = \{X_t, t \geq 0\}$, which is the unique solution of the stochastic differential equation

$$dX_t = \theta X_t \, dt + dB_t^H, \quad X_0 = x_0 \in \mathbb{R}. \tag{1}$$

Here $\theta \in \mathbb{R}$ is an unknown drift parameter. The noise is modelled as a fractional Brownian motion $B^H = \{B_t^H, t \geq 0\}$ with a known Hurst index $H \in (0, 1)$. First, we investigate the problem of the drift parameter estimation by continuous and discrete observations of the trajectory of X. We construct several types of estimators and prove their strong consistency. It turns out that the methods for constructing the estimators and their asymptotic properties substantially depend on the sign of the unknown parameter. This motivates the hypothesis testing of the sign of drift parameter in the model (1). We propose a comparatively simple test for testing the null hypothesis $H_0 \colon \theta \leq 0$ against the alternative $H_1 \colon \theta > 0$ and prove its consistency. Contrary to the previous works, our approach is applicable for all $H \in (0, 1)$. The test is based on the observations of the process X at two points: 0 and T. The distribution of the test statistic is computed explicitly, and the power of test can be found numerically for any given simple alternative. Also we consider the hypothesis testing $H_0 \colon \theta \geq \theta_0$ against $H_1 \colon \theta \leq 0$, where $\theta_0 \in (0, 1)$ is some fixed number. As an auxiliary result of independent interest we compute the covariance function of the fractional Ornstein–Uhlenbeck process.

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Integration-by-Parts Characterizations of Gaussian Processes

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The Stein's lemma characterizes the one-dimensional Gaussian distribution via an integration-by-parts formula. We show that a similar integration-by-parts formula characterizes a wide class of Gaussian processes, the so-called Gaussian Fredholm processes. Examples include the Brownian motion and fractional Brownian motions.

This talk is based on article Azmoodeh, E., Sottinen, T., Tudor, C.A. et al. Integration-by-parts characterizations of Gaussian processes. *Collect. Math.* (2020). https://doi.org/10.1007/s13348-019-00278-x

On mixed fractional SDEs with discontinuous drift coefficient

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We prove existence and uniqueness of the solution for a class of mixed fractional stochastic differential equations with discontinuous drift driven by both standard and fractional Brownian motion. Additionally, we establish a generalized Itô rule valid for functions with absolutely continuous derivative and applicable to solutions of mixed fractional stochastic differential equations with Lipschitz coefficients, which plays a key role in our proof of existence and uniqueness. The proof of such a formula is new and relies on showing the existence of a density of the law under mild assumptions on the diffusion coefficient.

Prediction of missing functional data with memory

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We consider optimal prediction of functional observations $X^i = (X^i_t)_{t\in[0,1]}, i = 1, \ldots, n$ that are realisations of some Gaussian subordinated process. We assume that parts of the paths are unobservable, and our aim is to fill in the missing information as accurately as possible. One natural approach is to predict some missing value X^k_s by using the information provided by $X^i_s, i \neq k$ of those functions X^i for which X^i_s is observed. However, under memory the unobserved X^k_s relies heavily on that particular functional observation X^k directly, and thus applying other observations X^i may be missleading, even if they are drawn from the same stochastic process. In this talk we present a novel approach for accurate prediction of missing information X^k_s that is based on applying combined information provided by the observed part of the path X^k and the observed values $X^i_s, i \neq k$.

Decomposition formula for rough Volterra stochastic volatility models

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Coauthors: Raúl Merino, Jan Pospisil, Tomas Sobotka, Tommi Sottinen

The research presented in this paper provides an alternative option pricing approach for a class of rough fractional stochastic volatility models. These models are increasingly popular between academics and practitioners due to their surprising consistency with financial markets. However, they bring several challenges alongside. Most noticeably, even simple nonlinear financial derivatives as vanilla European options are typically priced by means of Monte–Carlo (MC) simulations which are more computationally demanding than similar MC schemes for standard

stochastic volatility models. In this paper, we provide a proof of the prediction law for general Gaussian Volterra processes. The prediction law is then utilized to obtain an adapted projection of the future squared volatility — a cornerstone of the proposed pricing approximation. Firstly, a decomposition formula for European option prices under general Volterra volatility models is introduced. Then we focus on particular models with rough fractional volatility and we derive an explicit semiclosed approximation formula. Numerical properties of the approximation for a popular model — the rBergomi model — are studied and we propose a hybrid calibration scheme which combines the approximation formula alongside MC simulations. This scheme can significantly speed up the calibration to financial markets as illustrated on a set of AAPL options.

Approximating expected value of an option with non-Lipschitz payoff in fractional Heston-type model

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Coauthor: Yuliya Mishura

In this research, we consider option pricing in a framework of the fractional Heston-type model with stochastic volatility being a fractional modification of the Cox-Ingersoll-Ross process with $H > 1/2$. As it is impossible to obtain an explicit formula for the expectation $\mathbb{E}f(S_T)$ in this case, where S_T is the asset price at maturity time and f is a payoff function, we provide a discretization schemes $\hat{Y}^n$ and $\hat{S}^n$ for volatility and price processes correspondingly and study convergence $\mathbb{E}f(\hat{S}^n_T) \to \mathbb{E}f(S_T)$ as the mesh of the partition tends to zero. As we allow f to be non-Lipschitz and/or to have discontinuities of the first kind which can cause errors if S_T is replaced by $\hat{S}^n_T$ under the expectation straightforwardly, we use Malliavin calculus techniques to provide an alternative formula for $\mathbb{E}f(S_T)$ with smooth functional under the expectation. In this case, the rate of convergence is calculated.

Minisymposium

Stochastic Evolution Equations (MS-68)

Organized by Carlo Marinelli, *University College London, United Kingdom*

- Nonlinear parabolic stochastic evolution equations in critical spaces, *Antonio Agresti*
- Ito formulae for singular SPDEs, *Carlo Bellingeri*
- Random multiple-fragmentation and flow of particles on a surface, *Lucian Beznea*
- High-frequency analysis for parabolic stochastic PDEs, *Carsten Chong*
- Stochastic quantization of exponential-type quantum field theories, *Francesco Carlo De Vecchi*
- Non-equilibrium fluctuations in interacting particle systems and conservative stochastic PDE, *Benjamin Fehrman*
- Invariant measures for 2D inviscid fluids with linear damping and stochastic forcing term, *Benedetta Ferrario*
- Singular quasilinear SPDEs, *Máté Gerencsér*
- Optimal control of path-dependent McKean-Vlasov SDEs in infinite dimension, *Fausto Gozzi*
- Stochastic heat equations with distributional drifts, *Khoa Le*
- Regularization by noise of semilinear stochastic damped wave equations with Hölder continuous coefficients, *Federica Masiero*
- On a construction of martingale solutions of SPDEs, *Martin Ondreját*
- Weighted Energy–Dissipation principle for nonlinear stochastic evolution equations, *Luca Scarpa*
- Local characteristics and tangent martingales in Banach spaces, *Ivan Yaroslavtsev*
- Invariant measures for a stochastic nonlinear and damped 2D Schrödinger equation, *Margherita Zanella*

Nonlinear parabolic stochastic evolution equations in critical spaces

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Critical spaces for non-linear equations are important due to scaling invariance, and in particular this plays a central role in fluid dynamics. In this talk we introduce and discuss local/global well-posedness, and blow-up criteria for stochastic parabolic evolution equations in critical spaces. Our results extend the celebrated theory of Prüss, Wilke and Simonett for deterministic PDEs. Due to the presence of noise it is unclear that a stochastic version of the theory is possible, but we will show that a suitable variation of the theory remains valid. We will also explain some features which are new in both the deterministic and stochastic framework. Our theory is applicable to a large class of semilinear and quasilinear parabolic problems which includes many of the classical SPDE. Applications to stochastic Navier-Stokes equations with gradient noise will be also discussed.

This is a joint work with Mark Veraar (TU Delft).

Ito formulae for singular SPDEs

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During the last years, the theory of regularity structure has become a complete theory to prove the existence and uniqueness of a wide family of stochastic partial differential equations. In this talk, we will discuss how to combine this theory with other probabilistic constructions to derive some explicit Ito formula on a particular class of equations. We will focus on the additive stochastic heat equation with additive noise and the well-known KPZ equation.

Random multiple-fragmentation and flow of particles on a surface

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We investigate a stochastic fragmentation processes for particles with spatial position. The mathematical problem models the time evolution of a system of particles which move on an Euclidean surface driven by a given force and split in fragments with smaller masses and velocities. We establish a multiple-fragmentation process and we solve the corresponding stochastic integro-differential equation. Finally, we present several numerical simulations of such processes. The talk is based on a joint work with Ioan R. Ionescu (Paris) and Oana Lupaşcu-Stamate (Bucharest).

High-frequency analysis for parabolic stochastic PDEs

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We consider the stochastic heat equation driven by an additive or multiplicative Gaussian noise that is white in time and spatially homogeneous in space. Assuming that the spatial correlation function is given by a Riesz kernel of order $\alpha \in (0, 2 \wedge d)$, where d is the spatial dimension, we prove a central limit theorem for the power variations of the solution in the additive case. We further show that the same central limit theorem is valid with multiplicative noise if $\alpha \in (0, 1)$ but fails in general if $\alpha = 1$ (and $d \geq 2$) or if the noise is a space-time white noise (and $d = 1$). If time permits, we discuss applications of our results to statistical estimation for the stochastic heat equation.

Stochastic quantization of exponential-type quantum field theories

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Universität Bonn, Germany

Stochastic quantization is a method, proposed by Parisi and Wu, of constructive Euclidean quantum field theory for building the Schwinger functions of a quantum model from the invariant solutions of suitable (parabolic, hyperbolic or elliptic) stochastic partial differential equations (SPDEs). In the talk we provide an introduction to the topic and to the recent developments in the field, focusing on the analytic and probabilistic aspects of the problem. We propose a more detailed analysis of the SPDEs related to the two-dimensional exponential-type models such as the Høegh-Krohn, or Liouville quantum gravity, quantum field theory and the massive $\sinh(\varphi)_2$ interaction. The talk is mainly based on the joint work [1] with Sergio Albeverio and Massimiliano Gubinelli, and a paper in preparation with Nikolay Barashkov.

References

[1] Albeverio, Sergio, Francesco C. De Vecchi, and Massimiliano Gubinelli. "The elliptic stochastic quantization of some two dimensional Euclidean QFTs." *arXiv preprint arXiv:1906.11187*, to appear in *Annales de l'Institut Henri Poincaré*.

Non-equilibrium fluctuations in interacting particle systems and conservative stochastic PDE

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University of Oxford, United Kingdom
Coauthor: Benjamin Gess

Interacting particle systems have found diverse applications in mathematics and several related fields, including statistical physics, population dynamics, and machine learning. We will focus, in particular, on the zero range process and the symmetric simple exclusion process. The large-scale behavior of these systems is essentially deterministic, and is described in terms of a hydrodynamic limit. However, the particle process does exhibit large fluctuations away from its

mean. Such deviations, though rare, can have significant consequences—such as a concentration of energy or the appearance of a vacuum—which make them important to understand and simulate.

In this talk, which is based on joint work with Benjamin Gess, I will introduce a continuum model for simulating rare events in the zero range and symmetric simple exclusion process. The model is based on an approximating sequence of stochastic partial differential equations with nonlinear, conservative noise. The solutions capture to first-order the central limit fluctuations of the particle system, and they correctly simulate rare events in terms of a large deviations principle.

Invariant measures for 2D inviscid fluids with linear damping and stochastic forcing term

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Coauthor: Hakima Bessaih

We study the two-dimensional Euler equations, damped by a linear term and driven by an additive noise. We prove existence of an invariant measure in the space L^∞, where the problem has a unique global solution. This requires to deal with the weak-$*$ topology and the associated Markov semigroup is proven to be sequentially weakly Feller.

Singular quasilinear SPDEs

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TU Vienna, Austria

We overview some recent results on quasilinear stochastic PDEs. The developments for fully parabolic (nondegenerate) equations have led to expansions of the theories of regularity structures and paracontrolled distributions, as well as to some unexpected connections to renormalisation symmetries. On the other hand, degenerate equations are far less understood, and even for equations with an Itô structure often only martingale solutions are available, natural uniqueness questions are still open. Based on joint works with Y. Bruned, K. Dareiotis, B. Gess, M. Hairer.

Optimal control of path-dependent McKean-Vlasov SDEs in infinite dimension

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Luiss University, Italy

Coauthors: Andrea Cosso, Idris Kharroubi, Huyen Pham, Mauro Rosestolato

We study the optimal control of path-dependent McKean-Vlasov equations valued in Hilbert spaces motivated by non Markovian mean-field models driven by stochastic PDEs. We first establish the well-posedness of the state equation, and then we prove the dynamic programming principle (DPP) in such a general framework. The crucial law invariance property of the value function V is rigorously obtained, which means that V can be viewed as a function on the Wasserstein space of probability measures on the set of continuous functions valued in Hilbert space. We then define a notion of pathwise measure derivative, which extends the Wasserstein derivative due to P.L.Lions (2006), and prove a related functional Itô formula in the spirit of Dupire (2009) and Wu-Zhang (2020). The Master Bellman equation is derived from the DPP by means of a suitable notion of viscosity solution. We provide different formulations and simplifications of such a Bellman equation notably in the special case when there is no dependence on the law of the control.

Stochastic heat equations with distributional drifts

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Coauthors: Siva Athreya, Oleg Butkovsky, Leonid Mytnik

We consider stochastic heat equations on the real line with space-time white noise and distributional drifts in negative-indexed Besov spaces. It is shown that a weak solution exists if the regularity index is at least -3/2 while pathwise uniqueness holds of the regularity index is at least -1. In the particular case when the drift is a Dirac mass at a point, the equation has a unique strong solution. These results are obtained using the stochastic sewing lemma introduced by the speaker in 2018. This is a joint work with S. Athreya, O. Butkovsky and L. Mytnik.

Regularization by noise of semilinear stochastic damped wave equations with Hölder continuous coefficients

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Università di Milano Bicocca, Italy

We prove that semilinear stochastic abstract wave equations and damped wave equations are well-posed in the strong sense with an α-Hölder continuous drift coefficient, if $\alpha \in (2/3, 1)$.
The uniqueness may fail for the corresponding deterministic PDE and well-posedness is restored by considering an additive pertubation of white noise type which describes an external random forcing. This shows that a kind of regularization by noise holds for the semilinear wave equation.
In the proof we adopt an approach based on backward stochastic equations and use non-standard regularizing properties for the transition semigroup associated to the stochastic wave equation; these properties are based on control theoretic results.
We finally briefly discuss how our method applies also to stochastic evolution of parabolic type
The talk is based on joint works with D. Addona and E. Priola

On a construction of martingale solutions of SPDEs

Martin Ondreját, ondrejat@utia.cas.cz
Institute of Information Theory and Automation, v.v.i., Czech Academy of Sciences, Czech Republic

We will discuss a simplified method of proving existence of martingale solutions to SPDEs based on a compactness argument but not using the Skorokhod representation theorem. The method works both for theoretical and numerical approximations of solutions.

Weighted Energy–Dissipation principle for nonlinear stochastic evolution equations

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Coauthor: Ulisse Stefanelli

We present the Weighted Energy–Dissipation (WED) principle for nonlinear stochastic evolution equations in variational form. The approach consists in minimizing suitable convex WED functionals, defined on spaces of entire trajectories, and depending on an approximation parameter. The corresponding Euler–Lagrange equation is characterized as an elliptic-in-time regularization of the original problem, which can be equivalently seen as a forward–backward nonlinear stochastic evolution system. Finally, WED minimizers are shown to converge to the solution of the original nonlinear evolution equation as the approximation parameter vanishes.

This study is based on a joint work with Ulisse Stefanelli (University of Vienna, Austria).

Local characteristics and tangent martingales in Banach spaces

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Max Planck Institute for Mathematics in the Sciences, Germany

Let L be a real-valued Lévy martingale. Then we know that by the Lévy-Khinchin formula

$$\mathbb{E}e^{i\theta L_t} = \exp\Big\{t\Big(-\frac{1}{2}\sigma^2\theta^2 + \int_{\mathbb{R}} e^{i\theta x} - 1 - i\theta x\nu(dx)\Big)\Big\}, \quad t \geq 0, \quad \theta \in \mathbb{R},$$

where $\sigma \geq 0$ is responsible for the *continuous* part of L and the measure ν on $\mathbb{R}$ in responsible for the *discontinuous* part of L, i.e. if we decompose $L = W + N$ into a sum of a Brownian motion W and a purely discontinuous Poisson martingale N, then the distributions of W and N are uniquely determined by σ and ν respectively.

Any real-valued martingale M has a analogue of (σ, ν) which is called the *local characteristics* of M and which is defined as the pair $([M^c], \nu^M)$ of a quadratic variation $[M^c]$ of the continuous part M^c of M and the compensator ν^M of the jump measure of M. The local characteristic have been defined and intensively studied in a number of works by Jacod, Kallenberg, Kwapień, Shiryaev, and Woyczyński, and in particular it turned out that the local characteristics uniquely determine the distribution of the corresponding martingale if and only if they are constant. Therefore the notion of *tangent* martingales (i.e. martingales with the same local characteristics) was introduced. In 2017 Kallenberg have shown sharp L^p bounds for tangent real-valued martingales.

In the present talk we will discuss the generalisation of local characteristics to Banach space-valued martingales. In particular, we will prove sharp L^p estimates for tangent martingales with values in infinite dimensions. This will help us to provide new sharp bounds for vector-valued stochastic integrals with respect to a general martingale.

Invariant measures for a stochastic nonlinear and damped 2D Schrödinger equation

Margherita Zanella, margherita.zanella@polimi.it
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We consider a two-dimensional stochastic nonlinear defocusing Schrödinger equation with zero-order linear damping, where the stochastic forcing term is given by a combination of a linear multiplicative noise in Stratonovich form and a nonlinear noise in Itô form. We work at the same time on compact Riemannian manifolds without boundary and on compact smooth domains with either Dirichlet or Neumann boundary conditions. We construct a martingale solution using a modified Faedo-Galerkin's method, then by means of suitable Strichartz estimates we show the pathwise uniqueness of solutions. Finally, we prove the existence of invariant measures by means of a version of the Krylov-Bogoliubov method, which involves the weak topology, as proposed by Maslowski and Seidler . Some remarks on the uniqueness in a particular case are provided as well. The talk is based on a joint work with B. Ferrario and Z. Brzeźniak.

Minisymposia in

STATISTICS AND FINANCIAL MATHEMATICS

- Mathematical challenges in insurance (MS-41)

Minisymposium

Mathematical challenges in insurance (MS-41)

Organized by Mihael Perman, *University of Primorska, Slovenia*
Coorganized by
Bor Harej, *PRS Prime Re Solutions AG, Switzerland*
Claudia Klüppelberg, *TU München, Germany*
Thomas Mikosch, *University of Copenhagen, Denmark*
Ermanno Pitacco, *University of Trieste, Italy*
Tomaž Košir, *University of Ljubljana, Slovenia*

- Linking risk management under expected shortfall to loss-averse behavior (joint work with Thai Nguyen), *An Chen*
- On the validation of internal models, *Michel Dacorogna*
- Can machine learning algorithms outperform traditionally used methods in insurance pricing?, *Bor Harej*
- Explainability - when, why, and how; For what use cases explainability is needed; when it is not needed; high level overview of approaches for explainability, *Diego Klabjan*
- Pro-Cyclicality of Traditional Risk Measurements: Quantifying and Highlighting Factors at its Source, *Marie Kratz*
- Tree-based learning methods for extreme value regression with applications to cyber-insurance (joint work with Maud Thomas and Sébastien Farkas, Sorbonne Université), *Olivier Lopez*
- Monte Carlo Valuation of the Initiation Option in a GMWB Variable Annuity, *Pietro Millossovich*
- How would a Solvency II regulated insurer against riverine flooding have fared during the past 7000 years? A hypothetical case study for NE Austria to inform risk modelling in the face of climate change, *Franz Prettenthaler*
- Internal Model Approach for Risk Management II, *Vincenzo Russo*
- Optimal Drawdowns in Insurance, *Hanspeter Schmidli*
- Refined Doob Inequalities for σ-Integrable Submartingales, *Uwe Schmock*
- Internal Model Approach for Risk Management I, *Johannes Schoenenwald*

Linking risk management under expected shortfall to loss-averse behavior (joint work with Thai Nguyen)

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University of Ulm, Germany

We introduce and solve an optimal asset allocation problem under a weighted expected shortfall (WES) constraint, which contains the risk management problem under an expected shortfall constraint of Basak and Shapiro (2001) as a special case. Furthermore, we link our risk management problem under the WES constraint to an optimal asset allocation with a multiple-reference-based preference (MRBP) and find that the optimal wealth with MRBP owns the same form as the optimal solution under the WES constraint. For the degenerate case with a fixed reference level, we are able to determine the critical maximal allowed expected shortfall constraint as a function of the loss aversion parameters to achieve equivalence. In case of multiple thresholds with the same probability weights, we show that while no equivalence can be in general obtained between the WES and the MRBP solution, the optimal terminal wealth of the WES problem can be made to coincide with the MRBP terminal wealth in the most favorable and in the worst market states. More interestingly, if different probabilities can be assigned to the thresholds, the solutions of the two problems can be made identical.

On the validation of internal models

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Prime Re Solutions, Switzerland

The development of risk models for managing portfolios of financial institutions and insurance companies requires, both from the regulatory and management points of view, a strong validation of the quality of the results provided by internal risk models. In Solvency II for instance, regulators ask for independent validation reports from companies who apply for the approval of their internal models. Unfortunately, the usual statistical techniques do not work for the validation of risk models as we lack sufficient data to significantly test the results of the models. Indeed, we will never have enough data to statistically estimate the significance of the VaR at a probability of 1 over 200 years, which is the risk measure required by Solvency II. Instead, we need to develop various indirect strategies to test the relevance of the model. These indirect methods comprise various steps that we list and discuss. In this presentation, we analyze various ways to enable management and regulators to gain confidence in the quality of models. It all starts by ensuring a good calibration of the risk models and the dependencies between the various risk drivers. Then by applying stress tests to the model and various empirical analysis, in particular the probability integral transform, we can build a full and credible framework to validate risk models.

Can machine learning algorithms outperform traditionally used methods in insurance pricing?

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Prime Re Solutions, Switzerland

The comparison of different algorithms for insurance pricing exercise is a task that relies heavily on the data sample used. There are two options: real data and synthetic data. A critical issue with the real data is the lack of information of the exact underlying rate that we want to predict. That makes the comparison of different algorithms on real data less clear. We decide for an analysis on synthetic data to avoid this issue. Based on synthetic data sample, several different machine learning algorithms are calibrated to estimate the appropriate basic premium rate to cover expected claims: GLM, GAM, Random forest, XGBoost, Light GAM and Neural networks. We compare the predictions with the true underlying rates and try to find the best fit. Finally, we find remarkable results evaluating the profit of the insurers that would use a particular algorithm in comparison with others and the effect of winner's curse.

