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Anticipated Expansions of Life Expectancy and Their Long-Run Growth Effects

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Abstract. We analyse the long-run economic growth effects of an anticipated rise of longevity and a simultaneous drop of fertility within the general equilibrium of an R&D-based endogenous growth model with horizontal innovations and overlapping generations. In anticipation of the rise in longevity consumers increase their savings and reduce consumption, long before the rise of longevity actually happens. Nevertheless, the change of the balanced growth path is not smooth; there is still a small sharp fall of consumption because of the drop in fertility.

Keywords: population aging, economic growth, overlapping generations

1 Introduction

Most industrialized countries have aging populations due to decreasing birth rates and increasing life expectancy. All G7 countries experienced substantial demographic changes during the last 60 years resulting in an older population structure. R&D-based endogenous growth models predict that population aging fosters long-run growth. These models consider the reaction of economic agents to unexpected changes in demographic parameters, such as mortality and fertility, and convergence to a new balanced growth path. However, agents in developing countries may predict these changes by observing the situation in developed countries such that they could adjust the consumption/saving decision in advance. In order to describe such behaviour, one needs a model with rational expectations and predicted demographic changes. In this contribution, we extend the overlapping generations (OLG) model of Prettner (2013) that builds upon the R&D-based endogenous growth model of Romer (1990) such that it accounts for anticipated changes of mortality and fertility. We consider the Romer (1990) benchmark case in which population growth is zero such that fertility adjusts to equal mortality. The balanced growth path is derived as the only feasible stationary solution. We assume that the solution of the model stays in the vicinity of the balanced growth path, so that we evaluate changes in the solution by the changes in the balanced growth path due to increased longevity. The central strength of the model is that it allows the analysis of anticipated changes in mortality and fertility. We use the framework to pose the question of whether a sudden unanticipated drop of mortality and fertility by the same amount leads to same change of the GDP growth rate as the anticipated change. We find that anticipation makes the change in the balanced growth path smooth before the drop in fertility and mortality but that there is still a discontinuous change at the time of the drop.

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2 The model

2.1 Basic assumptions on population dynamics

We assume mortality $\mu(t)$ and fertility $\beta(t)$ to be independent of age. Then the aggregate population dynamics are given by

$$\frac{\dot{N}(t)}{N(t)} = \beta(t) - \mu(t), \quad (1)$$

where $N(t)$ is the total number of agents at time t. The size as of time t of the cohort born at time τ is then equal to

$$n(\tau, t) = e^{-\int_\tau^t \mu(\theta) d\theta} \beta(\tau)\, N(\tau). \quad (2)$$

For the sake of simplicity, we consider only synchronous changes in fertility and mortality such that $\beta(t) = \mu(t)$ holds in (1) and the population size N stays constant.

2.2 Optimization problem

An agent born at time τ maximizes her discounted life-time utility

$$u(\tau) \equiv \int_\tau^\infty e^{-\rho(t-\tau) - \int_\tau^t \mu(\theta)\mathrm{d}\theta} \log c(\tau, t)\, \mathrm{d}t \to \max_{c(\tau,\cdot)} \quad ,$$

where ρ is the discount rate, by choosing her consumption $c(\tau, t)$ for all instants $t \geq \tau$. Each agent belonging to the cohort born in τ is endowed at time t with one unit of labor, which she inelastically supplies on the labor market to earn the going wage rate $w(t)$. We also assume that individuals are insured against the risk of dying with positive assets at a fair life insurance company in the spirit of Yaari (1965). Such a life insurance company redistributes the wealth of individuals who died amongst those who are still alive. Thus, the real rate of return $r(t)$ on assets $a(\tau, t)$ is augmented by the mortality rate $\mu(t)$. Taking final goods as the numéraire, the wealth constraint of individuals belonging to cohort τ reads as

$$\frac{\partial}{\partial t} a(\tau, t) = [r(t) + \mu(t)]a(\tau, t) + w(t) - c(\tau, t).$$

The solution of the optimization problem has to obey the no-bequest condition $a(\tau, t) = 0$ and the no-Ponzi game condition $\lim_{t\to\infty} e^{-\int_\tau^t [r(\theta)+\mu(\theta)]\mathrm{d}\theta} a(\tau, t) \geq 0$. Optimal consumption and asset profiles are

$$c(\tau, t) = e^{-\rho(t-\tau) + \int_\tau^t r(\theta)\, \mathrm{d}\theta} \sigma(\tau) h(\tau), \qquad a(\tau, t) = \frac{c(\tau, t)}{\sigma(t)} - h(t), \quad (3)$$

where we introduce life-time human wealth $h(\tau)$, that equals discounted life-time consumption

$$h(\tau) = \int_\tau^\infty e^{-\int_\tau^z [r(\theta)+\mu(\theta)]\mathrm{d}\theta} w(z)\, \mathrm{d}z.$$

