

Cultural heritage as an instrument for the tourism sector: a cultural network approach

Francesca Amendola

University of Salerno, Department of Political Science and Communication, Italy
famendola@unisa.it

Anna Papaccio

University of Salerno, Department of Economics and Statistics, Italy
apapaccio@unisa.it

Abstract. The cultural heritage valorisation problem led to the development of new organizational models based on the relationship between cultural heritage, the economic system and the territory. One of the most useful solution has been the network structure, based on the interaction between different types of resources, goods and operators. This paper analyses the formation and stability of potential cultural networks among heterogeneous cultural and territorial actors. The formation of cultural (informal or formal) agreements amongst heterogeneous actors has been studied by means of the network formation theory.

Keywords: Cultural Economics, Cultural Heritage, Non-profit Institutions, Network formation games

1. Introduction

The definition of appropriate strategies for the management and promotion of cultural heritage protection and valorisation represented the subject of economic analysis and investigation on organizational models for several years.

Cultural tourism has become one of the emerging niches in the tourism sector (Richards, 2018), driven by the “heritage boom” (Hewison, 1987), seen as an “asset for all the community” (du Cros, 2001). In this sense, “tourism and cultural heritage management (CHM) are neither natural allies, nor natural enemies” (McKercher et al., 2005) and the positive or negative outcome of the encounter depends on the stakeholders’ behaviour, which indicates how much they value the heritage assets and how well their roles are defined within the relationship (McKercher et al., 2005). Therefore, it is necessary to take into account the “conflicting goals and power imbalances” (De Carlo and Dubini, 2010) between the different stakeholders and to analyse the relationship between heritage assets and economic growth (Settis, 2002, 2005). In order to analyse this relationship, following De Carlo and Dubini (2010) we look at cultural heritage as a resource for local development consistent with the resource-based view (RBV). The potential link between cultural heritage, economic development and tourism has previously been analysed in a territorial and economic context like Italy, where culture represents a fundamental asset for socio-economic development (Leon & Tuccini, 2011; Causi & Leon, 1990; Trimarchi, 2003).

The evolution of the regulatory structure in recent years in Italy has deeply changed the management and institutional structures in the cultural sector. The transformation process focuses on a different distribution of powers between the various levels of government (vertical subsidiarity), adopting a progressive decentralization of the valorisation functions towards peripheral bodies, based on the awareness that most of the links between resources and communities are expressed at the local-level. This change stems from the need to search for new and different forms of organization of cultural

institutions. The economic approach to the management problem for cultural institutions focuses on the relationship between cultural heritage, economy and territory. The main solutions to solve the problem of the dispersion of resources on the territory (Valentino et al., 1999) are based on networks, integration and territorial cultural systems. In particular, the networking of resources and all public and private entities who interact with them has been seen as an instrument to achieve the integration of objectives (preservation, valorisation and promotion of heritage and land use, socio-economic development).

This paper attempts to study the formation and the strength of a cultural network, analysing the incentives and defining roles and modalities of interaction between the players involved.

We analyse the formation of cultural (informal or formal) agreements amongst heterogeneous cultural (territorial) actors by means of the network formation theory. In order to characterize and distinguish between bilateral (e.g. endogenous network formation) and multilateral outcomes (e.g., induced cultural systems) we have used two different network stability notions, which are pairwise stability and hypergraph stability. This choice depends on the fact that hypergraph stability better describes a situation in which cultural and non-cultural actors decide to join an association (e.g., collective trademark) considering also potential multilateral deviations. To the best of our knowledge the valorisation of cultural heritage has not been analysed in the literature by means of network formation theory but rather looking at its impact on the economy (Grefe, 2004; Bowitz and Ibenholt, 2009).

We develop a cultural network formation game in a setting with three heterogeneous entities (nodes): a well-known cultural institution, considered a major touristic attractor of the territory (e.g., a very popular Museum), a minor cultural attractor (e.g., a rural ethnographic museum) and a touristic firm taking advantages from the presence of the attractors in the territory. The payoff function of each agent depends on the equilibrium values of prices and quantities that in turn depend on the network configuration that shapes the market structure in which players operate. Thus, the decision to form a link depends on the balance between the utility generated by the proposed agreement and the one generated by the status-quo solution, given other players choice.

The paper proceeds as follows. In the next section, we develop the main approaches used in the management of cultural heritage and tourism relation, introducing some important definitions. In section 3, we analyse the cultural network formation in a mixed oligopolistic model. In section 4, we show the results respectively on network stability of bilateral and hyperlink settings. In section 5, we conclude with some remarks.