Explainability - when, why, and how; For what use cases explainability is needed; when it is not needed; high level overview of approaches for explainability

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Northwestern University, United States

Knowledge graphs organize information in a graph form by relying on the notion of objects, subjects, and relationships. They play an important role in healthcare insurance use cases by capturing interactions among patients, symptoms, medications, and treatments. We consider several use cases in insurance and then focus on how to construct them and how to infer missing relationships. We discuss challenges most resulting from limited data availability.

Pro-Cyclicality of Traditional Risk Measurements: Quantifying and Highlighting Factors at its Source

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ESSEC Business School, France

There is an accepted idea that risk measurements are pro-cyclical: in times of crisis, they overestimate the future risk, while they underestimate it in quiet times. We lay down a methodology to evaluate the amount of pro-cyclicality in the way financial institutions measure risk, and identify two main factors explaining this pro-cyclical behavior: the clustering and return-to-the-mean of volatility, as it could have been anticipated but not yet quantified, and, more surprisingly, the very way risk is measured, independently of this return-to-the-mean effect. We provide CLTs and FCLT's for functionals of quantile and dispersion estimators to support theoretically those empirical findings.

Tree-based learning methods for extreme value regression with applications to cyber-insurance (joint work with Maud Thomas and Sébastien Farkas, Sorbonne Université)

Olivier Lopez, olivier.lopez@sorbonne-universite.fr
Sorbonne Universite, France

Regression trees are well known models introduced by Breiman (1984) which allow to perform clustering and regression at the same time. On the other hand, their instability endangers the analysis that can be performed using these tools. In this work, we consider the application of regression trees to the analysis of heavy-tailed phenomenon, introducing them as a valuable tool to perform extreme value regression. We derive theoretical results and consider the stabilization of the method using random forests and gradient boosting. We show how these techniques can be applied to the analysis and evaluation of cyber risk for insurance portfolios. The growing field of cyber-insurance is particularly challenging due to the extreme volatility of the losses, the heterogeneity of cyber claims, and the fast evolution of the risk. The tools we develop may be used to contribute to draw a line between what can be insured by private companies, and what type of claims is unbearable through a classical insurance mechanism.

Monte Carlo Valuation of the Initiation Option in a GMWB Variable Annuity

Pietro Millossovich, pietro.millossovich.1@city.ac.uk
City, University of London, United Kingdom

We focus on the initiation option featured in many Guaranteed Lifelong Withdrawal Benefit variable annuity contracts, granting their owner the right to decide the age at which lifetime withdrawals should begin. Such contracts have been successfully analysed using a PDE approach, see Huang et al. (IME, 56(2014), 102-111). While the latter method is elegant, it becomes less viable when the valuation model is more involved and other guarantees are considered. We exploit the Least Square Monte Carlo method and explore the interaction of the initiation option with lapses and other riders, and the effect of stochastic volatility, interest rates and mortality.

How would a Solvency II regulated insurer against riverine flooding have fared during the past 7000 years? A hypothetical case study for NE Austria to inform risk modelling in the face of climate change

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Joanneum Research Forschungsgesellschaft mbH, Austria

Knowledge about changes of flood occurrence patterns is important for risk estimation of the future. Robust and well-calibrated paleoflood records, derived e.g. from lake sediments, are excellent natural archives to investigate flood variability of the past and to use the data for further modelling. In this paper, we analyse a 7100 year summer flood record recovered from Lake Mondsee (NE Alps), using a statistical approach. We identify a point process of renewal type, with a significant change-point of the occurrence pattern around 350 AD, switching from the overlay of two mechanisms of event recurrences of 5 and 50 years before to 2 and 17 years after this change-point. This change-point approach enables a comparison to other flood records, and possibly to relate event frequencies to climatic conditions. We also highlight how lower temporal resolution of flood records can hamper the analysis of relations to climatic signals. Hence high-resolution records with robust chronologies and flood information (e.g. seasonality and event characteristics) are essential to improve the understanding of the interplay between climatic signals and flood occurrences, which is an important ingredient for proper risk estimation and risk management.

Internal Model Approach for Risk Management II

Vincenzo Russo, vincenzo.russo@generali.com
Generali Group, Italy

In the context of risk management needs in the insurance industry, mathematical and statistical methods are widely used to evaluate, measure and manage risks that may arise from the insurance business.

In particular, since Solvency II regulation entered in force in 2016 across the European Union, insurance companies have been entitled to develop and implement proprietary Internal Models for the evaluation and monitoring of their risk profile. Such Internal Models are prescribed by the regulation to produce in output the full multivariate statistical distribution of the losses that a company or group might experience over a 1-year time horizon, across all risks and all types of business.

The development of an Internal Model represents several methodological and computational challenges, and the specific methodological and technical aspects that characterize the design of the Generali Internal Model will be presented, stressing also the ability to take business decisions from the selected approach.

The modelling starts from an appropriate identification and representation of the main risks affecting the assets and liabilities. These risk factors need to be described by means of standalone theoretical or empirical distributions, and the dependency structure that delineates their mutual interactions has then to be established and modelled, typically by using a Copula approach. Finally, the functional relationship between the risk factors and the movement of the values of assets and liabilities detained by the company needs is determined in order to derive the correspondent distribution of the potential losses of the Own Fund.

The model heavily relies on numerical procedures, like Monte Carlo simulations, that have been implemented at many levels to address the lack of closed-form solutions for most of the quantities that need to be evaluated. The techniques applied to overcome the computational challenges underlying these numerical methods will be described, focusing on the procedures adopted to ensure their computational efficiency, numerical stability and robust convergence.

Optimal Drawdowns in Insurance

Hanspeter Schmidli, schmidli@math.uni-koeln.de
University of Cologne, Germany

In the last years, drawdown measures have attained attention in financial mathematics. Recently, these measures had been applied to actuarial mathematics. Not all of the measures considered make sense in an insurance context. We therefore consider the expected discounted time in the critical area for an infinite time horizon. More specifically, let X_t be the surplus process of an insurance portfolio and denote by $\bar{X}_t = \max\{\bar{x}, \sup_{s \le t} X_s\}$ the running maximum. The drawdown process is then $D_t = \bar{X}_t - X_t$. The expected discounted time in drawdown is defined as $v^1(\bar{x} - x) = \mathbb{E}[\int_0^\infty \mathrm{e}^{-\delta t} \mathbb{I}_{D_t > d} \, \mathrm{d}t]$. Here, δ is a preference parameter meaning drawdowns tomorrow are preferred to drawdowns today. In addition, the insurer can buy proportional reinsurance. We aim to find the optimal reinsurance strategy minimising the expected discounted time in drawdown. We find an explicit solution for a diffusion approximation and also discuss the classical Cramér–Lundberg model.

Refined Doob Inequalities for σ-Integrable Submartingales

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Vienna University of Technology, Austria

Using the notion of conditional expectation extended to σ-integrable functions and σ-finite measure spaces, we consider σ-integrable submartingales with totally ordered index sets. Without requiring path properties, using the essential supremum of the process, we prove refined versions of Doob's maximum inequality and Doob's L^p-inequality for the cases $p > 1$, $p = 1$ and $p \in (0, 1)$. Tightness of bounds is shown by examples involving stopped Brownian motion or a martingale increasing continuously until jumping down and stopped.

Internal Model Approach for Risk Management I

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Assicurazioni Generali S.p.A., Italy

In the context of risk management needs in the insurance industry, mathematical and statistical methods are widely used to evaluate, measure and manage risks that may arise from the insurance business.

In particular, since Solvency II regulation entered in force in 2016 across the European Union, insurance companies have been entitled to develop and implement proprietary Internal Models for the evaluation and monitoring of their risk profile. Such Internal Models are prescribed by the regulation to produce in output the full multivariate statistical distribution of the losses that a company or group might experience over a 1-year time horizon, across all risks and all types of business.

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Minisymposia in

TOPOLOGY

- Low-dimensional Topology (MS-11)

Minisymposium

Low-dimensional Topology (MS-11)

Organized by
Sašo Strle, *University of Ljubljana, Slovenia*
Brendan Owens, *University of Glasgow, School of Mathematics and Statistics, United Kingdom*

- Combinatorial cusp counting in curves, *Sebastian Baader*
- Exotic 4-manifolds with signature zero, *Inanc Baykur*
- Quasi-morphisms on Surface Diffeomorphism Groups, *Jonathan Bowden*
- L-space links and link Floer homology, *Alberto Cavallo*
- Virtual Morse-Bott index, moduli spaces of pairs, and applications to topology of smooth four-manifolds, *Paul Feehan*
- Non-orientable slice surfaces and inscribed rectangles, *Peter Feller*
- The slope of a link computed via C-complexes, *Ana G. Lecuona*
- Bonded knots: a topological model for knotted proteins, *Boštjan Gabrovšek*
- Some 3-manifolds do not bound definite 4-manifolds, *Marco Golla*
- Flattening knotted surfaces, *Eva Horvat*
- New Heegaard Floer slice genus and clasp number bounds, *Andras Juhasz*
- Closed geodesics and Frøyshov invariants of hyperbolic three-manifolds, *Francesco Lin*
- Non-loose negative torus knots, *Irena Matkovič*
- Complex vs convex Morse functions and geodesic flow, *Burak Özbağci*
- Embedding spheres in knot traces, *Arunima Ray*
- Khovanov homology and the cinquefoil, *Steven Sivek*
- On the cosmetic surgery conjecture, *Andras Stipsicz*
- Algebraic fibrations of surface-by-surface groups, *Stefano Vidussi*
- Obstructing Stein fillings by filtering the Heegaard Floer contact invariant, *Andrew Wand*
- SU(2)-representations of fundamental groups of 3-manifolds, *Raphael Zentner*

Combinatorial cusp counting in curves

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A classic open problem in algebraic geometry asks for the maximal number of cusps in plane algebraic curves of degree d. This is closely related to determining the minimal genus of smooth cobordisms between torus links and connected sums of trefoil knots. As we will see, the signature function is strikingly effective at estimating the latter.

Exotic 4-manifolds with signature zero

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We will talk about our recent construction of the smallest closed exotic 4-manifolds with signature zero known to date. Our novel examples are derived from fairly special small Lefschetz fibrations we build, with spin and non-spin monodromies. This is joint work with N. Hamada.

Quasi-morphisms on Surface Diffeomorphism Groups

Jonathan Bowden, jpbowden1981@yahoo.com.au
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Coauthors: Sebastian Hensel, Richard Webb

We show that the identity component of the group of diffeomorphisms of a closed oriented surface of positive genus admits many unbounded quasi-morphisms. As a corollary, we also deduce that this group is not uniformly perfect and its fragmentation norm is unbounded, answering a question of Burago–Ivanov–Polterovich. As a key tool we construct a hyperbolic graph on which these groups act, which is the analog of the curve graph for the mapping class group. (joint with S. Hensel and R. Webb)

L-space links and link Floer homology

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It is a standard result in knot Floer homology that L-space knots are fibered and strongly quasi-positive. The same does not hold for L-space links; in fact, we can provide examples of L-space links where neither of the two properties hold.

In a work joint with Beibei Liu, we show that if a restriction is put on the family of L-space links then we can prove exactly the same result. Namely, we consider only the ones for which the H-function, a concordance invariant from the link Floer homology group, has a specific shape.

The goal of this talk is to present a proof of the mentioned statement. This will be done by using the τ-set of a link and the fact that it characterizes fibered and strongly quasi positive links in S^3.

Virtual Morse-Bott index, moduli spaces of pairs, and applications to topology of smooth four-manifolds

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Rutgers University, New Brunswick, United States

Coauthor: Thomas Leness

n Feehan and Leness (2020), we introduced an approach to Morse-Bott theory, called virtual Morse-Bott theory, for Hamiltonian functions of circle actions on closed, real analytic, almost Hermitian spaces. In the case of Hamiltonian functions of circle actions on closed, smooth, almost Kaehler (symplectic) manifolds, virtual Morse-Bott theory coincides with classical Morse-Bott theory due to Bott (1954) and Frankel (1959). Positivity of virtual Morse-Bott indices implies downward gradient flow in the top stratum of smooth points in the analytic space. In this monograph, we apply our method to the moduli space of SO(3) monopoles over a complex, Kaehler surface, we use the Atiyah-Singer Index Theorem to compute virtual Morse-Bott indices of all critical strata (Seiberg-Witten moduli subspaces), and we prove that these indices are positive in a setting motivated by the conjecture that all closed, smooth four-manifolds of Seiberg-Witten simple type obey the Bogomolov-Miyaoka-Yau inequality.

Non-orientable slice surfaces and inscribed rectangles

Peter Feller, peter.feller@math.ch
ETH Zurich, Switzerland

We consider the complexity of non-orientable locally-flat surfaces in the four-ball B^4 and in $S^1 \times B^3$ with boundary a prescribed torus knot and discuss differences between the locally-flat and smooth setup.

Our investigation is motivated by the following old metric problem posed by Toeplitz over a hundred years ago: Does every Jordan curve (the image of a continuous injection from the circle to the Euclidean plane), contain four points that form the corners of a square.

Based on joint work with M. Golla.

The slope of a link computed via C-complexes

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University of Glasgow, United Kingdom

Together with Alex Degtyarev and Vincent Florence we introduced a new link invariant, called slope, of a colored link in an integral homology sphere. In this talk I will define the invariant, highlight some of its most interesting properties as well as its relationship to Conway polynomials and to the Kojima–Yamasaki eta-function. The stress in this talk will be on our latest computational progress: a formula to calculate the slope from a C-complex.

Bonded knots: a topological model for knotted proteins

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We introduce bonded knots, oriented knots together with a set of properly embedded coloured arcs. Such structures can be used to topologically model protein structures, where the knots correspond to closed protein backbone chains and the bonds correspond to non-local interactions between the amino acids. The bond colours encode the interaction type (disulphide bridges, ionic bonds,...) that may appear in the conformation of the protein.

We will define the HOMFLYPT skein module of rigid and non-rigid coloured bonded knots and show that the rigid version if freely generated by coloured Θ-curves and handcuff links, whereas the non-rigid version is generated by trivial coloured Θ-curves. In other words, there exits a well-defined invariant of rigid and non-rigid coloured bonded knots that respects the HOMFLYPT skein relation.

Some 3-manifolds do not bound definite 4-manifolds

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Many construction of 3-manifolds automatically come with a 4-manifold with definite intersection form. Using Heegaard Floer correction terms and an analysis of short characteristic covectors in bimodular lattices, we give an obstruction for a 3-manifold to bound a definite 4-manifold, and produce some concrete examples. This is joint work with Kyle Larson.

Flattening knotted surfaces

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A knotted surface $\mathcal{K}$ in the 4-sphere admits a projection to a 2-sphere, whose set of critical points coincides with a hyperbolic diagram of $\mathcal{K}$. We apply such projections, called flattenings, to define three invariants of embedded surfaces: the width, the trunk and the partition number. These invariants are studied for satellite 2-knots.

New Heegaard Floer slice genus and clasp number bounds

Andras Juhasz, juhasza@maths.ox.ac.uk
University of Oxford, United Kingdom

I will discuss several concordance invariants definted using knot Floer homology that give improvements over known slice genus and clasp number bounds from Heegaard Floer homology. I also explain how the involutive correction terms of Hendricks and Manolescu give both a slice genus and a clasp number bound. This is joint work with Ian Zemke.

Closed geodesics and Frøyshov invariants of hyperbolic three-manifolds

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Coauthor: Michael Lipnowski

Frøyshov invariants are subtle numerical topological invariants of rational homology three-spheres derived from gradings in monopole Floer homology. In this talk I will look at their relation with invariants arising from hyperbolic geometry (such as volumes and lengths of closed geodesics), using an odd version of the Selberg trace formula and ideas from analytic number theory. In particular, for the class of minimal L-spaces, I will discuss how to obtain effective upper bounds purely in terms of volume and injectivity radius. Furthermore, I will describe (again for minimal L-spaces) a procedure to compute them taking as input explicit geometric data, and show for example how this can be used to determine all the Frøyshov invariants for the Seifert-Weber dodecahedral space. This is joint work with M. Lipnowski.

Non-loose negative torus knots

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University of Oxford, United Kingdom

The Legendrian invariant in knot Floer homology, defined by Lisca, Ozsváth, Stipsicz and Szabó, is torsion for knots in overtwisted structures, and it is non-zero only if the knot is strongly non-loose as a transverse knot. Using a correspondence between the knot invariants and invariants of contact surgeries, I will show that strongly non-loose transverse realizations of negative torus knots are classified by their invariants and that their U-torsion order equals one.

Complex vs convex Morse functions and geodesic flow

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Koç University, Turkey

Suppose that S is a closed and oriented surface equipped with a Riemannian metric. In the literature, there are four seemingly distinct constructions of open books on the unit (co)-tangent bundle of S, having complex, symplectic, contact and Riemannian geometric flavors, respectively. We observe that all of these constructions are based on a suitable choice of a Morse function on S and show that once such a Morse function is fixed, the resulting open books are equivalent. This is a joint work with Pierre Dehornoy.

Embedding spheres in knot traces

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Max Planck Institute for Mathematics, Germany
Coauthors: Peter Feller, Allison Miller, Matthias Nagel, Patrick Orson, Mark Powell

We characterise when the generator of the second homotopy group of a knot trace can be represented by a locally flat embedded 2-sphere with abelian fundamental group of the complement, in terms of classical and computable invariants of the corresponding knot.

Khovanov homology and the cinquefoil

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Imperial College London, United Kingdom

In this talk I will outline a proof that Khovanov homology detects the (2,5) torus knot. The proof makes use of deep results in Floer homology and many recent developments in Khovanov homology and homotopy, but, perhaps surprisingly, it does not require us to know that knot Floer homology detects $T_{2,5}$. This is joint work with John Baldwin and Ying Hu.

On the cosmetic surgery conjecture

Andras Stipsicz, stipsicz.andras@renyi.hu
Alfréd Rényi Institute of Mathematics, Hungary

We discuss recent advances in the cosmetic surgery conjecture.

Algebraic fibrations of surface-by-surface groups

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University of California, Riverside, United States

An algebraic fibration of a group G is an epimorphism to the integers with a finitely generated kernel. This notion has been studied at least since the '60s, and has recently attracted renewed attention. Among other things, we will study it in the context of fundamental groups of surface bundles over a surface, where it has some interesting relations with some classical problems about the mapping class group.

Obstructing Stein fillings by filtering the Heegaard Floer contact invariant

Andrew Wand, andy.wand@glasgow.ac.uk
University of Glasgow, United Kingdom

The 'contact invariant' in Heegaard floer homology has been one of the most widely-used tools in the field of contact topology since its introduction 2 decades ago by Ozsvath and Szabo.

In the talk, we will discuss an interpretation of this invariant in the framework of open book decompositions of 3-manifolds, and a new refinement of the invariant which measures in some sense non-(Stein/symplectic)-fillability of a contact structure. We will show how this can be used to provide an obstruction to existence of Stein fillings of such structures. This is joint work with C. Kutluhan, G. Matic, and J. Van Horn-Morris.

SU(2)-representations of fundamental groups of 3-manifolds

Raphael Zentner, raphael.zentner@mathematik.uni-regensburg.de
Universität Regensburg, Germany

We will review some known facts about SU(2)-representations of fundamental groups of 3-manifolds, and explain how tools from gauge theory permit conclusions of existence. Part of this will be about joined work with Steven Sivek, and part about joined work with Tye Lidman and Juanita Pinzon Caicedo.

Minisymposia in

General Topics

- A journey from pure to applied mathematics (MS-53)
- Mathematics in the Digital Age of Science (MS-50)

Minisymposium

A JOURNEY FROM PURE TO APPLIED MATHEMATICS (MS-53)

Organized by Elena Resmerita, *Alpen-Adria Universität Klagenfurt, Austria*
Coorganized by Carola Schönlieb, *Cambridge University, United Kingdom*

- Linear non-degeneracy and uniqueness of the bubble solution for the critical fractional Hénon equation in $\mathbb{R}^N$, *Begoña Barrios*
- On the p-adic geometry of Shimura varieties, *Ana Caraiani*
- Very weak solutions to PDEs in inhomogeneous and anisotropic spaces, *Iwona Chlebicka*
- Flows of nonsmooth vector fields: new results on non uniqueness and commutativity, *Maria Colombo*
- Independent sets in random subgraphs of the hypercube, *Gal Kronenberg*
- Identifying 3D Genome Organization in Diploid Organisms via Euclidean Distance Geometry, *Kaie Kubjas*
- On arithmetic functions orthogonal to deterministic sequences, *Joanna Kułaga-Przymus*
- Hilbert's Trip to the Casino, *Stefanie Petermichl*
- The fast p-Laplacian evolution equation. Global Harnack principle and fine asymptotic behaviour, *Diana Stan*
- On New Approaches for Nonsmooth Optimization, *Andrea Walther*

Linear non-degeneracy and uniqueness of the bubble solution for the critical fractional Hénon equation in $\mathbb{R}^N$

Begoña Barrios, bbarrios@ull.edu.es
Universidad de La Laguna, Spain

In this talk we show a linear non-degeneracy result of positive radially symmetric solutions of

$$(-\Delta)^s u = |x|^\alpha u^{\frac{N+2s+2\alpha}{N-2s}} \text{ in } \mathbb{R}^N,$$

where $(-\Delta)^s$ is the fractional Laplacian operator , $0 < s < 1$, $\alpha > -2s$ and $N > 2s$. Moreover, as a consequence, a uniqueness result of those solutions with Morse index equal to one is obtained. In particular, we get that the ground state solution is unique. Our approach follows some ideas developed in the deep, and celebrated, papers done by R. Frank and E. Lenzmann (Acta Math. 2013) and R. Frank, E. Lenzmann, L. Silvestre (Comm. Pure Appl. Math. 2016) but, of course, our proofs are not based on ODE arguments as occurs in the local case. Our non-degeneracy result extends, in the radial setting, some known theorems proved by J. Dávila, M. del Pino and Y. Sire (Proc. Amer. Math. Soc. 2013) and by F. Gladiali, M. Grossi and S.L.N. Neves (Adv. Math. 2013). However, due to the nature of the fractional operator and the weight in nonlinearity, we also argue in a different way than these authors do.

The results presented in this talk have been obtained in collaboration with S. Alarcón and A. Quaas.

On the p-adic geometry of Shimura varieties

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The Langlands program is a vast network of conjectures that connect many areas of pure mathematics, such as number theory, representation theory, and harmonic analysis. Shimura varieties are certain algebraic-geometric objects that play an important role in the Langlands program. They have a nice moduli interpretation and they provide, in many cases, a geometric realisation of the global Langlands correspondence. I will illustrate the beautiful geometry of Shimura varieties using the simplest example, that of the modular curve. I will then mention some recent results that use p-adic geometry, specifically the theory of perfectoid spaces.