At this stage, we define the new parameter

$$\sigma(\tau) = \frac{1}{\int_\tau^\infty e^{-\rho(z-\tau) - \int_\tau^z \mu(\theta)\mathrm{d}\theta} \mathrm{d}z}, \quad (4)$$

which is the marginal propensity to consume out of total wealth and which would be equal to the simple expression $\sigma = \mu + \rho$ in the case of constant mortality μ.

Let us define aggregate consumption and assets as

$$C(t) = \int_{-\infty}^t c(\tau, t)\, n(\tau, t) d\tau\,, \qquad A(t) = \int_{-\infty}^t a(\tau, t)\, n(\tau, t)\, d\tau.$$

Time differentiation of aggregated consumption C, taking into account (2) and (3), yields

$$\dot{C}(t) = [r(t) - \rho - \mu(t)]\, C(t) + c(t, t)\, n(t, t).$$

It follows from (3) that

$$c(t,t) = \sigma(t)\, h(t), \qquad A(t) = \frac{C(t)}{\sigma(t)} - h(t)\, N.$$

Then, with (2), we have $c(t,t)\, n(t,t) = \sigma(t)\, h(t)\, \beta(t)\, N = \beta(t)\, C(t) - \beta(t)\, \sigma(t)\, A(t)$, so that

$$\dot{C}(t) = [r(t) - \rho]\, C(t) - \beta(t)\, \sigma(t)\, A(t), \tag{5}$$

where we take into account that $\beta(t) = \mu(t)$.

2.3 Production side

The **final goods sector** produces output of the consumption aggregate according to the production function

$$Y(t) = [L_Y(t)]^{1-\alpha} \int_0^{Q(t)} [x(t,q)]^{\alpha}\, \mathrm{d}q, \tag{6}$$

where L_Y refers to labor used in final goods production, $Q(t)$ is the technological frontier, $x(t,q)$ is the amount of the blueprint-specific machine $q \in (0, Q(t)]$ used in final goods production, and α is the elasticity of output with respect to machines. Profit maximization and the assumption of perfect competition in the final goods sector imply that the production factors are remunerated according to their marginal products

$$w(t) = (1-\alpha)\frac{Y(t)}{L_Y(t)}, \qquad p(t,q) = \alpha\, [L_Y(t)]^{1-\alpha}[x(t,q)]^{\alpha-1}, \tag{7}$$

where w refers to the wage rate and p to the price paid for machines. The production function in the **intermediate goods sector** is linear with a unitary capital input coefficient such that $x(t,q) = k(t,q)$. The firm producing good q has variable costs of production consisting of the rental payments to capital $r(t)\, k(t,q)$. The profits of firm q are therefore given by

$$\pi(t,q) = p(t,q)\, k(t,q) - r(t)\, k(t,q) = \alpha\, [L_Y(t)]^{1-\alpha}\, [k(t,q)]^{\alpha} - r(t)\, k(t,q) \to \max_{k(t,q)},$$

and profit maximization yields optimal quantities and prices of machines as well as maximal profit

$$k(t) = \left[\frac{\alpha^2}{r(t)}\right]^{\frac{1}{1-\alpha}} L_Y(t), \qquad p(t) = \frac{r(t)}{\alpha}, \qquad \pi(t) = \frac{1-\alpha}{\alpha}\, r(t)\, k(t). \tag{8}$$

Since k and p are the same for all firms, we omit the technology index q from now on. Then the output in (6) and the wage in (7) can be written as follows

$$Y(t) = \frac{p(t)k(t)Q(t)}{\alpha}, \qquad w(t) = \frac{1-\alpha}{\alpha}\,\frac{p(t)k(t)Q(t)}{L_Y(t)}. \tag{9}$$