2. Cultural heritage management (CHM) and tourism: strategies for a collaboration

A correct usage of cultural heritage serving local development should respond to: (i) the multidimensional nature of cultural heritage, (ii) the huge number of actors involved in the process (institutions, organizations etc.) and (iii) the difficulties in implementing cultural heritage policies, which surely arise problems of coordination (Amendola and Papaccio, 2014). The relationship between tourism, cultural heritage and economic development can be better managed by using cultural structures integrating different type of goods, resources and economic activities and different sectors, such as public and private operators. In particular, the networking of resources and all public and private entities involved can be an effective means to achieve the integration of the objectives of CHM preservation and valorisation and of socio-economic development.

As an organizational structure, located between the organization and the market, the Network is a model of cooperative relationships (or transactions) among individual and collective agents, itself constituting a new collective agent (Pichierri, 2002). The agents in the net are linked by interdependent relationships and coordination mechanisms, whereas the set of actors (nodes) and relationship shapes the network structure itself. Therefore, a network can be defined through a collections of nodes (e.g. enterprises)

interconnected by a number of links representing interactions or collaboration relationships between nodes. The first goal of a cultural network is to strengthen the relationship among cultural heritage, economy and the territory, which can be reached through different levels of integration among parties.

2.1. Management models of cultural heritage: the role of Cultural Attractors

Models that focus on the organization of CHM in territorial networks can be divided into three types, depending on the level of strategic integration: Cultural Networks, Cultural Systems and Cultural Districts (Hinna & Minuti, 2009; Bagdadli, 2001).

Cultural Networks provide forms of external integration: agents in the network preserve their decisional autonomy, but they share the cultural supply service and the communication and information policies, in order to form a specific imagine of the local cultural supply. The type of agreement connecting agents in the general network defines a collection of bilateral and interdependent exchange contracts (Esposito, 2013). Typical examples of external supply integration are: the museum cards (Great Britain Heritage Pass, Scotland Explorer card), tourist card (Luxemburg card, Campania Artec card) and season tickets (Schwizer Museumpass, Torino Piemonte abbonamento musei, Museumjaarkart, Passeport des Musées du Rhin Supérieur).

Cultural Systems represents strategic forms of integration that regulate directly cultural sites sharing a special theme (e.g., archaeology, art, science, etc.) and/or a territory. Members of the system generally integrate plans and policies through a joint management i.e. by adopting forms of internal integration. Cultural Systems, for their close connection with the territory (or a subject) and for their integration processes of internal type, tackle and try to overcome the limitations of a parochial (or localistic) and fragmented management of cultural heritage. Examples include the Territorial System of National Heritage and Cultural Affairs of the Province of Cuneo, The Museum System of the Province of Ravenna, the Museum System of the Province of Modena, in Italy, and the system of the Museu de la ciència de la tècnica de Catalunya (MCTC) in Spain.

Cultural Districts are accomplished forms of lattice structures, tying the development of the cultural system to the highest ambition of local development (Pichierri, 2002). We can distinguish between at least four different types of cultural districts: Industrial Cultural Districts, the Institutional Cultural District, the Museum Cultural Districts and Metropolitan Cultural District (Santagata, 2001). Some examples of the first type of district are Como for silk, the manufacture of jewellery and gold to the municipality of Valenza, or Murano glass. The Institutional Cultural Districts generally provide for the presence of formal institutions, such as protected copyrights and trademark rights, generally allocated to a restricted area of production (Santagata, 2001). The Langhe in Piedmont, the Chianti region in Tuscany, the consortiums for the protection of a trademark are classic examples of this type of CD.

As mentioned above, the management of cultural tourism has been based on its capacity of generate sustainable well-being for residents along the economic, social, political, cultural and ecological dimension (Sainaghi, 2004). An effective management of the link between resources (attractors) and products, which is crucial for the competitiveness of an area, requires re-establishing the cultural attractors (or resources) as central players in the system.

To improve the quality of life of residents and promote new forms of sustainable economic development it is possible to turn into competitive advantage the collection of natural resources, landscapes and both material and immaterial cultural heritage, that increase also the touristic attractiveness of the territory. Cultural Attractors can be classified by their notoriety to national and international audiences in three levels: a first level (major attractors), consisting of sites and places of importance; a second level (attractors), formed from the collection of cultural events of various kinds and importance, but known especially for the specific type of tourist; and a third level (minor attractors), formed from the collection

of testimonies of local popular culture in rural and marine territory, but vaguely known to the general public.