Very weak solutions to PDEs in inhomogeneous and anisotropic spaces

Iwona Chlebicka, i.chlebicka@mimuw.edu.pl
University of Warsaw, Poland

I will discuss well-posedness and regularity to nonlinear PDEs of simple divergent form

$$-\text{div}A(x, \nabla u) = f \quad \text{or} \quad \partial_t u - \text{div}A(t, x, \nabla u) = f,$$

where the datum f is merely integrable or even is a measure, whereas the growth of A is governed by an inhomogeneous and fully anisotropic N-function $M(x, \nabla u)$. Inhomogeneity means space-dependence and anisotropy yields dependence of M on ∇u not necessarily via its length $|\nabla u|$.

The datum is too poorly regular for weak solutions to exist, thus a more delicate notion of very weak solutions is necessary. Besides existence and uniqueness they share some regularity properties of the weak ones. The generality we admit covers classical linear and polynomial growth operators possibly with measurable coefficients, generalizations of p-Laplacian involving log-Hölder continuous variable exponent, as well as problems posed in fully anisotropic Orlicz spaces (under no growth conditions), and double-phase spaces within the range of parameters sharp for density of smooth functions. The conditions for the density and their importance will be stressed.

The talk will be based on series of joint papers with: Youssuf Ahmida, Angela Alberico, Andrea Cianchi, Piotr Gwiazda, Ahmed Youssfi, and Anna Zatorska-Goldstein.

Flows of nonsmooth vector fields: new results on non uniqueness and commutativity

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Given a vector field in R^d, the classical Cauchy-Lipschitz theorem shows existence and uniqueness of its flow (namely, the solution X(t) of the ODE $X'(t) = b(t, X(t))$ from any initial datum $x \in R^d$) provided the vector field is sufficiently smooth. The theorem looses its validity as soon as v is slightly less regular. However, in 1989, Di Perna and Lions introduced a generalized notion of flow, consisting of a suitable selection among the trajectories of the associated ODE, and they showed existence, uniqueness and stability for this notion of flow for much less regular vector fields.

The talk presents an overview and new results in the context of the celebrated DiPerna-Lions and Ambrosio's theory on flows of Sobolev vector fields, including a negative answer to the following long-standing open question: are the trajectories of the ODE unique for a.e. initial datum in R^d

for vector fields as in Di Perna and Lions theorem? We will exploit the connection between the notion of flow and an associated PDE, the transport equation, and combine ingredients from probability theory, harmonic analysis, and the "convex integration" method for the construction of nonunique solutions to certain PDEs.

Independent sets in random subgraphs of the hypercube

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University of Oxford, United Kingdom

Independent sets in bipartite regular graphs have been studied extensively in combinatorics, probability, computer science and more. The problem of counting independent sets is particularly interesting in the d-dimensional hypercube $\{0,1\}^d$, motivated by the lattice gas hardcore model from statistical physics. Independent sets also turn out to be very interesting in the context of random graphs. In this talk we will review some fundamental results, and discuss new results on random subgraphs of the hypercube. This talk is based on joint work with Yinon Spinka.

Identifying 3D Genome Organization in Diploid Organisms via Euclidean Distance Geometry

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Aalto University, Finland

The 3D organization of the genome plays an important role for gene regulation. Chromosome conformation capture techniques allow one to measure the number of contacts between genomic loci that are nearby in the 3D space. In this talk, we study the problem of reconstructing the 3D organization of the genome from whole genome contact frequencies in diploid organisms, i.e. organisms that contain two indistinguishable copies of each genomic locus. In particular, we study the identifiability of the 3D organization of the genome and optimization methods for reconstructing it. This talk is based on joint work with Anastasiya Belyaeva, Lawrence Sun and Caroline Uhler.

On arithmetic functions orthogonal to deterministic sequences

Joanna Kułaga-Przymus, joanna.kulaga@gmail.com
Nicolaus Copernicus University in Toruń, Poland

The arithmetic Möbius function μ is believed to behave randomly. One way to express such a behaviour is the content of Sarnak's conjecture from 2010 on the Möbius disjointness from all deterministic sequences. In 2015 Veech formulated a (dynamical) property of the Möbius function itself and he conjectured that this property is equivalent to Sarnak's conjecture. My talk will be devoted to the strategy of the proof of Veech's conjecture and some consequences of this equivalence.

The talk is based on a joint work with Adam Kanigowski, Mariusz Lemańczyk and Thierry de la Rue.

Hilbert's Trip to the Casino

Stefanie Petermichl, stefanie.petermichl@mathematik.uni-wuerzburg.de
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We discuss a central object in harmonic analysis - the Hilbert transform and its generalisations, the Riesz transforms. The Hilbert transform turns sinus waves into cosinus waves and is even used (mechanically) in antennas. In this talk, we discuss various probabilistic models and tools that can be used to precisely describe or control these objects in a variety of settings. We present results spanning 1999 to 2019. The talk targets general audiences.

The fast p-Laplacian evolution equation. Global Harnack principle and fine asymptotic behaviour

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Coauthors: Matteo Bonforte, Nikita Simonov

We study fine global properties of nonnegative solutions to the Cauchy Problem for the fast p-Laplacian evolution equation on the whole Euclidean space, in the so-called "good fast diffusion range". It is well-known that non-negative solutions behave for large times as B, the Barenblatt (or fundamental) solution, which has an explicit expression. We prove the so-called Global Harnack Principle (GHP), that is, precise global pointwise upper and lower estimates of nonnegative solutions in terms of B. This can be considered the nonlinear counterpart of the celebrated Gaussian estimates for the linear heat equation. To the best of our knowledge, analogous issues for the linear heat equation, do not possess such clear answers, only partial results are known. Also, we characterize the maximal (hence optimal) class of initial data such that the GHP holds, by means of an integral tail condition, easy to check. Finally, we derive sharp global quantitative upper bounds of the modulus of the gradient of the solution, and, when data are radially decreasing, we show uniform convergence in relative error for the gradients.

On New Approaches for Nonsmooth Optimization

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Humboldt Universität zu Berlin, Germany

Numerous optimization tasks exhibit a nonsmooth behavior. In contrast to the classical smooth case, where optimality conditions are well studied and understood, criteria to determine whether a given point is optimal or even just stationary are still the subject of ongoing research for nonsmooth functions to be minimized. In this presentation, first we discuss new optimality conditions for a large class of piecewise smooth functions using so-called kink qualifications. Here, also the computational complexity to verify the new criteria is covered. Next, we present optimization algorithms resulting from these findings. Finally, we present some applications that fit into the considered problem class.

Minisymposium

Mathematics in the Digital Age of Science (MS-50)

Organized by Katja Berčič, *University of Ljubljana, Slovenia*
Coorganized by
Thierry Bouche, *Université Grenoble Alpes, France*
Patrick Ion, *IMKT & University of Michigan, United States*
Olaf Teschke, *zbMATH, Germany*

- Infrastructure for mathematical data, *Katja Berčič*
- Digital Collections of Examples in Mathematical Sciences, *James Davenport*
- Digitised Mathematics for the Working Mathematician, *Klaus Hulek*
- Automated Reasoning for Experimental Mathematics, *Alexei Lisitsa*
- The centre Mersenne for diamond open access publication, *Evelyne Miot*
- Standard and Custom APIs for Mathematical Information Retrieval, *Fabian Müller*
- zbMATH Open as a hub for the Global Digital Mathematics Library, *Olaf Teschke*
- Mathematical Libraries and Knowledge Management, or There and Back Again, *Stephen Watt*

Infrastructure for mathematical data

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Producing and using data is becoming an increasingly useful part of doing mathematics. Unlike large projects, such as the OEIS or LMFDB, small projects have less (or zero) infrastructural support. Addressing this gap would be beneficial for the mathematical community, while following broader trends in research data, such as the FAIR principles. At the same time, improving the infrastructure for mathematical data has potential for synergy with proof assistants: on one hand, the proof assistants might be able to use mathematical databases as sources of concrete examples or counterexamples, on the other, the correctness of the data could be at least partially machine verified.

Digital Collections of Examples in Mathematical Sciences

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Examples, positive or negative, are the lifeblood of mathematics. This is particularly important when there is generic behaviour, but also lots of special cases. For example, almost all polynomials are irreducible, but nevertheless greeat effort is spent on factoring algorithms. A single example can be represented in various ways, but collections of examples are more difficult. Not that many collections exist, and those that do are often not in machine-processable forms. This means that the sort of contest that has powered the SAT and SMT communities is not available in general computer algebra, or many other fields of computational mathematics. We will make some recommendations here.

Digitised Mathematics for the Working Mathematician

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Mathematics has been both a driving force and a beneficiary of digitisation from the very beginning. Computers entered mathematical institutes early on, enabling previously unattainable calculations, and mathematical research literature was among the first to be made available electronically. Extensive collections of mathematical objects have been available electronically for decades. Platforms are widely used for collaborative work and for rapid dissemination and discussion of results.

The rapid development of digitised resources comes along with enormous benefits and challenges for the working mathematician: In an increasingly fragmented ecosystem, how do we ensure the completeness and reliability of mathematical knowledge essential for our research? We outline solutions that became feasible due to the recent success of Open Access/Open Data initiatives, and discuss which next steps might be the most useful to support the everyday work.

Automated Reasoning for Experimental Mathematics

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In this talk we present recent developments in the application of automated reasoning methods and techniques, such as automated theorem (dis)proving, constraint and SAT solving, in Experimental Mathematics. We argue that automated reasoning is a very useful tool for the development of mathematical knowledge which, in particular, can help to discover new mathematical results [1], can be used as a powerful alternative to mathematical algorithms [2] and can be used to verify and falsify published mathematical results [3].

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The centre Mersenne for diamond open access publication

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We shall present the centre Mersenne, a comprehensive publishing infrastructure for diamond open access scientific journals written in LaTeX (mainly in mathematics so far, but not only). The centre Mersenne is developed by Mathdoc, a service unit of CNRS and Grenoble University. Launched in 2018 with about ten mathematical journals, the centre Mersenne currenty disseminates more than 20 journals in a larger and connected scientific area. We will discuss the main achievements of the centre Mersenne since its creation as well as its challenges and objectives for the next future.

Standard and Custom APIs for Mathematical Information Retrieval

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Industry-standard APIs like OAI-PMH exist for harvesting metadata from repository providers like zbMATH Open. They contain a common baseline functionality, which can however be extended in ways specific to the provider. Going further in this direction, customized REST APIs enable users to access data in a more structured fashion, as well as depositing data instead of just reading. We give an overview of our vision for zbMATH Open and its current status.

zbMATH Open as a hub for the Global Digital Mathematics Library

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2021 marks the transition of zbMATH to an open platform, which is not just freely accessible , but provides open data and APIs that facilitate the connection of diverse facets of mathematical information. We report on the achievements so far, and outline possible next steps in the future.

Mathematical Libraries and Knowledge Management, or There and Back Again

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As its statements can be both deep and unambiguous, mathematical knowledge is both well-suited and promising for machine treatment. At the same time, mathematical results have a longevity of utility that gives their systematic handling a practical motivation. It has now been twenty years since the first International Workshop on Mathematical Knowledge Management and, since then, the subject has grown and matured to become a rich field of study. But we do not yet have a comprehensive, accessible digital library of the mathematical literature. We examine the state of digital mathematical literature collections, the series of developments necessary to give immediate practical impact, and the longer path where mathematical knowledge management enhances the utility of these digital collections.

Special Sessions

- Algebra
- Algebraic and Complex Geometry
- Analysis and its Applications
- Combinatorics and Discrete Mathematics
- Differential Geometry and Applications
- Dynamical Systems and Ordinary Differential Equations and Applications
- Logic and Mathematical Aspects of Computer Science
- Mathematical Physics
- Mathematics in Science and Technology
- Number Theory
- Numerical Analysis and Scientific Computing
- Optimization and Control
- Partial Differential Equations and Applications
- Probability
- Statistics and Financial Mathematics
- Topology
- General Topics

Session

ALGEBRA

Domination of blocks, fusion systems and hyperfocal subgroups

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Coauthor: Constantin-Cosmin Todea

In the context of modular representation theory of finite groups, considering a finite group G, an algebraically closed field k of characteristic p, a block b of kG and a maximal Brauer (D, e), the block b is inertial if b and e lie in a special type of Morita equivalence. A particular situation of this equivalence makes b into a nilpotent block. For a normal p-subgroup P of G, setting $\bar{G} := G/P$, the G-acted epimorphism of group algebras $\pi : kG \to k\bar{G}$ determines the connection between b and its dominating blocks. We investigate the connections between some properties of blocks and of their dominating blocks. We find conditions to verify that a block is inertial if and only if its dominating block is inertial. In some situations the equality of the factor fusion systems associated with a block and with its Brauer correspondent block give information about the hyperfocal subgroups.

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On the length of matrix algebras

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By the length of a finite system of generators for a finite-dimensional algebra over an arbitrary field we mean the least positive integer k such that the products of length not exceeding k span this algebra (as a vector space). The maximum length for the systems of generators of an algebra is referred to as the *length of the algebra*. Apart from intrinsic algebraic importance, the length function has applications, for example, in computing methods of the mechanics of isotropic continua and matrix rational procedures.

The length evaluation can be a difficult problem, since, for example, the length of the full matrix algebra is still unknown (Paz's Problem, 1984). Paz conjectured that the length of any generating set for the algebra of n by n matrices is at most $2n-2$. The question about the length determination was later extended on different matrix sets and subalgebras by Laffey (1986).

First we present a survey of our results on the main properties of length function. In particular, we provide a construction of series of matrix algebras demonstrating that a length of a subalgebra can be larger than the length of the algebra and the difference of their lengths can be arbitrary large. This result partially explains the difficulty of length evaluation.

In this talk we will also show that Paz's conjecture holds under the assumption that the generating set contains a nonderogatory matrix or a matrix with minimal polynomial of degree $n-1$. We will also present linear bounds for the length of generating sets that include a matrix with some restrictions on its Jordan normal form. Having an upper bound, we also provide examples of matrix sets of different type which length achieve the bound $2n-2$.

This talk is based on joint research with Alexander Guterman (Moscow State University), Thomas Laffey and Helena Šmigoc (University College Dublin).

On middle Bol loops and the total multiplication groups

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Let $(Q,\cdot)$ be a loop and $a \in Q$. The mappings $x \to ax, x \to xa, x \to a/x$ are denoted by L_a, R_a, D_a, respectively. The multiplication and total multiplication groups $Mlt(Q) = \{L_a, R_a; a \in Q\}$ and $TMlt(Q) = \{L_a, R_a, D_a; a \in Q\}$ of $(Q,\cdot)$ along with their subgroups $Inn(Q)$ and $TInn(Q)$ - the stabilizers of the unit in $Mlt(Q)$ and $TMlt(Q)$, respectively, are important tools in studying such properties of loops as normality of subloops, solvability, nilpotency etc. It is known that $Mlt(Q)$ is invariant under the isotopy of loops (A. Albert) while $TMlt(Q)$ is invariant under the isostrophy of loops (announced by the author and A. Drapal). Characterizations of the mentioned groups for some classes of loops with inverse properties are obtained, including their representation, general properties and systems of generators for $TInn(Q)$. Necessary and sufficient conditions when $TMlt(Q)$ is nilpotent are given.

An open problem regarding middle Bol loops is if this class includes the class of loops with universal (i.e. invariant under the isotopy of loops) flexibility. If this conjecture is true, then the loops with invariant flexibility under the isostrophy are Moufang loops. It is shown that commutative loops with invariant flexibility under the isostrophy are Moufang loops.

Real Cayley-Dickson algebras: doubly alternative elements and zero divisor graphs

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Due to the non-alternativity of Cayley-Dickson algebras with dimension at least 16, there appear zero divisors which are hard to study and to classify, except for some particular cases. At present most of the authors restrict their attention to the algebras of the main sequence where all parameters which determine the Cayley-Dickson construction are assumed to be equal to -1. We mention in particular the works by Moreno and Biss, Dugger, and Isaksen. Moreno's key idea was to study doubly alternative zero divisors, that is, such elements that their components are both alternative elements of the previous algebra. This notion was then extended to the Cayley-Dickson split-algebras and led to similar results.

The talk is devoted to zero divisors in arbitrary real Cayley-Dickson algebras whose components satisfy some additional conditions on the norm and alternativity. We are interested in the patterns which they form in orthogonality and zero divisor graphs, and these patterns appear to be hexagonal. In the case of the algebras of the main sequence, these hexagons can be extended to the so-called double hexagons. Moreover, the vertices of a double hexagon have a convenient multiplication table which has a block structure.

Session

Algebraic and Complex Geometry

- About new examples of Serret's curves, *Aleksandar Lipkovski*

About new examples of Serret's curves

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Abel's theorem claims that the lemniscate can be divided into n equal arcs by ruler and compass iff $n = 2^k p_1 ... p_m$, where p_j are pairwise distinct Fermat primes. The proof relies on the fact that the lemniscate can be parametrised by rational functions and the arc length is an elliptic integral of the first kind of the parameter. In 1845, Serret proposed a method to describe all such curves. He found a series of such curves and described its important properties. Since then, no new examples of curves with rational parametrisation, such that arc length is an elliptic integral of the first kind of the parameter are known. In this note we describe anew example of such curve.

Session

Analysis and its Applications

- Cuntz–Pimsner algebras associated to finite rank vector bundles twisted by a homeomorphism, *Maria Stella Adamo*
- A glimpse to the Berezin numbers inequality, *Mojtaba Bakherad*
- Numerical solution of Optimal Transport Problem on graphs, *Enrico Facca*
- On system of split generalised mixed equilibrium problem and fixed point problems for multivalued mappings with no prior knowledge of operator norm, *Oluwatosin Temitope Mewomo*
- Some new refinements of Hardy-type inequalities, *James Adedayo Oguntuase*
- The q-Steffensen inequality and some related generalizations, *Ksenija Smoljak Kalamir*
- On the spectral gap of one-dimensional Schrödinger operators on large intervals, *Matthias Taeufer*
- On rapidly varying sequences, *Valentina Timotić*
- On semi-discrete sub-partitions of vector-valued measure, *Gershon Wolansky*

Cuntz–Pimsner algebras associated to finite rank vector bundles twisted by a homeomorphism

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Coauthors: Dawn Archey, Magdalena Georgescu, Marzieh Forough, Ja A Jeong, Karen Strung, Maria Grazia Viola

In this talk, we will discuss the structural properties of Cuntz-Pimsner algebras arising by the continuous sections $\Gamma(V, \alpha)$ of a complex locally trivial vector bundle V on a compact Hausdorff space X twisted by a minimal homeomorphism $\alpha : X \to X$. We tackle this problem by identifying "large enough" C*-subalgebras that capture the fundamental properties of the containing Cuntz-Pimsner algebra but are more tractable. Lastly, we will examine conditions when these C*-algebras can be classified using the Elliott invariant.

A glimpse to the Berezin numbers inequality

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A reproducing kernel Hilbert space (RKHS for short) $\mathcal{H} = \mathcal{H}(\Omega)$ is a Hilbert space of complex valued functions on a (nonempty) set Ω, which has the property that point evaluations are continuous i.e. for each $\lambda \in \Omega$ the map $f \mapsto f(\lambda)$ is a continuous linear functional on $\mathcal{H}$. The Riesz representation theorem ensure that for each $\lambda \in \Omega$ there is a unique element $k_\lambda \in \mathcal{H}$ such that $f(\lambda) = \langle f, k_\lambda \rangle$, for all $f \in \mathcal{H}$. The collection $\{k_\lambda : \lambda \in \Omega\}$ is called the reproducing kernel of $\mathcal{H}$. If $\{e_n\}$ is an orthonormal basis for a functional Hilbert space $\mathcal{H}$, then the reproducing kernel of $\mathcal{H}$ is given by $k_\lambda(z) = \sum_n \overline{e_n(\lambda)} e_n(z)$. For $\lambda \in \Omega$, let $\hat{k_\lambda} = \frac{k_\lambda}{\|k_\lambda\|}$ be the normalized reproducing kernel of $\mathcal{H}$. For a bounded linear operator A on $\mathcal{H}$, the function $\widetilde{A}$ defined on Ω by $\widetilde{A}(\lambda) = \langle A\hat{k_\lambda}, \hat{k_\lambda} \rangle$ is the Berezin symbol of A, which firstly have been introduced by Berezin. The Berezin set and the Berezin number of the operator A are defined by

$$\mathbf{Ber}(A) := \{\widetilde{A}(\lambda) : \lambda \in \Omega\} \qquad \text{and} \qquad \mathbf{ber}(A) := \sup\{|\widetilde{A}(\lambda)| : \lambda \in \Omega\},$$

respectively. Namely, the Berezin transform has been investigated in detail for the Toeplitz and Hankel operators on the Hardy and Bergman spaces; it is widely applied in the various questions of analysis and uniquely determines the operator(i.e., for all $\lambda \in \Omega$, $\widetilde{A}(\lambda) = \widetilde{B}(\lambda)$ implies $A = B$).

The objective of this paper is to present a generalized Berezin number inequality and refine the new inequalities. We also present some results of Berezin number inequalities involving f-connection of operators.

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Numerical solution of Optimal Transport Problem on graphs

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The Dynamical Monge-Kantorovich (DMK) model is dynamical system of equations whose steady state has been related to the solution of the Optimal Transport Problem with cost equal to the Euclidean distance. In this talk we present a re-adaption into a graph setting of the DMK model, that can be rewritten in the form of a Gradient Flow. Using this formulation, we can solve the OTP on graphs with cost given by the shortest path distance looking at the long-time solution of the Gradient Flow equations. To this aim, we discretized the Gradient Flow equation via backward Euler time-stepping, in order to use larger time steps, getting faster convergence toward the optimal solution. The non-linear equations arising from such implicit time-stepping scheme are solved via Newton-Raphson Method. Thus, the optimization problem is reduced to the solution of a sequence of large and sparse saddle point linear systems, for which efficient preconditioners have to be build. In this talk we present different preconditioning approaches to tackle this problem.

On system of split generalised mixed equilibrium problem and fixed point problems for multivalued mappings with no prior knowledge of operator norm

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In this paper, we introduce system of split generalised mixed equilibrium problem, which is more general than the existing well known split equilibrium problems and their generalisations, split variational inequality problems and several other related problems. We propose a new iterative scheme with inertial term, which is independent on the operator norm and obtain strong convergence result for approximating a common solution of the problem and fixed point of finite family of multivalued demicontractive mappings. We obtain consequent results which complement several existing results in this direction in the current literature. We also apply our results to approximate the solution of split convex minimisation problems and present numerical examples to demonstrate the efficiency of our algorithm in comparison with some existing algorithms in the literature.

Keywords: Inertial algorithm, system of split generalised mixed equilibrium problems, fixed point problems, multivalued demicontractive mappings, strong convergence.