2.4 R&D sector

New technologies are produced in the R&D sector according to the production function

$$\dot{Q}(t) = \lambda\, Q(t)\, L_Q(t), \qquad Q(0) = Q_0 > 0 \tag{10}$$

where $L_Q(t)$ is the amount of scientists employed at time t, Q_0 is the initial technological frontier, and λ is the productivity of scientists. Since the market for patents is competitive, the R&D industry as a whole makes no profit, and the firm who buys a patent pays the discounted stream of operating profit to the R&D sector as a setup cost. The reason is that there is free entry such that lower (higher) setup costs would induce market entry (exit) driving operating profits down (up) until the setup costs were again equal to the discounted stream of future operating profits. Note that the setup costs are financed by the owners of the intermediate goods producing firms via the emission of new shares. Thus, the market price $v(t)$, of the patents produced at time t is determined by the zero-profit condition

$$v(t)\, \dot{Q}(t) = w(t)\, L_Q(t), \tag{11}$$

which results from the fact that the R&D industry creates $\dot{Q}(t)$ patents in a unit of time with the associated cost being the wage bill for the employed scientists $L_Q(t)$. We define $v(t)$ as the present value of the real profit flow for the firm producing any product $q \in [0, Q(t)]$ over the interval $[t, +\infty)$

$$v(t) = \int_t^\infty \exp\left[-\int_t^s r(\theta)\mathrm{d}\theta\right] \pi(s)\, \mathrm{d}s, \tag{12}$$

thus excluding speculative bubbles in the patent market, see, e.g. (Romer, 1990), (Grossman & Helpman 1991, Chapter 3).

2.5 Market clearing

The **labor market clearing** condition is

$$L_Q(t) + L_Y(t) = N, \tag{13}$$

where the population size N represents aggregate labor endowment, because each agent inelastically supplies one unit of labor.

The **goods market clearing** condition states that net investment is the rest of output after consumption

$$\frac{d}{dt}\big(k(t)\, Q(t)\big) = Y(t) - C(t), \qquad k(0) = k_0 > 0. \tag{14}$$

Finally, the **financial market clearing** condition states that all assets are either invested in physical capital or in shares of the intermediate goods producers such that

$$A(t) = k(t)\, Q(t) + v(t)\, Q(t), \tag{15}$$

where $k(t)Q(t)$ and $v(t)Q(t)$ are the total amounts of capital and cost of patents in the economy.

3 Equations describing the general equilibrium

Using the equation for aggregate consumption (5) together with the balance of assets (15) and $r(t) = \alpha\, p(t)$ in (8), we get

$$\frac{\dot{C}(t)}{C(t)}\, Z(t) = [\alpha\, p(t) - \rho]\, Z(t) - \beta(t)\, \sigma(t)\left(1 - \frac{v(t)}{k(t)}\right), \tag{15}$$

where we denote $Z(t) = \frac{C(t)}{Q(t)k(t)}$. The market price of a patent in (12) along with the expression for profits in (6) reads as

$$v(t) = \frac{1-\alpha}{\alpha}\int_t^\infty \exp\left[-\int_t^s r(\theta)\mathrm{d}\theta\right] r(s)\, k(s)\, \mathrm{d}s. \tag{16}$$

Assuming that $L_Q(t) > 0$, we have from (10) and (11) that $\frac{w(t)}{Q(t)} = \lambda\, v(t)$. On the other hand, from the last expression in (9), we get $\frac{w(t)}{Q(t)} = \frac{1-\alpha}{\alpha}\, \frac{p(t)k(t)}{L_Y(t)}$. Then we arrive at

$$\lambda\, L_Y(t)\, v(t) = \frac{1-\alpha}{\alpha}\, p(t)\, k(t). \tag{17}$$

Equation (10) together with the labor market clearing condition (13) yields the growth rate of the technological frontier

$$\frac{\dot{Q}(t)}{Q(t)} = \lambda\,\big(N - L_Y(t)\big). \tag{18}$$

Finally, the goods market clearing condition (14) along with (9) can be written in the form

$$\frac{\dot{k}(t)}{k(t)} + \frac{\dot{Q}(t)}{Q(t)} = \frac{p(t)}{\alpha} - Z(t). \tag{19}$$

4 Balanced growth path

Along the balanced growth path consumption, assets, and technology grow at the constant rate g, while the other variables in (15)-(19) are constants. Then equations (16) and (17) yield

$$v = \frac{1-\alpha}{\alpha}\,k, \qquad \lambda\, L_Y = p.$$

Equation (18) takes the form

$$\frac{\dot{Q}(t)}{Q(t)} = \lambda N - p.$$

so that we have from (15) and (19) the following system of two equations with two unknowns p and Z

$$(\lambda N - p)\, Z = [\alpha\, p - \rho]\, Z - \frac{\beta\sigma}{\alpha}, \qquad \lambda N - p = \frac{p}{\alpha} - Z,$$

that yields the quadratic equation $\alpha^2 Z^2 - (\lambda\, N\,(1-\alpha) + \rho)\,\alpha\, Z - \beta\,\sigma = 0$. As a consequence, there are two roots

$$\alpha\, Z_{\pm} = \frac{\rho}{2} + \frac{(1-\alpha)\,\lambda\, N}{2} \pm \sqrt{\left(\frac{\rho}{2} + \frac{(1-\alpha)\,\lambda\, N}{2}\right)^2 + \beta\,\sigma}.$$