2010 Mathematics Subject Classification: 65K15, 47J25, 65J15, 90C33.

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Some new refinements of Hardy-type inequalities

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We obtain some further refinements of Hardy-type inequalities via superqudraticity technique. Our results both unify and further generalize several results on refinements of Hardy-type inequalities in the literature.

The q-Steffensen inequality and some related generalizations

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Since quantum calculus has many applications not only in mathematics but also in other sciences, results concerning q-integrals are important in many applications. In this talk we present q-analogue of the Steffensen inequality and prove some generalizations and refinements of q-Steffensen's inequality which were extensively studied in the case of the classical Steffensen inequality.

On the spectral gap of one-dimensional Schrödinger operators on large intervals

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We study the effect of non-negative potentials on the spectral gap of one-dimensional Schrödinger operators in the limit of large intervals. In particular, we derive upper and lower bounds on the gap for different classes of potentials which characterize its asymptotic behaviour. This talk is based on joint work with Joachim Kerner; arXiv:2012.09060.

On rapidly varying sequences

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In this paper we investigate the connection between the class $R_{\infty,s}$, of rapidly varying sequences (in the sense of de Haan) and the rapid equivalence, selection principles and game theory.

On semi-discrete sub-partitions of vector-valued measure

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We introduce a concept of optimal transport for vector-valued measures and its dual formulation, and concentrate on the semi-discrete case and show some fundamental differences between the scalar and vector cases. A manifestation of this difference is the possibility of non-existence of optimal solution for the dual problem for feasible primer problems.

Session

Combinatorics and Discrete Mathematics

- Semirings and temporal network analysis, *Vladimir Batagelj*
- A novel non-statistical methodology for detecting gerrymandering in parallel voting systems, *Radu Buzatu*
- The general position number of classical Sierpiński Graphs, *Khadijeh Fathalikhani*
- Entropy of Ribbon Tilings, *Vladislav Kargin*
- On the structure of maximum matchings in vertex-weighted graphs, *Miklós Krész*
- 1-skeleton of the polytope of pyramidal tours with step-backs, *Andrei Nikolaev*

Semirings and temporal network analysis

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A semiring is a "natural" structure to formalize computations with link weights in networks. In a temporal network, the presence/activity and properties/weights of nodes and links can change through time. To describe temporal networks we introduce the notion of temporal quantities. We define the addition and multiplication of temporal quantities in a way that can be used for the definition of addition and multiplication of temporal networks. The corresponding algebraic structures are semirings. We developed fast algorithms for both operations [2]. Large networks are usually sparse. We answer the question when the product of sparse networks is sparse itself [1, 4]. The proposed approach enables us to treat as temporal quantities also other network characteristics such as degrees, connectivity components, centrality measures, Pathfinder skeleton, etc. It is supported by the Python library Nets [5].

As a special case, we present two ways (instantaneous and cumulative) to transform bibliographic networks, using the works' publication year, into corresponding temporal networks based on temporal quantities. We also show how to use the addition of temporal quantities to define interesting temporal properties of nodes, links and their groups thus providing an insight into the evolution of bibliographic networks. Using the multiplication of temporal networks we obtain different derived temporal networks providing us with new views on studied networks [3].

The proposed approach is illustrated with examples from the analysis of Franzosi's violence network, Corman's Reuters terror news network, and a collection of bibliographic networks from WoS.

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A novel non-statistical methodology for detecting gerrymandering in parallel voting systems

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Gerrymandering is a practice intended to establish a political advantage for a particular party or group by manipulating electoral district boundaries. Switching from one electoral system to another one is frequently criticized by the opposition and is viewed as a means for the ruling party to stay in power. In particular, when the new electoral system is a parallel voting (or a single-member district) system, the ruling party is usually suspected of applying gerrymandering to increase the chance to win in a maximum possible number of districts.

Since it is extremely challenging to detect gerrymandering by using statistical methods, we propose a novel non-statistical methodology that has proven effective for detecting gerrymandering and computing fair districting under parallel voting systems. Our methodology is based on identifying the set of all feasible electoral outcomes and comparing the corresponding efficiency scores' values. For identifying all feasible electoral outcomes, we formulate and solve several gerrymandering problems as integer linear programming problems.

We showcased the application of our approach to the Moldovan parliamentary elections of 2019. Our results suggest that contrary to previous studies' arguments, there is no clear evidence to consider that the districting map used in those elections was unfair. Importantly, we also provide an example of the most equitable districting map that does not advantage any political party over another.

The general position number of classical Sierpiński Graphs

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Let G be a graph and S be a subset of its vertex set. If no three vertices of S lie on a common geodesic, then S is called a general position set. Such a set with largest size is a gp-set of G and its size is the gp-number of G. In this paper, we try to find gp-number of Sierpiński Graphs.

Entropy of Ribbon Tilings

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I will talk about ribbon tilings, which have been originally introduced and studied by Pak and Sheffield. These are tilings of a connected region in the square lattice by n-ribbons, – the sequences of n squares, with the next square appearing only on the right or the upper side of the previous square. These tilings are a generalization of well-studied domino tilings. I will explain how ribbon tilings are connected to acyclic orientations on graphs, and present some results about enumeration of these tilings on some simple regions.

In particular, we have a bound for the growth in the number of ribbon tilings on general regions and an explicit asymptotic expression for the number of tilings of "thin and long" n-by-N rectangular regions. As a consequence, we show that the entropy of a random ribbon tiling of a "thin and long" region is asymptotically $\log n$.

Joint work with Yinsong Chen.

On the structure of maximum matchings in vertex-weighted graphs

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Matching theory is a classical field of Combinatorics, however, despite the well-developed theory of efficient algorithms and structural results for the unweighted and edge-weighted cases, the Maximum Vertex-Weighted Matching Problem (MVWMP) has received less attention. Even though it was observed by T. H. Spencer and E. W. Mayr (Node weighted matching, in Automata, Languages and Programming) as early as 1984 that most problems concerning vertex-weighted graphs are closer to their unweighted counterparts than the edge-weighted ones, only a few specific theoretical problems have been considered since then.

In this talk we will present the most fundamental structure theorems for MVWMP. The counterpart of the classical Gallai-Edmonds Structure Theorem is worked out for vertex-weighted graphs, first for non-negative assignments, and then for the general case where negative weights are also allowed. These theorems characterize the structure and composition of maximum weight matchings, and also specify the deficiency of such matchings. Building on the Gallai-Edmonds decomposition, two Berge-type formulas are presented to estimate the deficiency of graphs incurring in one particular weight and also globally at all weights.

1-skeleton of the polytope of pyramidal tours with step-backs

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The 1-skeleton of the polytope P is the graph whose vertex set is the vertex set of P and edge set is the set of 1-dimensional faces of P. We consider the following 3 characteristics of a 1-skeleton: the vertex adjacency, the diameter, and the clique number. The adjacency relation is the key for the study and analysis of 1-skeleton, as well as the basis for the edge-following algorithms. While the diameter and the clique number serve as lower bounds on complexity in some computation models and classes of algorithms (see [1,3], for example).

We consider a classic asymmetric traveling salesperson problem: for a given complete weighted digraph K_n it is required to find a Hamiltonian cycle of minimum weight. For a Hamiltonian cycle τ we denote by $\tau(i)$ the successor of a vertex i and by $\tau^{-1}(i)$ the predecessor of i. A vertex i, satisfying $\tau^{-1}(i) < i$ and $\tau(i) < i$, is called a *peak*. A *step-back peak* is the vertex i, such that

$$\tau^{-1}(i) < i,\ \tau(i) = i-1,\ \tau^2(i) > i,\ \text{or}\ \tau^{-2}(i) > i,\ \tau^{-1}(i) = i-1,\ \tau(i) < i.$$

A *proper peak* is a peak i which is not a step-back peak. A *pyramidal tour with step-backs*, introduced by Enomoto, Oda and Ota [2], is a Hamiltonian cycle with exactly one proper peak

n. These tours are of interest, since, on the one hand, the minimum cost tour can be determined in $O(n^2)$ time by dynamic programming, and, on the other hand, there are known restrictions on the distance matrix that guarantee the existence of an optimal solution that is a pyramidal tour with step-backs.

We define the traveling salesperson polytope $\mathrm{TSP}(n)$ and the polytope of pyramidal tours with step-backs $\mathrm{PSB}(n)$ as the convex hulls of characteristic vectors of all corresponding tours in the complete digraph K_n. It is known that for the polytope $\mathrm{TSP}(n)$ the question of whether two vertices are nonadjacent is NP-complete [4], the diameter equals 2 for asymmetric polytope [3] and is at most 4 for symmetric polytope [5], and the clique number of 1-skeleton is superpolynomial in n [1].

Let x and y be two pyramidal tours with step-backs. We denote by x^v and y^v the corresponding vertices of the $\mathrm{PSB}(n)$ polytope and by $x \cup y$ a regular directed multigraph that contains all edges of both tours x and y.

Lemma 1 (Sufficient condition for nonadjacency)**.** *Given two tours x and y, if the multigraph $x \cup y$ includes a pair of edge-disjoint pyramidal tours with step-backs, different from x and y, then the corresponding vertices x^v and y^v of the polytope $\mathrm{PSB}(n)$ are not adjacent.*

Lemma 2 (Necessary condition for nonadjacency)**.** *If the vertices x^v and y^v of the polytope $\mathrm{PSB}(n)$ are not adjacent, then the multigraph $x \cup y$ includes at least two pyramidal tours with step-backs, different from x and y.*

Our main result is that both necessary and sufficient conditions for nonadjacency can be verified in linear time.

Theorem 1. *The question of whether two vertices of the polytope $\mathrm{PSB}(n)$ are adjacent or nonadjacent can be verified in linear time $O(n)$.*

Based on the vertex adjacency relation, we can investigate other characteristics of 1-skeleton of the polytope $\mathrm{PSB}(n)$.

Theorem 2. *The diameter of 1-skeleton of the polytope $\mathrm{PSB}(n)$ is bounded above by 4. The clique number of 1-skeleton of the polytope $\mathrm{PSB}(n)$ is quadratic in n:*

$$\omega(\mathrm{PSB}(n)) = \Theta(n^2).$$

Thus, the polyhedral characteristics of the traveling salesperson problem on Hamiltonian cycles and pyramidal tours with step-backs are fundamentally different and correspond to the computational complexity of the problem.

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Session

DIFFERENTIAL GEOMETRY AND APPLICATIONS

On PNDP-manifolds

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We provide a possible way of constructing new kinds of manifolds which we will call Partially Negative Dimensional Product manifold (PNDP-manifold for short).
In particular a PNDP-manifold is an Einstein warped product manifold of special kind, where the base-manifold B is a Remannian (or pseudo-Riemannian) product-manifold $B = \Pi_{i=1}^{q'} B_i \times \Pi_{i=(q'+1)}^{\widetilde{q}} B_i$, with $\Pi_{i=(q'+1)}^{\widetilde{q}} B_i$ an Einstein-manifold, and the fiber-manifold F is a derived-differential-manifold (i.e., F is the form: smooth manifold ($\mathbb{R}^d$)+ obstruction bundle, so it can admit "virtual" negative dimension).
Since the dimensions of a PNDP-manifold is not related with the usual geometric concept of dimension (we consider they as "virtual" dimensions), from the speculative and applicative point of view, we use special projections/desuspensions, to identify the PNDP with another type of "object" of the same dimension, thus introducing a new type of "hidden" dimensions.
We have consider three kinds of PNDP-manifolds:

Type I) the $(n, -n)$-PNDP manifold that has overall, zero-dimension ($dimM = dimB + dimF = n + (-n) = 0$). The speculative result may be interpreted as an "invisible" manifold but made up of two manifolds with n and $-n$ dimensions, respectively, then we try to consider it as a kind of "point-like manifold" (zero-dimension), but with "hidden" dimensions, and

Type II) the $(n, -d)$-PNDP manifold, where n (the base-manifold dimension) is different from d (with d a positive integer number and the fiber-manifold dimension is $m = -d$) such that $dim = n + (-d) > 0$. The particular speculative feature of this manifold is that it appears as another Einstein-manifold with $(n + (-d))$-dimension.

Type III) it is like the *Type II*, but $dim = n + (-d) < 0$. It has the speculative feature of being considered, through our projection, like $||(n - d)||$-th desuspension of a point.

These manifolds, introducing this new concept of "hidden" dimensions, could have some applications, in particular in the description of "point-like" structures, as a structure of superconducting graphene or in the MOG theory without anomalies, but recently also in the econophysical field, in the description of financial markets influenced by ghost fields such as dark volatility.

2010 Mathematics Subject Classification: 53C25

Keywords and phrases: PNDP-manifolds; Einstein warped product manifolds; negative dimensional manifolds; derived-manifold; desuspension; point-like manifold; virtual dimensions.

Vertex algebra approach to cohomology of foliations

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Using the reduction cohomology theory for grading-restricted vertex algebras, we construct vertex algebra bicomplexes and introduce a cohomology theory for foliations of smooth manifolds. We then define corresponding characteristic classes.

Session

Dynamical Systems and Ordinary Differential Equations and Applications

The minimality of Sturm-Liouville problems with a boundary condition depending quadratically on the eigenparameter

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We study minimality of root functions of the following Sturm-Liouville problem

$$-y'' + q(x)y = \lambda y,\ 0 < x < 1, \tag{0.1}$$

$$y(0)\cos\beta = y'(0)\sin\beta,\ 0 \leq \beta < \pi, \tag{0.2}$$

$$y(1) = (a\lambda^2 + b\lambda + c)y'(1),\ a \neq 0, \tag{0.3}$$

where λ is the spectral parameter, $q(x)$ is a real valued and continuous function on the interval $[0, 1]$, and a, b, c are real. It is known that the eigenvalues of (0.1)-(0.3) form an infinite sequence, accumulating only at $+\infty$, and one of the following cases are possible:

(a) All the eigenvalues are real and simple;
(b) All the eigenvalues are simple and all, except a conjugate pair of non-real, are real;
(c) All the eigenvalues are real and all, except one double, are simple;
(d) All the eigenvalues are real and all, except one triple, are simple.

By constructing the biorthogonal system explicitly, it is possible to show that in the case (a) the system of eigenfunctions with any two eigenfunctions excluded forms a minimal system in space $L_2(0, 1)$. It is also possible to give similar results in the cases (b), (c) and (d). In particular, for the case (b) one can prove that the system of eigenfunctions without the eigenfunction, corresponding to the double eigenvalue, is a minimal system. Similarly, in the case (c) one can prove that the system of eigenfunctions with no excluded functions is a minimal system. Finally, in the case (d) it is possible to prove that the system of eigenfunctions, without the two eigenfunctions, corresponding to non-real eigenvalues, is minimal. These minimality results can then be extended to basis properties in $L_2(0, 1)$ and $L_p(0, 1)$ $(1 < p < \infty)$. But the study of these properties wouldn't be complete if we will not consider similar questions for the system of root functions which contain beside eigenfunctions some associated functions. If the associated functions are not excluded then these minimality properties do not always hold true. There are associated functions which when included make the system of root functions not minimal anymore. In the current work, we will give necessary and sufficient conditions for the system of root functions to be minimal in $L_2(0, 1)$ for all possible choices of the two excluded functions, including the cases when the excluded functions are eigenfunction and when they are associated functions.

On Linear Inhomogeneous Boundary-Value Problems for Differential Systems in Sobolev Spaces

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For the systems of ordinary differential equations of an arbitrary order on a compact interval, we study a character of solvability of the most general linear boundary-value problems in the Sobolev spaces W_p^n, with $n \in \mathbb{N}$ and $1 \leq p \leq \infty$. We find the indices of these Fredholm problems and obtain a criterion of their well-posedness. Each of these boundary-value problems relates to a certain rectangular numerical characteristic matrix with kernel and cokernel of the same dimension as the kernel and cokernel of the boundary-value problem. The condition for the sequence of characteristic matrices to converge is found. We obtain a constructive criterion under which the solutions to these problems depend continuously on the small parameter ε at $\varepsilon = 0$, and find the degree of convergence of the solutions. Also applications of these results to multipoint boundary-value problems are obtained.

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Exponential dichotomy conditions for difference equations with perturbed coefficients

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A system of linear difference equations with periodic coefficients is considered

$$y_{n+1} = (A(n) + B(n))y_n, \quad n \in \mathbb{Z}, \tag{1}$$

where $A(n)$ are non-degenerate matrices of size $m \times m$ and the matrix sequence $\{A(n)\}$ is N-periodic, i.e. $A(n+N) = A(n), \quad n \in \mathbb{Z}$. The sequence $\{B(n)\}$ is an N-periodic sequence of perturbations. We assume that the system

$$x_{n+1} = A(n)x_n, \quad n \in \mathbb{Z}, \tag{2}$$

is exponentially dichotomous. As shown in[1], this is equivalent to the fact that there are Hermitian matrices $H(0), H(1), \dots, H(N-1)$ and a matrix P satisfying the following boundary value problem

$$\begin{cases} H(l) - A^*(l)H(l+1)A(l) = \left(U_l^*\right)^{-1} P^* U_l^* U_l P U_l^{-1} \\ -\left(U_l^*\right)^{-1} (I-P)^* U_l^* U_l (I-P) U_l^{-1}, \quad l = 0, 1, \dots, N-1, \\ H(0) = H(N) > 0, \\ H(0) = P^* H(0) P + (I-P)^* H(0)(I-P), \\ P^2 = P, \quad P U_N = U_N P, \end{cases} \tag{3}$$

where U_l is the Cauchy matrix of (2). This criterion is analogous to the criterion of M. G. Krein for the exponential dichotomy of difference equations with constant coefficients [2].

Using the fact that the solution of the boundary value problem (3) is represented as

$$H(l) = \Big(U_l^*\Big)^{-1}\Bigg(\sum_{k=0}^{\infty}\Big(U_N^*\Big)^k P^*\Big(\sum_{i=l}^{N+l-1}U_i^*U_i\Big)PU_N^k\Bigg)U_l^{-1}$$
$$+\Big(U_l^*\Big)^{-1}\Bigg(\sum_{k=1}^{\infty}\Big(U_N^*\Big)^k (I-P)^*\Big(\sum_{i=l}^{N+l-1}U_i^*U_i\Big)(I-P)U_N^k\Bigg)U_l^{-1} = H^-(l) + H^+(l),$$

we can obtain conditions for perturbations $\{B(n)\}$ under which the system (1) is also exponentially dichotomous.

Theorem. *Let* $det(A(n)) \neq 0$ *and the matrix sequence of perturbations* $\{B(n)\}$ *satisfy the condition*

$$\max\{\|B(0)\|, \dots, \|B(N-1)\|\}$$
$$< \left(h^-\Big(\sqrt{1-\frac{1}{h^-}}+1\Big)\sqrt{h^-\|H(0)\|} + h^+\Big(\sqrt{1+\frac{1}{h^+}}+1\Big)\sqrt{h^+\|H(0)\|}\right)^{-1},$$

where

$$h^- = \max\{\|H^-(0)\|, \|H^-(1)\|, \dots, \|H^-(N-1)\|\},$$
$$h^+ = \max\{\|H^+(0)\|, \|H^+(1)\|, \dots, \|H^+(N-1)\|\},$$

then the perturbed system (1) is exponentially dichotomous .

This paper is a continuation of [1, 3–5].

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Multigrid Fast Sweep Method For Computation of Isostables and Isochrons

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We propose a fast iterative multigrid algorithm for the computation of isostables and isochrons for dynamical systems with stable limit cycles or fixed points in high dimensions. We solve a first-order static Hamilton–Jacobi equation with a constant source term using a Eulerian Fast Sweeping Method developed for this type of problem. We reduce the number of iteration of the standard Fast Sweeping Method using nested grids and demonstrate the speed up on several illustrative examples.

Keywords: isostables, isochrons, Hamilton–Jacobi, multigrid

Conditions of global solvability, Lyapunov stability, Lagrange stability and dissipativity for time-varying semilinear differential-algebraic equations, and applications

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Theorems on the existence and uniqueness of global solutions, Lagrange stability and instability, dissipativity (ultimate boundedness), Lyapunov stability and instability, and asymptotic stability for time-varying semilinear DAEs (nonautonomous degenerate ordinary differential equations) will be presented, and mathematical models of nonlinear time-varying electrical circuits will be considered in order to demonstrate the application of the presented theorems. The features and advantages of the obtained theorems will also be discussed. The talk is based on the results published in the journals “Differential Equations”, “Global and Stochastic Analysis”, and “Proceedings of the Institute of Mathematics and Mechanics”.

Non-uniformly hyperbolic dynamics for some classes of piecewise smooth systems

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We consider piecewise smooth systems (PWS) of ordinary differential equations. The phase space is supposed to be split into two (or more) subsets with a threshold being a peicewise smooth surface. For such systems, smoothly depending on a parameter, the so-called grazing bifurcation is considered. Roughly speaking, this bifurcation corresponds to the existence of a periodic solution tangent to the threshold.

Basing on the approaches of previous author’s works and some new ideas on estimating Lyapunov exponents for near-grazing periodic solutions, we describe non-uniformly hyperbolic invariant sets for some classes of PWS. In other words, local coexistence of infinitely many periodic solutions with distinct dimensions of stable/unstable manifolds is proved for those

classes of PWS. In addition, we discuss how the developed theory may be applied to study dynamics of strongly nonlinear systems e.g. the van der Pol equation.

Limit states of multi-component discrete dynamical systems

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Object. We study models of multicomponent discrete dynamic conflict systems with attractive interaction, which are characterized by a positive value that is called the attractor index. Consider the set of discrete probability measures $\mu_i \in M_1^+(\Omega)$ on finite space $\Omega = \{\omega_1, \dots, \omega_n\}$, $i = \overline{1, m}$. Each of these measures μ_i can be identified with a stochastic vector $\mathbf{p}_i = (p_{ij})_{j=1}^n$, where

$$p_{ij} = \mu_i(\omega_j), \quad i = \overline{1, m}, \quad j = \overline{1, n}.$$

Consider the mapping $\divideontimes$

$$\{\mathbf{p}_1^t, \mathbf{p}_2^t, \dots, \mathbf{p}_m^t\} \xrightarrow{\divideontimes, t} \{\mathbf{p}_1^{t+1}, \mathbf{p}_2^{t+1}, \dots, \mathbf{p}_m^{t+1}\}, \tag{1}$$

which generates multi-component discrete dynamical systems with trajectories (1), where the coordinates of each vector $\mathbf{p}_i^t = (p_{ij}^t)_{j=1}^n$ are changed according to equations

$$p_{ij}^{t+1} = \frac{1}{z^t}\big(p_{ij}^t(\theta^t + 1) + \tau_j^t\big), \quad t = 0, 1, \dots. \tag{2}$$

Here $\theta^t = \theta(\mathbf{p}_1^t, \mathbf{p}_2^t, \dots, \mathbf{p}_m^t)$ is a finite positive function, $\mathcal{T}^t = (\tau_j^t)_{j=1}^n$ is a vector with non-negative coordinates (attractor index), and $z^t = \theta^t + 1 + W^t$ is normalizing denominator, $W^t = \sum_{j=1}^n \tau_j^t$.

Main results.

Theorem 1. *Let all coordinates of vector $\mathbf{w}^t = (w_j^t)_{j=1}^n$, $w_j^t := \frac{\tau_j^t}{W^t}$ be bounded and monotonic (increase or decrease independently one to other). Then for all $i = \overline{1, m}$ there exist*

$$\mathbf{p}_i^\infty = \lim_{t\to\infty} \mathbf{p}_i^t$$

and all limit vectors $\mathbf{p}_i^\infty$ coincide with the vector $\mathbf{w}^\infty$, i.e.

$$p_{ij}^\infty = \frac{\tau_j^\infty}{W^\infty} \ \forall j.$$

Let us consider the different variants of attractor index $\mathcal{T}^t$:

$$\tau_j^t := \tau_{j,\min}^t = \min_i \{p_{ij}^t\}, \tag{3}$$

$$\tau_j^t := \tau_{j,\max}^t = \max_i \{p_{ij}^t\}, \tag{4}$$

$$\tau_j^t := \overline{\tau}_j^t = \frac{1}{m}\sum_{i=1}^n p_{ij}^t, \tag{5}$$

$$\tau_{j_1}^t = \tau_{j_2}^t > 0, \quad j_1, j_2 = \overline{1, n}. \tag{6}$$

Theorem 2. *Let coordinates of the attractor index* $\mathcal{T}^t$ *be given by one of the equations (3), (4), (5), (6). Then each trajectory of a dynamic system* (1) *with an initial state* $\{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_m\}$ *converges to the fixed point* $\{\mathbf{p}_1^\infty, \mathbf{p}_2^\infty, \ldots, \mathbf{p}_m^\infty\}$

$$\mathbf{p}_i^\infty = \lim_{t\to\infty} \mathbf{p}_i^t, \ \forall\, i = \overline{1,m},$$

where the coordinates of $\mathbf{p}_i^\infty$ *have a view:*

$$p_{ij}^\infty = \frac{\tau_j}{W} \ \ \forall\, i = \overline{1,m}, \ \ j = \overline{1,n}. \tag{7}$$

Stability. The limit state is unstable in cases (3)-(5), however it is stable only in the case (6), when all limit coordinates are equal to $\frac{1}{n}$.

Application. Such model of dynamic systems can describe the dynamics of real processes. Attractor index can describe a real external influence on a certain system (for example, information influence on a society). System behavior can be controlled or described by setting attractor index which can be exposed to such influence.

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Session

Logic and Mathematical Aspects of Computer Science

Aggregation of individual rankings through fusion functions: criticism and optimality analysis

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Our main idea is to analyze from a theoretical and normative point of view different methods to aggregate individual rankings. To do so, first we introduce the concept of a general mean on an abstract set. This new concept conciliates the Social Choice where well-known impossibility results as the Arrovian ones are encountered and the Decision-Making approaches where the necessity of fusing rankings is unavoidable. Moreover it gives rise to a reasonable definition of the concept of a ranking fusion function that does indeed satisfy the axioms of a general mean. Then we will introduce some methods to build ranking fusion functions, paying a special attention to the use of score functions, and pointing out the equivalence between ranking and scoring. To conclude, we prove that any ranking fusion function introduces a partial order on rankings implemented on a finite set of alternatives. Therefore, this allows us to compare rankings and different methods of aggregation, so that in practice one should look for the maximal elements with respect to such orders defined on rankings IEEE.

Towards Non-Presentable Models of Homotopy Type Theory

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One important aspect of homotopy type theory is the construction of *models*: $(\infty, 1)$-categories in which we can interpret the axioms of our type theory. Studying various models can help us discern which statements can and cannot be proven with our given axiomatization.

Due to a result by Shulman, we already know that every Grothendieck $(\infty, 1)$-topos is a model for homotopy theory. However, we do not expect all models to be a Grothendieck $(\infty, 1)$-topos and in particular, we anticipate non-presentable models of homotopy type theory.

The goal of this work is to take a first step towards showing the existence of non-presentable models of homotopy type theory, by constructing a non-presentable *elementary* $(\infty, 1)$*-topos* . Elementary $(\infty, 1)$-toposes share many features with Grothendieck $(\infty, 1)$-toposes (such as descent, universes, natural number objects, ...), but are not required to be presentable and thus can include examples that are not Grothendieck $(\infty, 1)$-toposes.

We will construct such examples via the *filter construction*. Generalizing a result from 1-category theory, we prove that for every elementary $(\infty, 1)$-topos $\mathcal{E}$ and filter of subobjects Φ, we can construct an elementary $(\infty, 1)$-topos $\prod_\Phi \mathcal{E}$, which is in fact not presentable if the filter Φ is not principal. We will apply this result to the $(\infty, 1)$-category of Kan complexes to construct non-presentable examples of elementary $(\infty, 1)$-toposes.

Session

MATHEMATICAL PHYSICS

- Asymptotics of eigenvalue fluctuations for random Schroedinger operators, *Yoel Grinshpon*
- Constructive Method to Solving 3D Navier - Stokes Equations, *Alexander Koptev*
- Removable singularities for anisotropic porous medium equations, *Savchenko (Shan) Mariia*
- Non-periodic ground states of one-dimensional, non-frustrated, two-body interactions, *Jacek Miękisz*

Asymptotics of eigenvalue fluctuations for random Schroedinger operators

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Random Schroedinger operators arise in mathematical physics as models of quantum particles in disordered media. Thus, the spectral properties of such operators have attracted a considerable amount of attention. In this talk, we will discuss some recent results regarding eigenvalue asymptotics on various scales, both in the random decaying case and for the celebrated Anderson model.

Some of the results are joint work with Jonathan Breuer and Moshe White.

Constructive Method to Solving 3D Navier - Stokes Equations

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We consider 3D Navier – Stokes equations for motiom of incompressible media. This equations are of mathematical interest and have a lot of applications to practical problems. For today many aspects connected with the Navier – Stokes equations have been studied not enough and need more profound investigation [1-2]. Main unresolved problem is the lack of a constructive method of solution. How to resolve the Navier – Stokes equations while preserving all nonlinear terms is the question that needs to be addressed.

The author proposes an approach to this problem the essence of which is to reduce the basic problem of solving the Navier – Stokes equations to a set of simple tasks. We face to five more simple tasks that you need to consistently allow. Each of the individual Navier – Stokes equations must be reduced to a free divergent form and integrated. Resulting equality can be converted so as to exclude some nonlinear and non-divergent terms. As the result we arrive to nine equations which link main unknowns u, v, w, p, associated unknowns Ψ_i where $i = 1, 2, \dots, 15$ and an arbitrary additive functions in three variables α_i, β_i, γ_i, δ_i. Considered together these nine ratios provide the first integral of 3D Navier – Stokes equations [3]. The well-known integrals of Bernoulli, Euler - Bernoulli and Lagrange - Cauchy are its special cases [4]. Four of the nine obtained equations represent expressions for basic unknowns. So they determine the overall structure of the solutions. The remaining five equations can be resolved relative to six associated unknowns Ψ_j where $j = 10, 11, \dots, 15$ only if two conditions of compatibility are hold. They reduce to two fifth order equations with respect to nine unknown Ψ_k where $k = 1, 2, \dots, 9$. Each set of functions Ψ_k satisfying a given system determines exact solution of the Navier - Stokes equations. To complete the solution you need to find the six remaining associated unknowns Ψ_j, where $k = 10, 11, \dots, 15$. Three of the last functions can be set arbitrarily whereas the remaining three are defined as solution of linear inhomogeneous equations.

As a result all values presents of the structure formula for main unknowns are defined and these unknowns are easy to find. Some exact solutions constructed in this way are given in [5-6]. A similar approach can be applied to construct solutions of the Euler equations for case of motion for inviscid incompressible media. In all relationships it is enough to put $\frac{1}{Re} = 0$.

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Removable singularities for anisotropic porous medium equations

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This paper is devoted to the obtaining conditions for removable singularity at the point for solutions of quasilinear parabolic equations model of which are

$$u_t - \sum_{i=1}^{n} \left(u^{m_i-1} u_{x_i}\right)_{x_i} = 0, \tag{1}$$

$$u_t - \sum_{i=1}^{n} \left(u^{m_i-1} u_{x_i}\right)_{x_i} + f(u) = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} - \sum_{i=1}^{n} \left(u^{m_i-1} u_{x_i}\right)_{x_i} + \sum_{i=1}^{n} |u_{x_i}|^{q_i} = 0, \tag{3}$$

We focus on the solutions which satisfy the initial condition

$$u(x,0) = 0, \;\; x \in \Omega \setminus \{0\}, \tag{4}$$

where Ω is a bounded domain in R^n, $n \geq 2, t \in (0,T), 0 < T < +\infty, 0 \in \Omega.$

We suppose that the exponents m_i, q_i $i = \overline{1,n}$ satisfy the following condition

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \ldots \leq m_n < m + \frac{2}{n}, m = \frac{1}{n} \sum_{i=1}^{n} m_i,$$

$$\frac{2+nm}{1+n} \leq q < 2, \quad \max_{0\leq i\leq n} q_i < q\left(1+\frac{1}{n}\right), \quad \frac{1}{q} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{q_i}.$$

The main difficulty lies in the fact that part of $m_i < 1$ (singular case), and another part of $m_i > 1$ (degenerate case). We found a universal approach to study the properties of solutions of the anisotropic porous medium equation which not depends on the values of the anisotropic exponents m_i. We established the pointwise condition for removability of the singularity for solutions of the equation (1) [1]. We also obtained the pointwise estimates of solutions, depending on the relations between the exponents m_i and q_i (for the equation (3) [3]), m_i and q (for the equation (2) in case $f(u) = u^q$ [2]) which guarantee that the point singularity is removable. The proof of removability result is based on the new a priori estimates of "large" type solutions. In particular, we obtain the Keller-Osserman type estimate of the solution to the problems (2), (4) and (3), (4).

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Non-periodic ground states of one-dimensional, non-frustrated, two-body interactions

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Since the discovery of quasicrystals, one of the problems in statistical mechanics is to construct microscopic models of interacting atoms or molecules in which all ground-state configurations minimizing energy are non-periodic.

We construct for the first time examples of one-dimensional classical lattice–gas models with non-frustrated, two-body, infinite-range interactions and without periodic ground-state configurations. Ground-state configurations of our models are Sturmian sequences defined by irrational rotations on the circle. We present minimal sets of forbidden patterns which define Sturmian sequences in a unique way. Our interactions assign positive energies to forbidden patterns and are equal to zero otherwise. We illustrate our construction by the well-known example of the Fibonacci sequences.

We will also discuss stability of one-dimensional non-periodic ground-state configurations with respect to finite-range perturbations of interactions. We will show that they are not stable for fast-decaying interactions.

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Session

Mathematics in Science and Technology

- A Mathematical Model for Low Grade Gliomas and the effects of chemotherapy, *María Vela Pérez*
- Image completion with approximate convex hull tensor decomposition, *Rafal Zdunek*

A Mathematical Model for Low Grade Gliomas and the effects of chemotherapy

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The optimal management of low grade gliomas remains an open problem. Although complete surgical resection is recommended, given the diffuse infiltrative nature of these tumors and the risk of major surgery in important areas of the brain, they often result in tumors that are incompletely resected or simply biopsied. For this reason, the use of TMZ combined with other options is increasingly used [1, 2]. We present a modification of the system appeared in [2] to understand the late response to the chemotherapy observed in LGG without using an excessive number of unknown parameters. In our approach we consider separately the proliferation process and natural death process [3]. This allows us to obtain a good agreement of model solutions with medical data which was impossible in the approach presented in [2].

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Image completion with approximate convex hull tensor decomposition

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Many structural image completion methods are based on a low-rank approximation of the underlying image using matrix or tensor decomposition models. In this study, we assume that the image to be completed is represented by a multi-way array and can be approximated by a conical hull of subtensors in the observation space. If an observed tensor is near-separable along at least one mode, the extreme rays, represented by the selected subtensors, can be found by analyzing the corresponding convex hull. Following this assumption, we propose a geometric algorithm to address a low-rank image completion problem. The extreme rays are extracted with a segmented convex-hull algorithm that is suitable for performing noise-resistant non-negative tensor factorization. The coefficients of a conical combination of such rays are estimated using Douglas-Rachford splitting combined with the rank-two update least-squares algorithm. The proposed algorithm was applied to incomplete RGB images and hyperspectral arrays with a large number of randomly missing entries. Experiments confirm its good performance with respect to other well-known image completion methods, such as NMCSA, TMac, TCTF, SiLRTC, FaLRTC, and HaLRTC. Its computational complexity can be upper-bounded by $O(\xi \prod_{n=1}^{N} I_n J_n)$ for incomplete tensor $M \in R^{I_1 \times \cdots \times I_N}$ given the multi-linear rank $(J_1, \ldots, J_N)$.

Session

NUMBER THEORY

- Generalization of proofs and codification of graph families, *Lorenzo Sauras Altuzarra*
- Frobenius number of relatively prime three Lucas numbers, *Boonrod Yuttanan*

Generalization of proofs and codification of graph families

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In this talk, I will explain how to apply Baaz's generalization method and a recent graph-theoretical technique to, respectively, formal proofs from elementary number theory and certain increasing families of simple graphs. These procedures have resulted to have a considerable potential in revealing arithmetical patterns, that got reflected in theorems (for example, sufficient conditions for a value to be a divisor of an arbitrary Fermat number) and conjectures (mainly about integer sequences).

Frobenius number of relatively prime three Lucas numbers

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Let $a_1, a_2, \ldots, a_n$ $(n \geq 2)$ be positive integers with $\gcd(a_1, a_2, \ldots, a_n) = 1$. Finding the largest positive integer N such that the Diophantine equation

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = N$$

has no solution in non-negative integers is known as the Frobenius problem. Such the largest positive integer N is called the Frobenius number of $a_1, a_2, \ldots, a_n$. Various results of the Frobenius number have been studied extensively, see [1]−[8]. In this talk, the Frobenius problem is discussed in the cases $n = 2$ and 3. In particular, we determine the formula for the Frobenius number of relatively prime three Lucas numbers other than results of S. Ýlhan and R. Kýper in [8].

Keywords and phrases: Frobenius number, Lucas numbers, Fibonacci numbers.

2010 Mathematics Subject Classification: 11D07, 11B39

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Session

NUMERICAL ANALYSIS AND SCIENTIFIC COMPUTING

- A singularly perturbed problem on a Duran-Lombardi mesh, *Mirjana Brdar*
- A numerical algorithm for solving problem for a system of essentially loaded differential equations, *Zhazira Kadirbayeva*
- A stochastic numerical scheme for SDEs with fBm under non-Lipschitz coefficient, *Minoo Kamrani*
- A numerical solution to a nonlinear boundary value problem for the Fredholm integro-differential equation, *Sandugash Mynbayeva*
- Approximate Calculation of Triple Integrals of Rapidly Oscillating Functions using Different Types of Information about Functions, *Olesia Nechuiviter*
- Approximation of the two variables Discontinuous Functions by Discontinuous Interlination Splines using Triangular Elements, *Iuliia Pershyna*
- Numerical computation of the complex zeros of Bessel and Hankel functions, *Diego Ruiz-Antolín*
- Neural network approximations for high-dimensional PDEs, *Diyora Salimova*

A singularly perturbed problem on a Duran-Lombardi mesh

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We consider a singularly perturbed convection-diffusion problem on unite square whose solution may have exponential and parabolic boundary layers. The problem is solved numerically by a finite element method with piecewise bilinear elements on a graded Duran-Lombardi mesh. We prove uniform convergence of this method in an energy norm. Furthermore, by using a streamline-diffusion version of the method (SDFEM) we are able to perform analysis of the supercloseness property of the SDFEM in the corresponding streamline-diffusion norm. Our analysis offers a choice of parameters which improves stability.

A numerical algorithm for solving problem for a system of essentially loaded differential equations

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In the present paper we consider the following linear boundary value problem for essentially loaded differential equations with multi-point conditions:

$$\frac{dx}{dt} = A(t)x + \sum_{j=1}^{m} M_j(t)\dot{x}(\theta_j) + \sum_{i=0}^{m+1} K_i(t)x(\theta_i) + f(t), \quad t \in (0,T), \tag{1}$$

$$\sum_{i=0}^{m+1} C_i x(\theta_i) = d, \quad d \in R^n, \quad x \in R^n, \tag{2}$$

where the $(n \times n)$ -matrices $A(t)$, $M_j(t)$ $(j = \overline{1,m})$, $K_i(t)$ $(i = \overline{0,m+1})$, and n-vector-function $f(t)$ are continuous on $[0,T]$, C_i $(i = \overline{0,m+1})$ are constant $(n \times n)$ - matrices, and $0 = \theta_0 < \theta_1 < \theta_2 < \ldots < \theta_{m-1} < \theta_m < \theta_{m+1} = T$; $\quad \|x\| = \max_{i=\overline{1,n}} |x_i|$.

Let $C([0,T], R^n)$ denote the space of continuous functions $x : [0,T] \to R^n$ with the norm $||x||_1 = \max_{t\in[0,T]} ||x(t)||$.

A solution to problem (1), (2) is a continuously differentiable on $(0,T)$ function $x(t) \in C([0,T], R^n)$ satisfying the essentially loaded differential equations (1) and the multi-point condition (2).

We offer algorithm for solving to linear multi-point boundary value problem for essentially loaded differential equations (1), (2). Using the properties of essentially loaded differential equation and assuming the invertibility of the matrix compiled through the coefficients at the values of the derivative of the desired function at load points, we reduce the considered problem to a multi-point boundary value problem for loaded differential equations. The parameterization method [1] is used for solving this problem. The linear boundary value problem for loaded differential equations is reduced to equivalent problem consisting the Cauchy problems for system of ordinary differential equations with parameters in subintervals, multi-point condition and continuity conditions. At fixed values of parameters the Cauchy problem for system of ordinary differential equations in subinterval has a unique solution. This solution is represented with fundamental matrix of system. Using these representations we compile a system of linear algebraic equations with respect to parameters. We proposed algorithm for finding of numerical solution to the equivalent problem [2]. This algorithm includes the numerical solving of the Cauchy problems for system of the ordinary differential equations and solving of the linear system of algebraic equations.

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A stochastic numerical scheme for SDEs with fBm under non-Lipschitz coefficient

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Our aim is to propose a numerical method for the solution of stochastic differential equations(SDEs) with fractional Brownian motion(fBM) which the diffusion coefficient is non-Lipschitz. We consider fractional Brownian motions with Hurst parameter $\frac{1}{2} < H < 1$.

The basic idea is to apply Lamperti transformation to obtain a stochastic differential equation with additive noise. The well-posedness of this equation is proved and by applying a numerical method which is based on Euler scheme, the solution of the transformed equation is derived. Therefore, by using the inverse of the Lamperti transformation, numerical solution of the main SDE is established.

Based on the properties of the Malliavin calculus, convergence of the proposed method is explained. Furthermore the strong rate of convergence is derived.

A numerical solution to a nonlinear boundary value problem for the Fredholm integro-differential equation

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It is considered the boundary value problem (BVP) for the Fredholm IDE

$$\frac{dx}{dt} = f(t,x) + \sum_{k=1}^{m} \varphi_k(t) \int_0^T \psi_k(\tau)x(\tau)d\tau, \quad t \in (0,T), \quad x \in \mathbb{R}^n, \tag{1}$$

$$g\left[x(0), x(T)\right] = 0, \tag{2}$$

where $f : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are continuous; the $n \times n$ matrices $\varphi_k(t)$, $\psi_k(\tau)$, $k = \overline{1,m}$, are continuous on $[0,T]$, $\|x\| = \max_{i=\overline{1,n}} |x_i|$.

Denote by $\mathbb{C}\big([0,T],\mathbb{R}^n\big)$ the space of continuous functions $x:[0,T]\to\mathbb{R}^n$ with the norm $\|x\|_1=\max_{t\in[0,T]}\|x(t)\|$. By a solution to problem (1), (2) we mean a continuously differentiable on $(0,T)$ function $x(t)\in\mathbb{C}\big([0,T],\mathbb{R}^n\big)$ that satisfies equation (1) and boundary condition (2).

Employing regular partition Δ_N (see [4, 5]) of the interval [0,T] the Δ_N general solution $x(\Delta_N,t,\lambda)$ to the linear nonhomogenous Fredholm IDE was introduced in cite1. In [7] the new concept of a general solution to the Fredholm IDE (1) was extended. By substituting the corresponding expressions of $x(\Delta_N,t,\lambda)$ into the boundary condition and continuity conditions of a solution to equation (1) at the interior points of Δ_N we construct a system of nonlinear algebraic equations in parameters. It is proved that the solvability of the BVP is equivalent to the solvability of this system.

In present communication, an algorithm for finding a numerical solution to BVP (1), (2) is proposed. To this end, we use the Dzhumabaev parameterization method [3] and results of [6, 4, 5, 1, 2, 7]. At applying the parameterization method to BVP, the special Cauchy problem for a system of nonlinear Fredholm IDEs with parameters and a system of nonlinear algebraic equations in parameters are the intermediate problems. In this case, iterative methods are used both for solving the special Cauchy problem and for solving the systems of nonlinear algebraic equations. The algorithm for solving the special Cauchy problem includes two auxiliary problems: the Cauchy problems for ordinary differential equations and the evaluation of definite integrals. The accuracy of the method that we propose to solve the BVP (1), (2) depends on the accuracy of methods applied to the auxiliary problems and does not depend on a number of the partition subintervals. To solve the Cauchy problems we use the fourth order Runge-Kutta method and to evaluate definite integrals we use Simpson's formula. Therefor, the accuracy of the numerical solution is definite trough the accuracy of these problems.

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Approximate Calculation of Triple Integrals of Rapidly Oscillating Functions using Different Types of Information about Functions

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Modern methods of digital signal processing are characterized by new approaches to obtaining, processing and analyzing information. There is a need to build mathematical models in which information can be given not only by the values of the function at points, but also as a set of traces of the function on the planes and as a set of traces of the function on the lines. Report is dedicated to the improvement of mathematical models of digital signal processing and imaging by the example of constructing cubature formulas of approximate calculation of integrals of highly oscillatory functions of three variables. The feature of the proposed cubature formulas is using the input information about function or as a set of traces of function on planes or a set of traces of function on lines and of course as a set of values of the function in the points. The cubature formulas are correlated with a formula of the Filon type. The error estimations of the approximation of the integrals from the highly oscillating function by the cubature formulas on the class of differential functions are obtained. The main advantages of methods are high exactness of approximation, the opportunity to decrease the amount of information about function during the calculation.