Only the positive root, Z_+, leads to an economically meaningful solution because consumption and capital ought to be positive. The associated growth rate $g = \frac{\dot{Q}(t)}{Q(t)} = \lambda N - p = \frac{1}{1+\alpha}\,(\lambda\, N\; - \alpha\, Z_+)$ is given by the following expression

$$g = \frac{\lambda\, N}{2} - \frac{\rho}{2\,(1-\alpha)} - \sqrt{\left(\frac{\rho}{2} + \frac{(1-\alpha)\,\lambda\, N}{2}\right)^2 + \beta\,\sigma},$$

which is strictly positive for sufficiently large $\lambda\, N$.

5 Anticipated aging

If an abrupt change in mortality and fertility occurs at time t^*, such that $\mu(t) = \beta(t) = \mu_1$ for all $t < t^*$ and $\mu(t) = \beta(t) = \mu_2$ for all $t \geq t^*$, and if this change is anticipated, then we obtain from (4) that $\sigma(t) = \mu_2 + \rho$ for all $t \geq t^*$, while for all $t < t^*$ the parameter $\sigma(t) = \frac{\rho+\mu_1}{1+\frac{\mu_1-\mu_2}{\rho+\mu_2}\, e^{-(\rho+\mu_1)\,(t^*-t)}}$ changes continuously from $\mu_1 + \rho$ as $t \to -\infty$, to $\mu_2 + \rho$ at $t = t^*$. The evolution of this parameter is depicted in the first graph of Fig. 1. The steady-state consumption to capital ratio Z that is related to the corresponding value of σ also features continuous adjustment until $t = t^*$ at which point there is an additional discountinuous drop. The evolution of the consumption to capital ratio Z is depicted in the second graph of Fig. 1.

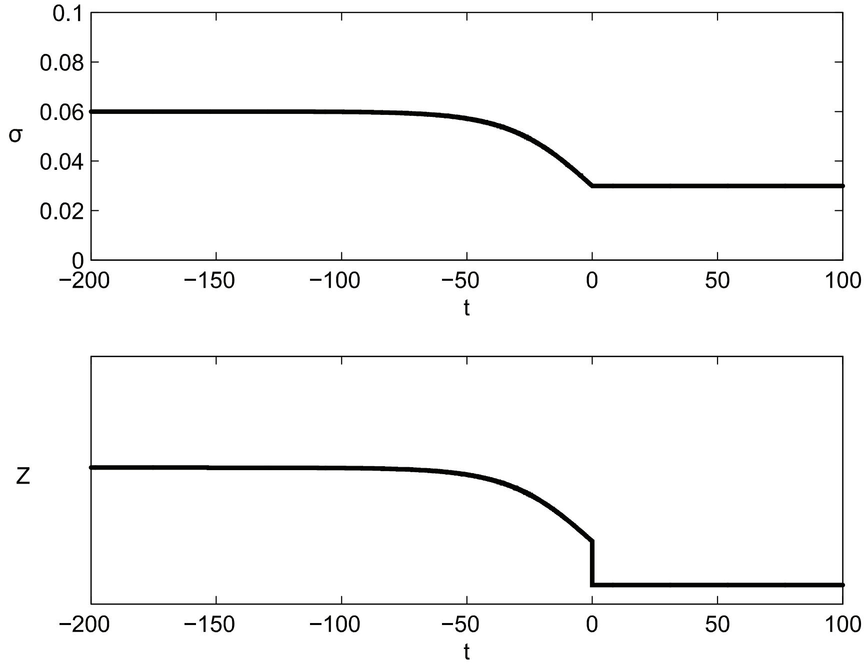

Figure 1: Parameter σ as a function of time (top). Steady state consumption to capital ratio Z related to the corresponding value of σ (bottom).

5 Conclusion

If the population is aging, i.e., $\beta\,(\rho\ +\mu)$ becomes lower, then the economic growth rate g increases. If aging is anticipated, then the increase of the growth rate starts slowly in advance, but it still jumps at the moment of the decreases of mortality μ and fertility β.

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