Approximation of the two variables Discontinuous Functions by Discontinuous Interlination Splines using Triangular Elements

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The problem of approximation of smooth functions by continuous construction with sufficient completeness was considered in the work of many researchers. However, as a rule, all these problems are only pleasant exceptions or the result of excessive idealization. More often, the objects of study are mathematically described by functions with discontinuities, kinks, and other violations of smoothness. Such nonsmooth functions include profile shock waves generated by powerful acoustic emitters, or the shape of dunes in the desert, which provides characteristic sharpening. Discontinued objects also very often arise in tasks using remote methods. Thus, the detection of cracks in industrial products using non-destructive testing is an important task in flaw detection, as is the determination of deviations of the internal structure of the product from the standard. In many tasks of geophysics, the determination of the location of bound aries separating blocks with different physical properties is the first step in further studies aimed at determining the physical quantities characterizing the internal structure of the Earth. In computed tomography, when studying the internal structure of a body, it is useful to take into account its heterogeneity, that is, different densities in different parts of the body. In general, remote methods are one of the most promising areas of environmental studies. They serve as the most important source of objective and operational information in various phenomena occurring in the geographic shell of the Earth, and are an effective tool for monitoring the state of the environment and solving environmental management problems. It is obvious that further progress in the development of natural sciences is inextricably linked with the use of remote research methods. This work belongs to a series of works by the authors aimed at the study and improvement of mathematical models in computed tomography. To date, tomography has developed many computational methods, algorithms and software tools aimed at restoring the internal properties of an object. They perform well when restoring objects with smooth properties, but give unsatisfactory results for objects with discontinuous characteristics. Therefore, there is a need to create mathematical methods for approximating discontinuous functions for a more accurate idea of the structure of the studied object. The mathematical foundations of tomography were laid at the beginning of the last century in the works of the German scientist J. Radon, who developed the theory of the transformation of functions of many variables (Radon transformation). According to these transformations, the function of many variables can be characterized not only by its values at points of multidimensional space, but also by integrals from this function taken over an infinite set of lines or planes. A series of works by authors [18-20] devoted to solving the flat problem of radon computed tomography using the heterogeneity of the internal structure of a two-dimensional body. For this purpose, it is advisable to use function interlination operators, since these operators restore (possibly approximated) functions on their known traces on a given system of lines. They provide an opportunity to construct operators whose integrals from these lines (linear integrals) will be equal to integrals from the most renewable function. That is, interlination is a mathematical apparatus, naturally related to the task of restoring the characteristics of objects according to their known projections. This article is a continuation of this article series. Paper is devoted to the development of a method for approximating two variables discontinuous functions by discontinuous interlination splines using arbitrary triangular elements. Experimental data are one-sided traces of a function along a system of given lines, such data are used in remote methods, in particular in tomography. The paper is also devoted to the development a method for approximating of two variables discontinuous functions by triangular elements that comprise one curved side. These methods make it possible to approximate the discontinuous function, using its more complex domains of definition and avoiding the Gibbs phenomenon

Numerical computation of the complex zeros of Bessel and Hankel functions

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The complex zeros of cylinder functions appear in several problems of applied mathematics and theoretical physics. For example, the complex zeros of Hankel functions are involved in quantum scattering problems by spheres and cylinders. An algorithm (with a Matlab implementation) for computing the complex zeros of the Bessel function of first kind $J_\nu(z)$, second kind $Y_\nu(z)$, Hankel functions $H_\nu^{(1)}(z)$, $H_\nu^{(2)}(z)$ and general combinations of Bessel functions $\alpha J_\nu(z) + \beta Y_\nu(z)$ and Hankel functions $\alpha H_\nu^{(1)}(z) + \beta H_\nu^{(2)}(z)$ is described in this presentation. The algorithm, based on the results obtained in [1], [2], allows to obtain with certainty and accuracy all the zeros of the selected function inside a box in the complex plane. The performance of the algorithm is illustrated with numerical examples. This is a joint work in collaboration with Amparo Gil and Javier Segura.

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Neural network approximations for high-dimensional PDEs

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Most of the numerical approximation methods for PDEs in the scientific literature suffer from the so-called curse of dimensionality (CoD) in the sense that the number of computational operations employed in the corresponding approximation scheme to obtain an approximation precision $\varepsilon > 0$ grows exponentially in the PDE dimension and/or the reciprocal of ε. Recently, certain deep learning based approximation methods for PDEs have been proposed and various numerical simulations for such methods suggest that deep neural network (DNN) approximations might have the capacity to indeed overcome the CoD in the sense that the number of real parameters used to describe the approximating DNNs grows at most polynomially in both the PDE dimension $d \in \mathbb{N}$ and the reciprocal of the prescribed approximation accuracy $\varepsilon > 0$. In this talk we show that for every $a \in \mathbb{R}$, $b \in (a, \infty)$ solutions of suitable Kolmogorov PDEs can be approximated by DNNs on the entire space-time region $[0, T] \times [a, b]^d$ without the CoD.

Session

Optimization and Control

- Optimization of parameter dependent structured Sylvester and T-Sylvester equations, *Ivana Kuzmanović Ivičić*
- Hyperbolic quadratic eigenvalue problem and frequency isolation, *Suzana Miodragović*
- New shape derivative formula for solving a free boundary problem of Bernoulli's type, *Azeddine Sadik*
- Real-time planning for the cooperative discovery of unknown graph by the multi-agent dynamical system, *Mila Zovko*

Optimization of parameter dependent structured Sylvester and T-Sylvester equations

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Sylvester and T-Sylvester equations are matrix equations of the form $AX + XB = E$ and $AX + X^TB = E$, respectively, where A, B and E are given and X is unknown matrix. Sylvester equations appear frequently in many areas of applied mathematics. For example, Sylvester equations play vital roles in matrix eigen-decompositions, control theory, model reduction, numerical solution of matrix differential Riccati equations and algebraic Riccati equation, image processing, and many more. On the other hand, T-Sylvester matrix equations have recently attracted attention of researchers because of their relationship with palindromic eigenvalue problems.

This talk will be focused on structured Sylvester and T- Sylvester equations, especially on structured problems with system matrices of the form $A = A_0 + U_1V_1$ and $B = B_0 + U_2V_2$ where U_1, U_2, V_1 and V_2 are small rank update matrices. Sherman-Morrison-Woodbury-type formula for the solutions of this type of equations will be given. The obtained formula is used for the construction of an algorithm that solves the equations of the above form much more efficiently than the standard algorithms.

Application of obtained algorithms will be illustrated on the damping optimization problem.

Hyperbolic quadratic eigenvalue problem and frequency isolation

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The solution of the forced system undergo large oscillations whenever some eigenvalue of the corresponding quadratic eigenvalue problem

$$\left(\lambda^2 M + \lambda C + K\right) x = 0, \quad 0 \neq x \in \mathbb{C}^n,$$

is close to the frequency of the external force. One way to avoid resonance is to modify matrices M, C and K in such a way that the new system has no eigenvalues close to these frequencies. This frequency isolation problem is considered for the hyperbolic QEP.

New shape derivative formula for solving a free boundary problem of Bernoulli's type

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In this paper, we deal with a new numerical method for the approximation of a class of free boundary problem reformulated as a shape optimizationone, which consist in minimizing an appropriate cost functional. We startby showing the existence of the shape derivative of the cost functional andexpress it by means of support functions, using the formulas proposed in [Boulkhemair, A. and Chakib, A., 2014. On a shape derivative formulawith respect to convex domains. Journal of Convex Analysis, 21(1), pp.67-87.], for a family of convex domains. Then the numerical discretization is performed using the boundary element method in order to avert the remeshingtask required when one use the finite element method. Finally, we give somenumerical results, based on the gradient method, showing the efficiency ofthe proposed approach.

Real-time planning for the cooperative discovery of unknown graph by the multi-agent dynamical system

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We propose a solution to the problem of discovering an unknown graph by the multi-agent dynamical system.

The basic idea for the proposed algorithm comes from the HEDAC (Heat Equation Driven Area Coverage) method introduced by Ivić, Crnković and Mezić in [1]. This method has already been successfully applied for motion control for multi-agent non-uniform spraying [2] and for motion control for autonomous heterogeneous multi-agent area search in uncertain conditions [3].

The proposed algorithm uses a potential field to discover an unknown graph with a built-in cooperative behavior of agents which includes collision avoidance, coverage coordination, and optimal path planning. The algorithm is robust, adaptive, scalable and computationally inexpensive which enables real-time planning.

We will present the application of the proposed algorithm for discovering different types of graphs.

As the problem of discovering an unknown graph is related to the examination of social networks, computer networks and maze exploration, the proposed algorithm will be applied in solving problems in this area.

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Session

Partial Differential Equations and Applications

- Some problems for a mixed type equation fractional order with non-linear loaded term, *Obidjon Abdullaev*
- Two-Dimensional Time-Fractional Telegraph Equation of Distributed order in Polar Coordinate, *Alireza Ansari*
- Relativistic Hydrodynamics: Geometric Analysis Meets Observational Astrophysics, *Shabnam Beheshti*
- Convergence estimates for abstract second order differential equations with two small parameters and lipschitzian nonlinearities, *Galina Rusu*
- Geometric Hardy inequalities on starshaped sets, *Bolys Sabitbek*
- Multiple entire solutions to the curl-curl problem with critical exponent, *Jacopo Schino*
- Asymptotic study of a thin layer of viscous fluid between two surfaces, *Raquel Taboada Vázquez*
- Method of energy estimates for studying of singular boundary regimes in quasilinear parabolic equations, *Yevgeniia Yevgenieva*
- Operator-norm asymptotics for thin elastic rods with rapidly oscillating periodic properties, *Josip Žubrinić*

Some problems for a mixed type equation fractional order with non-linear loaded term

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Note, that with intensive research on problems of optimal control of the agro-economical system, regulating the label of ground waters and soil moisture, it has become necessary to investigate BVPs for a loaded partial differential equations. Integral boundary conditions have various applications in thermo-elasticity, chemical engineering, population dynamics, etc. In this work we consider parabolic-hyperbolic type equation fractional order involving non-linear loaded term:

$$\begin{gathered} 0 = u_{xx} - {}_cD_{oy}^{\alpha}u + f_1(x, y; u(x, 0)), \quad x > 0,\, y > 0 \\ 0 = u_{xx} - u_{yy} + f_2(x, y; u(x + y, 0)), \quad x > 0,\, y < 0, \\ 0 = u_{xx} - u_{yy} + f_3(x, y; u_y(0, x + y)), \quad x < 0,\, y > 0 \end{gathered} \tag{1}$$

where $f_i(x, y, u)$, $(i = 1, 2, 3)$ are given functions, ${}_CD_{oy}^{\alpha}u$ Caputo operator fractional order α $(0 < \alpha < 1)$ (see [1].p.92):

$$\left({}_CD_{ax}^{\alpha}f\right)x = \frac{1}{\Gamma(1-\alpha)}\int_a^x \frac{f'(t)}{(x-t)^{\alpha}}dt, \quad x > a.$$

Let $\Omega \subset R^2$, be domain bounded with segments B_2A_2, A_2A_1 on the lines $x = l$, $y = h$ at $x > 0,\ y > 0$; and A_1C_2, C_2B_1 on the characteristics $x - y = l$, $x + y = 0$ of the Eq. (1) at $x > 0,\ y < 0$, also with the segments B_1C_1, C_1B_2 on the characteristics $y - x = h$, $x + y = 0$ of the Eq. (1) at $x < 0,\ y > 0$.

We denote asΩ_0 parabolic part of the mixed domain Ω, and hyperbolic parts through Ω_1 at $x > 0$ and Ω_2 at $x < 0$.

In the domain Ω ($\Omega = \Omega^+ \cup \Omega^- \cup (A_1B_1),$) we will investigate following

Problem I. To find a solution $u(x, y)$ of Eq. (1) from the class of functions: $u(x, y) \in C(\bar{\Omega}) \cap C^1\left(\{\overline{\Omega_2} \setminus \overline{A_1C_2}\} \cup \{\overline{\Omega_1} \setminus \overline{C_1B_2}\}\right) \cap C^2(\Omega_2 \cup \Omega_1)$; $u_{xx}, {}_CD_{oy}^{\alpha}u \in C\left(\Omega_0\right)$; satisfying boundary conditions:

$$u(l, t) = \varphi_1(y),\ \ 0 \le y \le h; \tag{2}$$

$$\frac{d}{dx}u\left(\theta_1(x)\right) = a_1(x)u_y(x, 0) + a_2(x)u_x(x, 0) + a_3(x)u(x, 0) + a_4(x),\ \ 0 \le x < l; \tag{3}$$

$$\frac{d}{dy}u\left(\theta_2(y)\right) = b_1(y)u_x(0, y) + b_2(y)u_y(0, y) + b_3(y)u(0, y) + b_4(y),\ 0 \le y < h; \tag{4}$$

and integral gluing condition:

$$\lim_{y\to+0} y^{1-\alpha}u_y(x, y) = \lambda_1(x)u_y(x, -0) + \lambda_2(x)u_x(x, -0) +$$

$$+ \lambda_3(x)\int_0^x r_1(t)u(t, 0)dt + \lambda_3(x)u(x, 0) + \lambda_4(x),\ 0 < x < l \tag{3}$$

$$u_x(-0,y)=\mu_1(y)u_x(+0,y)+\mu_2(y)u_y(0,y)+\mu_3(y)\int\limits_0^y r_2(t)u(0,t)dt+$$
$$+\mu_4(y)u(0,y)+\mu_5(y),\ 0<y<h \tag{4}$$

where $\theta_1(x)=\theta_1\left(\frac{x}{2};-\frac{x}{2}\right)$, $\theta_2(y)=\theta_2\left(-\frac{y}{2};\frac{y}{2}\right)$, $a_i(x),\ b_i(y),\ (i=1,2,3),\ \lambda_j(x),\ \ \mu_j(y)\ (j=1,2,3,4,5),\ \ \varphi_1(y)$ are given functions, besides $\sum_{k=1}^4\lambda_k^2(x)\neq 0$, $\sum_{k=1}^4\mu_k^2(x)\neq 0$, $\sum_{k=1}^3 a_k^2(x)\neq 0$ and $\sum_{k=1}^3 b_k^2(x)\neq 0$.

On the certain conditions to given function, we can prove uniqueness of solution of the **Problem I** applying the method of integral energy. Existence of solution, reduced to the Volterra and Fredholm type non linear integral equation respected to $u_y(0,y)=\tau_2'(y)$ and $u(x,0)=\tau_1(x)$ accordingly.

Keywords: Loaded equation, Caputo operator, non-local condition, integral gluing condition, non-linear integral equations.

Two-Dimensional Time-Fractional Telegraph Equation of Distributed order in Polar Coordinate

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In this paper, we propose an analytical approach for solving the two-dimensional time-fractional telegraph equation of distributed order in the sense of the Caputo fractional derivative

$$\frac{1}{\nu^2}\int_1^2 c(\alpha)^C D_t^\alpha u(r,t)\alpha+b\int_0^1 b(\alpha)^C D_t^\alpha u(r,t)d\alpha=a^2\Big(u_{rr}(r,t)+\frac{1}{r}u_r(r,t)\Big),\ t>0,r>0,$$

with the initial conditions $u(r,0)=f(r),u_r(r,0)=0$. The Hankel and the Laplace transforms are used to get an integral representation for the solution, particulary in terms of the Prabhakar function (the Mittag-Leffler function with three parameters) for the single order. Other special cases of the weight functions $c(\alpha),b(\alpha)$ are also investigated.

Relativistic Hydrodynamics: Geometric Analysis Meets Observational Astrophysics

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Relativistic hydrodynamics describes the motion of fluids in regimes including flow velocities close to the speed of light (e.g., relativistic plasmas) and fluids interacting with strong gravitational fields (e.g. neutron star mergers, black hole accretion disks). Mathematical research in this area serves as an essential tool in high-energy nuclear physics, cosmology, and astrophysics, offering opportunities for strong interplay between mathematical analysis, numerical simulation, theoretical and experimental physics. In this talk, I shall survey recent progress on well-posedness theorems for relativistic viscous hydrodynamics and discuss related open problems in both mathematics and astrophysics.

Convergence estimates for abstract second order differential equations with two small parameters and lipschitzian nonlinearities

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In a real Hilbert space H endowed with the scalar product $(\cdot,\cdot)$ and the norm $|\cdot|$, consider the following Cauchy problem:

$$\begin{cases} \varepsilon u''_{\varepsilon\delta}(t) + \delta\, u'_{\varepsilon\delta}(t) + Au_{\varepsilon\delta}(t) + B\big(u_{\varepsilon\delta}(t)\big) = f(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases} \qquad (P_{\varepsilon\delta})$$

where $A : V \subset H \to H$, is a linear self-adjoint operator, V is a real Hilbert space endowed with the norm $||\cdot||$, B is nonlinear $A^{1/2}$ lipschitzian opeartor, $u_0, u_1, f : [0,T] \to H$ and ε, δ are two small parameters.

We investigate the behavior of solutions $u_{\varepsilon\delta}$ to the problem ($P_{\varepsilon\delta}$) in two different cases: (i) $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$, relative to the solutions to the following unperturbed system:

$$\begin{cases} \delta l'_\delta(t) + Al_\delta(t) + B\big(l_\delta(t)\big) = f(t), \quad t \in (0,T), \\ l_\delta(0) = u_0; \end{cases} \qquad (P_\delta)$$

(ii) $\varepsilon \to 0$ and $\delta \to 0$, relative to the solutions to the following unperturbed system:

$$Av(t) + B\big(v(t)\big) = f(t), \quad t \in [0,T), \qquad (P_0)$$

Geometric Hardy inequalities on starshaped sets

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In this talk, we present the geometric Hardy inequalities on the starshaped sets in the Carnot groups. Also, we obtain the geometric Hardy inequalities on half-spaces for general vector fields.

Multiple entire solutions to the curl-curl problem with critical exponent

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We prove the existence of infinitely many solutions with diverging energy to the problem

$$\nabla \times \nabla \times \mathbf{U} = |\mathbf{U}|^4 \mathbf{U} \quad \text{in } \mathbb{R}^3.$$

We consider vector fileds of the form

$$\mathbf{U}(x) = \frac{u(x)}{r} \begin{pmatrix} -x_2 \\ x_1 \\ 0 \end{pmatrix}$$

with $r = \sqrt{x_1^2 + x_2^2}$ and $u(x) = u(r, x_3)$ to reduce the curl-curl operator to the vector Laplacian; at the same time we consider an isometric isomorphism between $\mathcal{D}^{1,2}(\mathbb{R}^3, \mathbb{R}^3)$ and $H^1(\mathbb{S}^3, \mathbb{R}^3)$ to recover compactness.

Asymptotic study of a thin layer of viscous fluid between two surfaces

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In this work, we are interested in studying the behavior of an incompressible viscous fluid moving between two closely spaced surfaces, also in motion.

We consider a three-dimensional thin domain, Ω_t^ε, filled by a fluid, that varies with time $t \in [0, T]$, given by

$$\begin{aligned}\Omega_t^\varepsilon \;=\; & \big\{(x_1^\varepsilon, x_2^\varepsilon, x_3^\varepsilon) \in R^3 : \\ & x_i(\xi_1, \xi_2, t) \leq x_i^\varepsilon \leq x_i(\xi_1, \xi_2, t) + h^\varepsilon(\xi_1, \xi_2, t) N_i(\xi_1, \xi_2, t), \\ & (i = 1, 2, 3),\ (\xi_1, \xi_2) \in D \subset \mathbb{R}^2\big\}\end{aligned} \tag{1}$$

where $\vec{X}(\xi_1, \xi_2, t)$ is the lower bound surface parametrization, $\vec{N}(\xi_1, \xi_2, t)$ is the unit normal vector and $h^\varepsilon(\xi_1, \xi_2, t)$ is the gap between the two surfaces in motion assumed to be small with regard to the dimension of the bound surfaces. We take into account that the fluid film between the surfaces is thin by introducing a small non-dimensional parameter ε, and setting that

$$h^\varepsilon(\xi_1, \xi_2, t) = \varepsilon h(\xi_1, \xi_2, t) \tag{2}$$

We assume that the fluid motion is governed by Navier-Stokes equations and using the asymptotic development technique, the following lubrication model in a thin domain with curved mean surface has been obtained:

$$\begin{aligned}\frac{1}{\sqrt{A^0}} \mathrm{div}\left(\frac{(h^\varepsilon)^3}{\sqrt{A^0}} M \nabla p^{-2,\varepsilon}\right) &= 12\mu \frac{\partial h^\varepsilon}{\partial t} + 12\mu \frac{h^\varepsilon A^1}{A^0}\left(\frac{\partial \vec{X}}{\partial t} \cdot \vec{N}\right) \\ &- 6\mu \nabla h^\varepsilon \cdot (\vec{W}^0 - \vec{V}^0) + \frac{6\mu h^\varepsilon}{\sqrt{A^0}} \mathrm{div}(\sqrt{A^0}(\vec{W}^0 + \vec{V}^0))\end{aligned} \tag{3}$$

It is a new generalized Reynolds equation where the pressure, p^ε, is approximated by $p^{-2,\varepsilon} = \varepsilon^{-2} p^{-2}$. The fluid velocities inside the domain are subsequently approximated from the pressure using the equations

$$u_1^0 = \frac{h^2(\xi_3^2 - \xi_3)}{2\mu A^0}\left(G \frac{\partial p^{-2}}{\partial \xi_1} - F \frac{\partial p^{-2}}{\partial \xi_2}\right) + \xi_3 (W_1^0 - V_1^0) + V_1^0 \tag{4}$$

$$u_2^0 = \frac{h^2(\xi_3^2 - \xi_3)}{2\mu A^0}\left(E \frac{\partial p^{-2}}{\partial \xi_2} - F \frac{\partial p^{-2}}{\partial \xi_1}\right) + \xi_3 (W_2^0 - V_2^0) + V_2^0 \tag{5}$$

$$u_3^0 = \frac{\partial \vec{X}}{\partial t} \cdot \vec{N} \tag{6}$$

where the velocity on the lower surface, $\vec{V}^0$, and on the upper surface, $\vec{W}^0$, are known. We denote by

$$A^0 = EG - F^2 \tag{7}$$
$$A^1 = -eG - gE + 2fF \tag{8}$$
$$M = \begin{pmatrix} G & -F \\ -F & E \end{pmatrix} \tag{9}$$

where E, F, G and e, f, g are the coefficients of the first and the second (respectively) fundamental forms of the surface parametrized by $\vec{X}$.

We have observed that, depending on the boundary conditions, other models can be obtained. We derive a shallow water model changing the boundary conditions that we had imposed: instead of assuming that we know the velocities on the upper and lower boundaries of the domain, we assume that we know the tractions on these upper and lower boundaries. We yield:

$$\begin{aligned}
&\frac{\partial V_i^0}{\partial t} + \sum_{l=1}^{2} \left(V_l^0 - C_l^0\right) \frac{\partial V_i^0}{\partial \xi_l} + \sum_{k=1}^{2} \left(R_{ik}^0 + \sum_{l=1}^{2} H_{ilk}^0 V_l^0 \right) V_k^0 \\
&= -\frac{1}{\rho_0} \left(\alpha_i^0 \frac{\partial \pi_0^0}{\partial \xi_1} + \beta_i^0 \frac{\partial \pi_0^0}{\partial \xi_2} \right) \\
&+ \nu \left\{ \sum_{m=1}^{2} \sum_{l=1}^{2} \frac{\partial^2 V_i^0}{\partial \xi_m \partial \xi_l} J_{lm}^0 + \sum_{k=1}^{2} \sum_{l=1}^{2} \frac{\partial V_k^0}{\partial \xi_l} (L_{kli}^0 + \psi(h)_{ikl}^0) \right. \\
&\left. + \sum_{k=1}^{2} V_k^0 (S_{ik}^0 + \chi(h)_{ik}^0) + \hat{\kappa}(h)_i^0 \right\} + F_i^0(h) - Q_{i3}^0 \left(\frac{\partial \vec{X}}{\partial t} \cdot \vec{N} \right) \\
&\qquad (i = 1, 2)
\end{aligned} \tag{10}$$

$$\frac{\partial h}{\partial t} + \frac{h}{\sqrt{A^0}} \mathrm{div} \left(\sqrt{A^0} \vec{V}^0 \right) + \frac{h A^1}{A^0} \left(\frac{\partial \vec{X}}{\partial t} \cdot \vec{N} \right) = 0 \tag{11}$$

where α_i^0, β_i^0, C_l^0, H_{ilk}^0, J_{lm}^0, L_{kli}^0, Q_{i3}^0, R_{ik}^0, S_{ik}^0 depend only on the parametrization $\vec{X}$ and $F_i^0(h)$, $\psi(h)_{ikl}^0$, $\chi(h)_{ik}^0$, $\kappa(h)_i^0$ depend on the parametrization $\vec{X}$ and on the gap h. The exact definition of these coefficients can be found in [5], where the complete derivation of both models is presented.

Once V_1^0, V_2^0 and π_0^0 (the approximation of the pressure on the lower bound) are calculated we have the following approximation of the velocities and the pressure

$$u_i^0 = W_i^0 = V_i^0 \quad i = 1, 2 \tag{12}$$
$$u_3^0 = \frac{\partial \vec{X}}{\partial t} \cdot \vec{N} \tag{13}$$
$$p^0 = \frac{2\mu}{h} \frac{\partial h}{\partial t} + \pi_0^0 \tag{14}$$

These models can not be found in the literature, as far as we know. We reach the conclusion that the magnitude of the pressure differences at the lateral boundary of the domain is key when deciding which of the two models best describes the fluid behavior.

Boundary conditions tell us which of the two models should be used when simulating the flow of a thin fluid layer between two surfaces: if the fluid pressure is dominant (that is, it is of order $O(\varepsilon^{-2})$), and the fluid velocity is known on the upper and lower surfaces, we must use the lubrication model; if the fluid pressure is not dominant (that is, it is of order $O(1)$), and the tractions are known on the upper and lower surfaces, we must use the shallow water model. In the first case we will say that the fluid is "driven by the pressure" and in the second that it is "driven by the velocity".

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Method of energy estimates for studying of singular boundary regimes in quasilinear parabolic equations

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In the cylindrical domain $Q = (0,T) \times \Omega,\ 0 < T < \infty$, where $\Omega \subset R^n$ is a bounded domain such that $\partial\Omega \in C^2$, the following problem is considered:

$$\begin{aligned} &(|u|^{q-1}u)_t - \Delta_p u = 0, \quad p \geqslant q > 0, \\ &u(0,x) = u_0 \ \text{ in } \Omega,\ \ u_0 \in L^{q+1}(\Omega), \\ &u(t,x)\Big|_{\partial\Omega} = f(t,x), \end{aligned} \tag{1}$$

where f generates boundary regime with singular peaking, namely,

$$f(t,x) \to \infty \quad \text{as } t \to T, \quad \forall\, x \in\ K \subset \partial\Omega, K \neq \emptyset. \tag{2}$$

Function f is called a localized boundary regime (S-regime) if

$$\overline{\Omega} \setminus \Omega_0 \neq \emptyset, \quad \text{where } \Omega_0 := \left\{ x \in \overline{\Omega} : \sup_{t \to T} u(t,x) = \infty \right\}$$

for an arbitrary weak solution u of problem (1). Sharp conditions of localization of boundary regime were obtained by some version of local energy estimates (see [1] and references therein).

Papers [2, 3] are devoted to investigation of the behavior of weak solutions in the case when the blow-up set $\Omega_0 \subset \partial\Omega$ (LS-regime). The precise estimates of the limiting profile of solutions were obtained, namely,

$$\sup_{t\to T} u(t,x) \leqslant \psi(x), \quad x \in \Omega,$$

where function ψ is determined by characteristics of peaking of the boundary regime f.

As an application of these results we study the following parabolic quasilinear equation with a nonlinear absorption term:

$$(|u|^{q-1}u)_t - \Delta_p u = -b(t,x)|u|^{\lambda-1}u, \quad (t,x) \in Q, \quad \lambda > p \geqslant q > 0, \tag{3}$$

Here $b(t,x) \geqslant 0$ is a degenerate absorption potential: $b(t,x) \to 0$ as $t \to T\ \forall\, x \in \Omega$. Precise upper estimates for all weak solutions of equation (3) near to $t = T$ (limiting profile of solution), depending on the behavior of function b, were obtained in the papers [2, 4]. It is important to underline that the obtained estimates don't depend on initial and boundary values and hold for large solutions of the equation (3) (if they exist).

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Operator-norm asymptotics for thin elastic rods with rapidly oscillating periodic properties

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We provide norm-resolvent estimates in $L^2 \to L^2$ and $L^2 \to H^1$ operator norms, for the class of problems in linear elasticity describing heterogeneous rods with rapidly oscillating coefficients in the regime of moderate contrast.

The estimates are provided with respect to the period of material oscillations in the setting of simultaneous homogenization and dimension reduction, while assuming that the period and the rod thickness are of the same order. These estimates are expected to provide also sharp estimates for the corresponding evolution problems.

The analysis is performed by the means of spectral analysis and Gelfand transform.

This is joint work with K. Cherednichenko and I. Velčić.

Session

PROBABILITY

- Hitting times for the Brownian motion and Bessel processes: some new algorithms, *Madalina Deaconu*
- Generation of first passage times for diffusion processes: an overview of simulation techniques, *Samuel Herrmann*
- The first exit problem of reaction-diffusion equations for small multiplicative Lévy noise, *Michael A. Hoegele*
- Joint functional convergence of partial sum and maxima processes, *Danijel Krizmanić*
- Markov chains in stationary and ergodic random environment, *Attila Lovas*
- Simulation of the time needed by a diffusion process in order to exit from a given interval (WOMS algorithm), *Nicolas Massin*
- Solving General Itô-Process Hitting-Time Problems with General Moving Boundaries, *Martin Nilsson*
- First passage times of subdiffusive processes over stochastic boundaries, *Pierre Patie*
- Level densities for general β-ensembles: An operator-valued free probability perspective, *Andrej Srakar*
- A CLT for degenerate diffusions with periodic coefficients, and application to homogenization of linear PDEs, *Ivana Valentić*
- Counterexamples for optimal scaling of Metropolis-Hastings chains with rough target densities, *Jure Vogrinc*

Hitting times for the Brownian motion and Bessel processes: some new algorithms

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The aim of this talk is to introduce new numerical methods for the hitting time of some boundaries for a class of stochastic processes. We will start by constructing the walk on moving spheres method for Bessel processes. We will apply this result for the simulation of the Brownian motion hitting times. This algorithm exhibits a new procedure which efficiently approximate both the hitting time and the hitting position of the stochastic process and can be used in problems arising from finance, geophysics or neuroscience.

We give also the convergence results and present some numerical examples that permit to emphasise the efficiency and accuracy of this new method.

This is a joint work with Samuel Herrmann (Dijon University, France).

Generation of first passage times for diffusion processes: an overview of simulation techniques

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Many biological or physical applications require to simulate random variables with a given probability distribution. The aim of our study is to focus on a particular random variable: the first passage time (FPT) of a diffusion process. We introduce (X_t) the unique solution of the following SDE:

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dB_t, \quad X_0 = x,$$

where (B_t) stands for a one-dimensional Brownian motion and define τ_L the first passage time through the level L. We propose an overview of several simulation techniques.

- The classical way is to use efficient algorithms for the simulation of sample paths, like discretization schemes. Such methods permit to obtain approximations of the first-passage times as a by-product.
- Another approach based on a random walk on spheroids permit in particular cases to express the first passage time as the limit of a random walk. It suffices therefore to describe precisely the convergence of this stochastic process and to introduce a stopping procedure.
- Finally we present a new rejection sampling algorithm which permits to perform an exact simulation of the first-passage time for general one-dimensional diffusion processes. The main ideas are based both on a previous algorithm pointed out by A. Beskos et G. O. Roberts which uses Girsanov's transformation and on properties of Bessel paths.

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The first exit problem of reaction-diffusion equations for small multiplicative Lévy noise

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In this talk we present the first exit problem from the vicinity of a stable state in a generic class of dissipative, scalar reaction diffusion equations perturbed by multiplicative, regularly varying Lévy noise in the limit of small noise intensity. In finite dimensions the asymptotic behavior of the first exit time and location have been determined in recent years by Pavylykevich, Imkeller, and H. and in infinite dimensions the additive case had been studied for the Chafee-Infante equation by Debussche, H. and Imkeller. We show how to obtain these results for perturbations by multiplicative regularly varying noise. For this aim we use some exponential estimates in infinite dimensions of the stochastic convolution with Lévy processes with bounded jump size. The main example covered by our results are the linear heat equation and multi-well gradient equations, such as the Chafee-Infante equation, perturbed by additive and multiplicative α-stable noise.

Joint functional convergence of partial sum and maxima processes

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For a strictly stationary sequence of random variables we study functional convergence of the joint partial sum and partial maxima process under joint regular variation with index $\alpha \in (0, 2)$ and weak dependence conditions. The convergence takes place in the space of $\mathbb{R}^2$–valued càdlàg functions on $[0, 1]$ with the Skorohod weak M_1 topology, and the limiting process consists of an α–stable Lévy process and an extremal process. We also show that the weak M_1 topology in general can not be replaced by the standard M_1 topology.

Markov chains in stationary and ergodic random environment

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Markov chains in stationary random environments (MCREs) with a general (not necessarily countable) state-space appear in several branches of applied probability including mathematical finance, queuing models with non-i.i.d. service times and statistical learning theory. Assuming suitable versions of the standard drift and minorization conditions, we prove the existence of limiting distributions for MCREs in cases when the system dynamics is contractive on the average with respect to the Lyapunov function and large enough small sets exist with large enough minorization constants. We also establish that a law of large numbers holds for bounded functionals of the process. Applications to queuing systems and to machine learning algorithms are also presented.

Simulation of the time needed by a diffusion process in order to exit from a given interval (WOMS algorithm)

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For many applications, it is important to describe or simulate the exit time from an interval for a stochastic process. In the particular context of diffusion processes which are solutions of stochastic differential equations, time discretization schemes like Euler scheme are usually used. They permit the simulation of a paths skeleton and lead to an approximation of the exit time as a by-product. The aim of the talk is to present a completely different approach based on the study of the Brownian paths. It is possible to find some domains, called for instance spheroids, such that both the distribution of Brownian exit time from this spheroid and its exit position are well-known.

From that point, we define an iterative procedure : a walk on spheres for the Brownian motion and so called WOMS algorithm. We can show that it is hard to find an algorithm as precise and efficient than this algorithm.

Starting from this particular case, we generalize the iterative procedure in order to deal with a family of diffusion written as functions of a changed time Brownian motion. This strong relation permits to define a new WOMS algorithm by deduction.Theoretical results and numerical illustrations point out the efficiency of such an algorithm. The particular Ornstein-Uhlenbeck case will be presented in details.

Solving General Itô-Process Hitting-Time Problems with General Moving Boundaries

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A spectral method for solving both first-passage time and first-exit time problems for general Itô processes is presented. The method handles general moving (*i.e.*, time-variable) boundaries including discontinuities. Results from the application to neuron modelling are given.

The method is built upon the idea of first expressing the problem as a Fokker–Planck equation over a non-rectangular strip. The Fokker–Planck equation is then spectrally reduced to a small set of ordinary differential equations which can be solved easily and quickly by standard solvers.

The reduction is the key step of the method. It observes that the right hand side of the Fokker–Planck equation

$$\frac{\partial}{\partial t} p = L\, p$$

can be used as the left-hand side of the Sturm-Liouville system

$$L\, p = -\lambda\, p,$$

which generates a moving orthogonal family of eigenfunctions. Fourier-transforming the Fokker–Planck equation in terms of these eigenfunctions produces a series of simple ordinary differential equations for the Fourier coefficients of the solution. For example, in the time-homogeneous first-passage time case, these equations can be expressed as

$$\frac{d\omega}{dt} = \left(C \frac{da}{dt} - \Lambda \right) \omega,$$

where ω is the vector of Fourier coefficients, a is the moving boundary, Λ is the diagonal matrix of the moving Sturm–Liouville eigenvalues λ_k, and C is the skew-symmetric matrix defined by

$$[C]_{km} = \frac{1}{\lambda_k - \lambda_m} \sqrt{\frac{\partial \lambda_k}{\partial a} \frac{\partial \lambda_m}{\partial a}}$$

for $k \neq m$.

First passage times of subdiffusive processes over stochastic boundaries

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Let X be the subdiffusive process defined by

$$X_t = Y_{D_t} \quad t \geq 0,$$

where Y is a Lévy process and $D_t = \inf\{s > 0;\ \sigma_s > t\}$ with σ a subordinator independent of Y.

We start by providing a composite Wiener-Hopf factorization to characterize the law of the pair $(T_a^{(\kappa)}, (X - \sigma)_{T_a^{(\kappa)}})$ where

$$T_a^{(\kappa)} = \inf\{t > 0;\ X_t > a + \kappa_t\}$$

with $a \in \mathbb{R}$ and κ a (possibly degenerate) subordinator independent of Y and σ. We proceed by providing a detailed analysis of the cases where either σ is a stable subordinator or X is spectrally negative. Our proofs hinge on a variety of techniques including excursion theory, change of measure, asymptotic analysis and establishing a link between subdiffusive processes and a subclass of semi-regenerative processes.

Level densities for general β-ensembles: An operator-valued free probability perspective

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Random point processes corresponding to β-ensembles for arbitrary $\beta > 0$, or, equivalently, log gases at inverse temperature β, are being subject to intense study. The orthogonal, unitary, and symplectic ensembles (β = 1, 2, or 4, respectively) are now well understood, but other values of β are believed to also be relevant in theory (e.g. relevant for the study of self-adjoint and Schrödinger operators) and applications (e.g. in logistics). For certain rational values of β, β-ensembles are related to Jack polynomials, but for general β much less is known. In a seminal article, Dumitriu and Edelman (2002) constructed tridiagonal random matrix models for general β-Hermite and β-Laguerre ensembles and defined open problems for research on general β-ensembles, including finding a unified formula for the level density in general β-case. In general, level density is defined as distribution of a random eigenvalue of an ensemble (by the Wigner semicircular law, the limiting distribution of the eigenvalue is semicircular). We derive the formula for the level density in the general β-case depending on the multivariate Fuss-Narayana polynomials (related to generalized Fuss-Narayana numbers) and homogenous polynomials from operator-valued free probability theory. In an alternate way, we derive the general β-case level density formulas using Malliavin-Stein fourth moment-based asymptotic calculus and study their perturbation invariability (Wang and Yan, 2005; Kozhan, 2017). Derivations allow us large possibilities of additional work, and we present extensions to problems of sampling general β-ensembles (referring to Li and Menon, 2012; Olver et al., 2013; Srakar and Verbic, 2020) and limiting entropy in β-ensembles related point processes (Mészaros, 2020).

A CLT for degenerate diffusions with periodic coefficients, and application to homogenization of linear PDEs

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Let $\mathcal{L}^\varepsilon$, $\varepsilon > 0$, be a second-order elliptic differential operator of the form $\mathcal{L}^\varepsilon = \big(a(\cdot/\varepsilon) + \varepsilon^{-1}b(\cdot/\varepsilon)\big)^{\mathrm{T}}\nabla + 2^{-1}\mathrm{Tr}\big(c(\cdot/\varepsilon)\,\nabla\nabla^{\mathrm{T}}\big)$ with a degenerate (possibly vanishing on a set of positive Lebesgue measure) diffusion coefficient $c(x)$. We first show that the diffusion process associated to $\mathcal{L}^\varepsilon$ satisfies a functional CLT with Brownian limit as $\varepsilon \to 0$, and then by employing probabilistic representation (the Feynman-Kac formula) of the solutions to the elliptic boundary-value and the parabolic initial-value problem we conclude the homogenization result. In the non-degenerate (uniformly elliptic) case these steps can be carried out by combining classical PDE results and the fact that the underlying diffusion process does not show a singular behavior in its motion, that is, it is irreducible. In the case of a degenerate diffusion part, this deficiency is compensated by the assumption that the underlying diffusion process with positive probability reaches the part of the state space where the diffusion term is non-degenerate. Also, in this case it is not clear that we can rely on PDE techniques therefore the proofs are completely based on stochastic analysis tools.

Counterexamples for optimal scaling of Metropolis-Hastings chains with rough target densities

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For sufficiently smooth targets of product form it is known that the variance of a single coordinate of the proposal in RWM (Random walk Metropolis) and MALA (Metropolis adjusted Langevin algorithm) should optimally scale as n^{-1} and as $n^{-1/3}$ with dimension n, and that the acceptance rates should be tuned to 0.234 and 0.574. We establish counterexamples to demonstrate that smoothness assumptions such as $\mathcal{C}^1(\mathbb{R})$ for RWM and $\mathcal{C}^3(\mathbb{R})$ for MALA are indeed required if these guidelines are to hold. The counterexamples identify classes of marginal targets, obtained by perturbing a standard Normal density at the level of the potential (or second derivative of the potential for MALA) by a path of fractional Brownian motion with Hurst exponent H, for which these guidelines are violated. For such targets RWM and MALA proposal variances should optimally be scaled as $n^{-1/H}$ and as $n^{-1/(2+H)}$ and will then obey anomalous optimal acceptance rate guidelines. We will discuss useful heuristic implications of the results. The talk is based on the preprint: https://arxiv.org/abs/1910.09485.

Session

Statistics and Financial Mathematics

- On change estimation in stochastic intensity-driven continuous time point processes through multiple testing, *Moinak Bhaduri*
- Consistency of Bayesian inference with Gaussian priors in an elliptic nonlinear inverse problem, *Matteo Giordano*
- Ranking of Baltic States II pillar pension funds by stochastic dominance ratio, *Audrius Kabašinskas*
- New insight into partial differentiation with non-independent variables, *Matieyendou Lamboni*
- Initial data analysis for longitudinal data – a general framework, *Lara Lusa*
- Recent directions in testing exponentiality: the right-censored data case, *Bojana Milošević*
- Spatial-Temporal Modelling of Temperature for Pricing Temperature Index Insurance, *Che Mohd Imran Che Taib*
- Special classes of multivariate generalised autoregressive conditional heteroskedasticity models, *Anthony Usoro*

On change estimation in stochastic intensity-driven continuous time point processes through multiple testing

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Point processes stand as convenient instruments to model count data, and the relevance of observation-reliant underlying intensities remains undeniable even in the face of seemingly tempting alternatives. Hawkes processes offer a sterling example, often leading to a branching process framework. We posit a genre of change detection algorithms, engineered through permutations of trend-switched statistics and a judicious application of false discovery rate control. Quick, accurate change detection on both the immigrant and offspring kernels, coupled with the scarcity of false positives are a few optimal properties. Certain members of this family that remain asymptotically consistent and close to the ground truth (evidenced through some Hausdorff-similarity) are isolated to pinpoint estimated change locations. Efficient forecasting proves to be a natural corollary.

Keywords: Point process, change detection, self-exciting intensity, Hawkes process, oceanography, global terrorism, economic announcements

Consistency of Bayesian inference with Gaussian priors in an elliptic nonlinear inverse problem

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We consider nonparametric Bayesian inference for nonlinear inverse problems based on Gaussian process priors. We present posterior consistency results for the problem of recovering the unknown conductivity in an elliptic PDE in divergence form from noisy discrete observations of its solution, and give a convergence rate for the reconstruction error of the associated posterior mean estimator. The analysis is based on a contraction rates theory for the induced regression problem, combined with a stability estimate for the solution map of the PDE.

Ranking of Baltic States II pillar pension funds by stochastic dominance ratio

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In this presentation we will introduce results of ranking of Lithuanian, Latvian and Estonian II pillar pension funds by Stochastic Dominance (SD) ratio. Insights of how to select non-dominated pension fund will be provided, comparison of pension systems and performance of fund managers will be discussed too. First, second and third order SD are used in this research. Pairwise SD is numerically computed using non-parametric and parametric approaches. The later one covers α-stable, hyperbolic, NIG and Student *t* probability distributions, while empirical SD is assumed to be non-parametric one.

New insight into partial differentiation with non-independent variables

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Summary. We use to work with models defined through a function and some equations connecting the input variables such as a given function subject to some constraints involving input variables. For such models, it is interesting to better determine the partial derivatives with respect to each input variable that comply with the constrained equations. As the equations connecting the input variables introduce some dependency structures among input variables, and the theory of probability allows for better characterizing the dependencies among variables, In this abstract, we propose new partial derivatives for functions with non-independent variables by making use of the formal definition of independence or dependence such as the cumulative distribution function (CDF). The proposed new partial derivatives are based on the classical gradient and the CDF. Such derivatives are uniquely defined and do not require any additional assumption. Our approach can be extended for determining cross-partial derivatives as well.

Main results. In this section, we include the distribution of inputs to derive the partial derivatives. It is to be noted that each initial input X_j lies in a given domain $\Omega_j \subseteq \mathbb{R}$ with $j = 1, \ldots, d$, and we assume that we are able to attribute to X_j a distribution. It is common to attribute a normal distribution with a higher variance when we do have much information about the variable, which comes down to make use of uniform distribution for a bounded domain Ω_j.

Therefore, the input variables $\mathbf{X} = (X_1, \ldots, X_d)$ have a given distribution F, and we are interested in a function given by $f(\mathbf{X})$ and $h(\mathbf{X}) = 0$. In what follows, we assume that $F = \prod_{j=1}^{d} F_j$ with F_j the CDF of X_j, which means that the initial input variables are independent. The equation $h(\mathbf{X}) = 0$ introduces some dependencies, and this yields to new dependent variables $\mathbf{X}^c \sim F^c$. It is worth noting that the inputs $\mathbf{X}^c$ must satisfies $h(\mathbf{X}^c) = 0$ and we have

$$Y = \begin{cases} f(\mathbf{X}) & \\ \text{s.t.} \quad h(\mathbf{X}) = 0 \end{cases} \stackrel{d}{=} f(\mathbf{X}^c),$$

provided that F^c is known.

Formally, $\mathbf{X}^c \stackrel{d}{=} \{\mathbf{X} \sim F \,:\, h(\mathbf{X}) = 0\}$, and we are able to find the distribution of $\mathbf{X}^c$. Indeed, some analytic derivation of F^c can be found in [1]. For complex function h, a copula-based approach is suitable to fit a distribution to simulated data. Based on F^c or the estimated distribution $\widehat{F^c}$, the multivariate conditional quantile transform (see [2,3,4]) implies a regression representation of $\mathbf{X}^c$ (see [5,6]), which also implies a dependency function of $\mathbf{X}^c$ given by ([1,7])

$$\mathbf{X}^c_{\sim j} = r_j\left(X^c_j, \mathbf{U}\right),$$

where X_j is independent of $\mathbf{U}$; $f_j : \mathbb{R}^d \to \mathbb{R}^{d-1}$ and $\left(X^c_j, \mathbf{X}^c_{\sim j}\right) \stackrel{d}{=} \left(X^c_j, r_j\left(X^c_j, \mathbf{U}\right)\right) \sim F^c$.

Now we have all the elements in hand to provide the partial derivatives (see Theorem 1). To that end, we use

$$\nabla_j f := \left[f'_{x_j}, f'_{x_{w_1}}, \ldots, f'_{x_{w_{d-1}}}\right]^T; \quad J^c_j := \left[1, \frac{\partial r_{w_1,j}}{\partial x_j}, \ldots, \frac{\partial r_{w_{d-1},j}}{\partial x_j}\right]^T,$$

for the gradient and the partial derivatives of each component of r_j w.r.t. x_j, respectively. Moreover, we use $J^c_j(\ell)$ the ℓ^{th} component of J^c_j and

$$J^c_{w_k} := \left[\frac{1}{J^c_j(k+1)}, \frac{J^c_j(2)}{J^c_j(k+1)}, \ldots, \frac{J^c_j(d)}{J^c_j(k+1)}\right]^T; \ \forall\, k \in \{1, \ldots d-1\}.$$

Theorem 1. If the functions f and r_j are differentiable w.r.t. their inputs, then we have

$$\frac{\partial f}{\partial x_j}(\mathbf{x}) = \nabla_j f(\mathbf{x})^T J_j^c(x_j, \mathbf{u}) \quad \text{or} \quad \frac{\partial f}{\partial x_j}(\mathbf{x}) = \nabla_j f(\mathbf{x})^T J_j^c\left(x_j, r_j^{-1}(\mathbf{x}_{\sim j})\right) ; \qquad (1)$$

$$\frac{\partial f}{\partial x_{w_k}}(\mathbf{x}) = \nabla_j f(\mathbf{x})^T J_{w_k}^c\left(x_j, r_j^{-1}(\mathbf{x}_{\sim j})\right) \forall k \in \{1, \ldots d-1\} . \qquad (2)$$

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Initial data analysis for longitudinal data – a general framework

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Systematic initial data analysis (IDA) and clear reporting of the findings is an important step towards reproducible research. A general framework of IDA for observational studies was proposed to include data cleaning, data screening, and possible refinements of the preplanned analyses (Huebner 2018). Longitudinal studies, where participants are observed repeatedly over time, have special features that should be taken into account in the IDA steps before addressing the research question. Our aim was to propose a framework for IDA in longitudinal studies. Based on the IDA framework from Huebner et al. (2018) we refined it for use in longitudinal studies and provided guidance on how to prepare an IDA plan for longitudinal studies. The framework includes several steps that are specific to longitudinal data, or bear greater importance when data are longitudinal. Appropriate numerical and graphical tools for longitudinal data allow the researchers to conduct IDA in a reproducible manner to avoid non-transparent impact on the interpretation of model results. For example, in the framework we propose how to summarize the time frame of the study, the characteristics of the participants, the outcome and the time varying covariates, how to summarize longitudinal average trends, how to explore the variation between individuals, how to characterize the correlation and the covariance of the outcome and of selected covariates, methods for the exploration of missing values and the description of drop-out. We provide an example on how to conduct IDA in the context of a longitudinal population cohort study (Boesch-Supan et al., 2013). The paper is presented on behalf of the Topic Group "Initial Data Analysis" of the STRATOS Initiative (STRengthening Analytical Thinking for Observational Studies, `http://www.stratos-initiative.org`).

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Recent directions in testing exponentiality: the right-censored data case

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Recently the characterizations of distributions have become a very powerful tool for the construction of goodness of fit tests. However, most of such tests have been designed for complete i.i.d. samples.

Here we introduce the adaptation of several recently proposed classes of characterization based exponentiality tests for the case of the randomly right-censored data and present their limiting and small sample properties. The results of a wide empirical study can be used as a benchmark for future tests proposed for this kind of data. In addition, we present an imputation procedure that can serve as an alternative approach to adaptation proposal.

Spatial-Temporal Modelling of Temperature for Pricing Temperature Index Insurance

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This paper discusses the pricing methodology of the temperature index insurance based on spatial temporal modelling of temperature. The crucial problem here is the location of the potential insurance buyer relative to the station where index is calculated. Since the observed temperatures at particular station are not always correlated to the temperature where the insurance holder lives, it is important to consider spatial issues in the pricing methodology. Thus, we model the temperature using spatial temporal stochastic processes and employ the universal Kriging method to predict the future temperature at some specific locations. Based on temperature index, we may then price the temperature insurance. We illustrate the pricing methodology using 20 years data from five stations in Malaysia. The findings are important for the development of weather index insurance.

Special classes of multivariate generalised autoregressive conditional heteroskedasticity models

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This paper considers Multivariate Generalised Autoregressive Conditional Heteroskedasticity Models with some conditions for identification of some special classes of the MGARCH(p,q) models. The volatility series of Nigeria Average, Urban and Rural Consumer Price Indices are used for the analysis. From the MGARCH (p,q) model, Isolated MGARCH(p,0) and Isolated MGARCH(0,q) models are established as new classes of Multivariate GARCH (p.q) models.

Session

TOPOLOGY

Chain connected pair of a topological space and its subspace

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In this talk we introduce the notion of chain connected set in a topological space given in [1,2,3]. A set C in a topological space X is chain connected in X if for every two elements x and y and every open covering $\mathcal{U}$ of X in X, there exists a chain in $\mathcal{U}$ that connects x and y. And by a chain in $\mathcal{U}$ that connects x and y we understand a finite sequence of elements of $\mathcal{U}$ such that x belongs to first element, y to the last, and any two consecutive elements from the chain have nonempty intersection. Also we introduced the notions of pair of chain separated sets [1] and weakly chain separated sets [3] in a space. The nonempty sets A and B in a topological space X are chain separated in X, if there exists an open covering $\mathcal{U}$ of X in X such that for every point $a \in A$ and every $b \in B$, there is no chain in $\mathcal{U}$ that connects x and y. The nonempty sets A and B in a topological space X are weakly chain separated in X, if for every point $a \in A$ and every $b \in B$, there exists an open covering $\mathcal{U}$ of X in X, such that there is no chain in $\mathcal{U}$ that connects x and y. Clearly, if two sets are chain separated in a topological space, then they are weakly chain separated in the same space. We give an example of weakly chain separated sets in a topological space which are not chain separated in the same space. Then we study the properties of these sets. Moreover we give a criteria for chain connected set in a topological space by using the notions of chain separatedness and weakly chain separatedness. A set C is chain connected in a topological space X if and only if it cannot be represented as a union of two chain (weakly chain) separated sets in X. Then we prove the properties of chain connected sets in a topological space by using the notions of chain separatedness and weakly chain separatedness. Furthermore, we give the criteria for two types of topological spaces using the notion of chain. The topological space is totally separated if any two different singletons are weakly chain separated in the space, and it is the discrete if they are chain separated. Moreover, we generalize the notion to a set in a topological space called totally chain separated set. A set C in a topological space X is totally chain separated in X if every pair of singletons of C are weakly chain separated in X. At the end we prove the properties of totally chain separated sets in a topological space. As a consequence, the properties of totally separated spaces are proven using the notion of chain.

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Sets with the Baire Property in Topologies Defined From Vitali Selectors of the Real Line

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Let $\mathcal{F}$ be the family of all countable dense subgroups of the additive topological group $(\mathbb{R}, +)$ of real numbers, and let V be a Vitali selector related to some element Q of $\mathcal{F}$. According to the generalized version of Vitali's theorem, it is well known that all elements of the family $\mathcal{P} := \{V + q : q \in Q\}$ of translated copies of V by points of Q, do not have the Baire property in $\mathbb{R}$, with respect to the Euclidean topology, and they are not measurable in the Lebesgue sense. In this paper, we consider the topological space $(\mathbb{R}, \tau(V))$, where $\tau(V)$ is a topology having $\mathcal{P}$ as a base. Apart from studying the topological properties of $(\mathbb{R}, \tau(V))$, we also look at the relationship between the families of sets with the Baire property in topologies defined from $\tau(V)$, by using distinct ideals of sets on $\mathbb{R}$. Moreover, we show that for any $Q_i \in \mathcal{F}$, the spaces $(\mathbb{R}, \tau(V_i)), i = 1, 2$, where V_i is Vitali selector related to Q_i, are homeomorphic. We further prove that the families of sets with the Baire property in the spaces $(\mathbb{R}, \tau(V_1))$ and $(\mathbb{R}, \tau(V_2))$ are Baire congruent.

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Session

General Topics

The Number of Solutions to $ax + by + cz = n$ and its Relation to Quadratic Residues

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We find a very efficient formula for calculating the number of solutions of the equation $ax + by + cz = n$ in non-negative integer triples (x, y, z), where a, b, c and n are given natural numbers. This formula involves some summations of floor functions of fractions. To quickly evaluate these sums, we find a reciprocity relation which generalizes a well-known reciprocity relation of Gauss, related to the law of quadratic reciprocity. Further, by counting the number of solutions of the equation $px + qy + z = \frac{q(p-1)}{2}$ in two different ways, we prove that the above result of Gauss is equivalent to a well-known result of Sylvester related to the Frobenius Coin Problem. This work has been published in the Journal of Integer Sequences.

A Unified Framework for Constructing Centralized Coded Caching Schemes

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In caching system, we prefer to design a coded caching scheme with the rate R and the packet number F as small as possible since the efficiency of transmission in the peak traffic times increases with the decreasing of R and the realizing complexity increases with the increasing of F. In this talk, we first introduce a framework for constructing coded caching schemes, which can represent almost all of the previously known schemes. Based on this framework, we obtain a new scheme, which generalizes the schemes constructed by Shangguan et al. (IEEE Trans. Inf. Theory, 64, 5755-5766, 2018) and Yan et al. (IEEE Commun. Lett., 22, 236-239, 2018) and has better performance compared with these two schemes since it has advantages on the user number, the coded gains and the flexible memory size. Then the relationships between a coded caching scheme and orthogonal array, covering array are derived respectively. Consequently a tight lower bound on the packet number F is derived since the packet number of the schemes constructed by Yan et al. (IEEE Trans. Inf. Theory 63, 5821-5833, 2017) gets this lower bound. Finally based on orthogonal array, we construct a new scheme which has the same user number, memory size and transmission rate as the scheme constructed by Shangguan et al. (IEEE Trans. Inf. Theory, 64, 5755-5766, 2018) but has smaller packet number.

The Construction of the Shortest Trajectory on a 2D Surface Using the Level Lines Information

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In the modern world, the capabilities to develop extensive transport networks are totally or almost exhausted in most of the big cities. Therefore, the issues of the optimal patterns of

networks, improvement of traffic management, optimisation of the transport route system are becoming especially important. The paper is dedicated to the method of constructing the shortest movement trajectory on a 2D surface, for which the formulas can be obtained according to GPS navigation or using the level lines information on the surface. The movement trajectory from point A to point B considers the angle of the greatest upturn (downturn) where the parts of the trajectory unfit for movement are eliminated. The mass, friction coefficient, tractive power of the car engine of the moving vehicle on the surface at each point of the trajectory are known. A numerical experiment was performed for the method.

Transformations of Hamiltonian systems connected with the fifth Painlevé equation

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The talk will be about the Painlevé equations, especially about the fifth one P_V. I am going to present three different Hamiltonians and Hamiltonian systems connected with P_V (KNY Hamiltonian, Okamoto's Hamiltonian and Rational Hamiltonian) and present a method how to match them by using algebraic geometry tools. I will show how that can be done by matching surface roots on the level of the Picard lattice. Moreover I will check whether our matching is cannonical.

This is a joint work with Galina Filipuk, Anton Dzhamay and Alexander Stokes.

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Dynamical analysis of rural-urban migration using a chaotic map

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Global and local migration patterns are generating a lot of attention in political circles. This is a challenge for mathematical scientists to develop and simulate the various impact of migration patterns using models. In this research, the dynamics of migration between two communities was investigated using the discrete logistic map. The specific case of rural-urban migration was considered. Phase space and Lyapunov exponents were employed to investigate the dynamical complexity of the model. Results obtained elucidate the implication of low and high rates of migration from rural areas on urban centres. For instance, a rural area with a population growth rate of 2.5% will cause chaotic population "explosion" in an urban centre with population growth of 3.8%, if the rural-urban migration rate is between 0.0 and 0.7. The model proposed will be useful for policy, infrastructural and social development planning by city administrators.

Geometry in space and orthogonal operators (Optimal linear prognosis)

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We will consider two facts (the facts are interesting from point of algebra, geometries and the theory of linear estimation).

We can use the two scalar productions: the first production is the source production $(x, y) = (x, y)_1$; the second production we can determine with help of the equality $(X_1, X_2)_2 = C_1R_1 + C_2R_2+C_3R_3, X_1 = C_1E_1+C_2E_2+C_3\delta_3, ||\delta_3|| = 1, X_2 = R_1E_1+R_2E_2+R_3\delta_3, , E_1, E_2, \delta_3$ are the vectors $(E_1, E_2) \neq 0, ||E_1|| = ||E_2|| = 1, E_1 = x_1/||x_1||$, where $E_2 = x_2/||x_2||, ||X|| = \sqrt{(X, X)_1} = \sqrt{(X, X)_2}$, on the x_1, x_2 sides of the rhombus $||x_1|| = ||x_2||$. We can write $\overline{x}_3 = \widehat{x}_3 + \Delta = e_1||\widehat{x}_3||_1 + c_3\delta, ||e||_1 = 1, \overline{x}_3 = \widehat{x}_3 + \Delta = e_2||\widehat{x}_3||_2 + c_3\delta$, $||e||_2 = 1$, if the $\widehat{x}_3$ vector is the projection of the x_3 vector on the plane with x_1, x_2 basis vector (the optimal linear estimation). The two equalities are possible, if and only if $||e_1|| \neq ||e_2||, ||\widehat{x}_3||_1 = \sqrt{(x_1, x_1)_1} \neq ||\widehat{x}_3||_2 = \sqrt{(x_1, x_1)_2}$, (from the linear independents of the vectors), but the length are the same for both metrics on the x_1, x_2 sides of the rhombus. It is the first fact.

To consider the second fact we can use the equality $(x_1, x_2) = (AQ_1, AQ_2) = (Q_1, Q_2) = 0$; the x_1, x_2 vectors are the result of the orthogonal A transformation of the two Q_1, Q_2 diagonals of rhombus to the x_1, x_2 sides of the rhombus: $A = A^{-1} = A^\top, AQ_1 = x_1, AQ_2 = x_2, A^{-1}x_1 = Q_1, A^{-1}x_2 = Q_2$,

$$A = (1/\sqrt{2}) \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, A^2 = E,$$

(with help of the $(AQ_1, AQ_2) = Q_{1k}^\top A^\top AQ_{2k} = (Q_1, Q_2) = 0$, $AA^\top = E$ equalities, where $Q_{1k}^\top$ and Q_{2k} are the vector-line and vector-column of the E_1, E_2 coordinates in $Q_1/||Q_1||$, $Q_2/||Q_2||$ basis). We obtain $x_1 \perp x_2$, but $(x_1, x_2) \neq 0$ as a primary assumption.

Poster Session

- Positive periodic solutions for nonlinear delay dynamic equations on time scales, *Kamel Ali Khelil*
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Positive periodic solutions for nonlinear delay dynamic equations on time scales

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In this work, we use fixed point theorem to study the existence of positive periodic solutions for delay dynamic equation on time scales. Transforming the equation to an integral equation enables to show the existence of positive periodic solutions by appealing to Krasnoselskii's fixed point theorem. The obtained integral equation is the sum of two mappings; one is a contraction and the other is compact.

Integration the loaded KdV equation in the class of steplike function

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Keywords: loaded Korteweg-de Vries equation, inverse scattering problem, the class of steplike function, scattering data, Lax pair, eigenvalue, eigenfunction.

It is known, that the Korteweg-de Vries equation can be integrated with Inverse Scattering Method [1]. In the works [2,3], the Korteweg-de Vries equations with a self-consistent source were integrated for a class of initial data of "step" type; in particular, laws of evolution of the scattering data were established. In applications of the method of inverse scattering transformation one looks for pairs of operators B and L such that the equation has some interesting nonlinear evolution equation for functions $u(x,t)$ that occur as potentials in the operator L. For the successful application of the method two further ingredients are needed: 1. the inverse scattering problem must be solved so that the potentials $u(x,t)$ can be reconstructed from scattering data; 2. and that one must be able to determine the evolution of the scattering data with$t$.

In this paper, we will consider the loaded Korteweg-de Vries equation

$$u_t - 6uu_x + u_{xxx} + \gamma(t)u(0,t)u_x = 0, \tag{1}$$

where $u = u(x,t),\ x \in R,\ t \geq 0,\ \gamma(t)$ - is an arbitrary, continuous function.

The function $u = u(x,t)$ is a sufficiently smooth and tending to its limits steplike $(c > 0)$

$$\int_{-\infty}^{0} (1-x)|u(x,t)|dx + \int_{0}^{\infty} (1+x)\left|u(x,t) - c^2\right|dx + \sum_{k=1}^{3} \int_{-\infty}^{\infty} \left|\frac{\partial^k u(x,t)}{\partial x^k}\right| dx < \infty \tag{2}$$

The equation (1) is considered with initial condition

$$u|_{t=0} = u_0(x), \quad x \in R^1, \tag{3}$$

where $u_0(x)$ function satisfies the conditions $(c > 0)$:

1. $\int_{-\infty}^{0} (1-x)|u_0(x)|dx < \infty, \qquad \int_{0}^{\infty} (1+x)|u_0(x) - c^2|dx < \infty,$

2. Suppose that, the equation $-y'' + u_0(x)y = \lambda y, \quad x \in R^1$ has $\lambda_1(0),\ \lambda_2(0),\ ...,\ \lambda_N(0)$ negative eigenvalues.

In this work the solution $u(x,t)$ of the loaded Korteweg-de Vries equation (1) in the class of steplike function (2) with initial condition (3) is obtained via Inverse Scattering Method.

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Covid-19 Transmission on long-term care facilities: optimizing control strategies

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Close-contact places such as Long-term facilities have been found to be high-risk and high-morbidity places in the US for COVID-19 outbreaks. The reasons include a vulnerable resident population, limited resources in facilities, close contacts with visitors and workers, contaminated resources, and ill-trained workers. In this study, such places are modeled to evaluate the impact of different transmission pathways of the COVID-19 outbreaks in the presence of various types of interventions. The model captures a coupled dynamics between three subpopulations (staff, the residents, and the visitors) and incorporates infection from infectious individuals and through the environment. Using parameterization of the models via reported cases surveillance data from such facilities in the US, we identified timely adaptive interventions that are critically effective for a vulnerable population. Finally, We study the trade-off between disease burden and prevention cost using cost-effectiveness analysis.

Maximal regularity for evolution equations and application to the Stefan problem

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Maximal regularity is a useful tool for solving abstract parabolic evolution equations. A variant of the one-phase quasistationary Stefan problem that we consider can be reduced to a single evolution equation. We tackle this problem using H^{∞}-calculus to show that the operator in the evolution equation has maximal regularity and then apply an existence theorem for this type of evolution equation. We use additionally an assumption that one particular result on the solvability of a degenerate oblique derivative problem extends in an appropriate way.

Generalizations of Steffensen's inequality by interpolating polynomials

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In papers $[1, 2, 3]$ we obtain new generalizations of Steffensen's inequality using two-point Abel-Gontscharoff polynomial, Hermite interpolating polynomials and Lidstone's polynomial. Here we present results obtained by one of the polynomials i.e. Lidstone's polynomial. Firstly, we give few valuable identities and then using these identities we prove new generalizations of Steffensen's inequality for $(2n)-$convex and $(2n+1)-$convex functions. Further, using Čebyšev and Grüss type inequalities we consider the bounds for the integrals in the perturben versions of the previously described identities.

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On 2-closures of rank 3 groups

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Let G be a permutation group on Ω. A *2-orbit* is an orbit of G in its induced action on $\Omega \times \Omega$. Recall that the number of 2-orbits is called the *rank* of G, and the largest permutation group on Ω having the same 2-orbits as G is called the *2-closure* of G.

If the order of G is even, then a nondiagonal 2-orbit of G induces a strongly regular graph called a *rank 3 graph*. The full automorphism group of that graph is precisely the 2-closure of the corresponding group.

We present the description of 2-closures of rank 3 groups of sufficiently large degree. Groups are organized into several families, based on the combinatorial structures preserved, and the full automorphism groups of the corresponding structures are given. The proof heavily relies on the classification of rank 3 groups and on known results about automorphisms of strongly regular graphs.

Mean value theorems for polynomial solutions of linear elliptic equations with constant coefficients in the complex plane

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The work is devoted to the mean value theorems for solutions of homogeneous linear partial differential equations with constant coefficients in the complex plane whose left hand side is represented in the form of the product of some non-negative integer powers of the formal Cauchy derivatives. We consider systems of special type homogeneous convolution equations defined on smooth functions in a disk, which generalize the classical mean value property over disks for harmonic functions. A sharp version of the uniqueness theorem for functions satisfying such a system in the case of one equation has been established. We also investigate the case of two equations and prove a theorem consisting the classical Delsarte's two-radii theorem for dimension two as a special case. The convolution equations generated by distributions with compact supports and the corresponding mean-value theorems were investigated by many authors: C. Berenstein and D. Struppa, L. Zalcman, V. Volchkov. In particular, V. Volchkov described a wide class of radial distributions with compact supports such that the solutions of the corresponding convolution equations in open Euclidean balls can be efficiently characterized in terms of the Bessel functions and proved the general uniqueness and two-radius theorems for solutions of these equations that go back to the classical results by F. John and J. Delsarte about spherical means, respectively.

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