The consolidation and enhancement of cultural networks of excellence, which include both major attractors and widespread heritage, the diversification and qualification of innovative cultural services, allowing to integrate local cultural supply in wide distribution markets, selling a combined set of services (including tourist attractions), is only one of the possible strategies for the valorisation and development of the territory. The basic idea is to focus on attractors that lie along the strong tourist flows or territorial structures in which the cultural heritage (or environmental and landscape) best known and visited, are located, (i.e. with a high concentration - in spatial or relational sense – of very important cultural resources) in order to spread positive externalities (including to other economic sectors). In addition, “young” micro-systems, territorial environments with high resources, either without a consistent touristic flow or with a significant concentration of minor cultural resources, (i.e. with high value goods but isolated with respect to their original environment and/or in highly problematic contexts), could benefit from the positive externalities generated by the proximity to major attractors. The main problem is represented by the convergence and cooperation between different actors and strategies, both public and private, within the tourist sector and with respect to other sectors, which can produce negative externalities.

3. The Model

3.1 The Actors of the network game

The presence of tourist and commercial actors and of positive externalities of production generated by the public nature of the cultural product enhances the problem of coordination between cultural actors. The market where tourist firms operate, generally has an oligopolistic structure or even a monopoly, since this kind of activities are connected to primary resources. Therefore, the market power of the touristic actors depends on the primary resources to which they are linked (territorial monopoly). On the other hand, the cultural and natural heritage, which represent the primary source of the touristic supply, are characterized by fixed amount (absolute scarcity), uniqueness and the connection with the territory where they arise (not reproducible) and the absence of a market, as public goods without a definable price (unmarketable) (Van Der Borg & Russo, 2006). We assume that *cultural attractors* hold strategic complementarity with other attractors and economic agents in the area and strategic substitutability with some touristic firms selling substitutes good or services. The loss generated by the touristic firm, selling substitute goods, is seen as a cost deriving by the positive externality of production generated by the cultural firm. In order to exploit the reciprocal positive externalities agents can build cooperative links instead to compete. The reciprocal economies of scales can be valorised through the network and accessibility facilities. The network can be formed (according to the *theorem of interdependencies for reciprocal external economies*)¹ only if the income generated by belonging to the network is greater than the externality gain. Moreover, Public Institutions (or formal agreements) can play an important role in strengthen the credibility of other agents’ actions in the net. In Italy, public subsidies² are supplied in order to contrast the externalities problem and to cover part of the high fixed costs borne by cultural

¹ We refer here to the “Theorem of Reassurance” (Sen, 1987).

² The subsidy can be supplied in different forms: *specific*, as a lump sum monetary subsidy (or in kind); or *ad valorem*, as incentive for fundraising activity (share of private funds), as a prize for the best known cultural organization (share of sold tickets), as a prize for the more appreciated cultural firms by customers (share of customers’ coupons).

firms. Public funding do not cover in average 30% of the revenues and in order to qualify for government support, cultural firms have to fulfil some minimum quality standard criteria³.

3.2 The Utility functions

We assume that there are only three stakeholders in a delimited territory: a Major (or Principal) Attractor (AP), a well-known cultural institution (e.g., the main Museum in a territory); a Minor Attractor (AM) an unexplored cultural heritage in the same territory and a commercial firm T , supplying touristic services or handcrafted goods (exploiting the cultural heritage proximity). We assume that the three agents first decide whether to link to any other cultural agent in the territory, by means of specific agreements, and then maximize their utilities getting a specific payoff depending by the resulting network structure. We proceed by backward induction: agents anticipate their payoffs related to the networks structure and take decisions about link creation correspondently.

As non-profit organizations, the two attractors do not maximize profits. As cultural firms, they try to maximize the number of visitors. However, they first face a survival problem: the budget balance.

We assume that AP and AM supply cultural goods, generating a partial market failure (*Dupuit* and *Hotelling* goods). In particular, AM offers Dupuit goods and bears only a fixed cost (due to high infrastructure investment and preservation) shared by the number of customers up to a saturation level⁴ and the AP offers Hotelling goods and bears also variable costs⁵.

AP maximizes the number of visitors (y) and provides also accessory goods or services (q_a) (competing with the private touristic firm T) to customers.

The utility of AP depends on benefits (B^{ap}) and linear costs (C^{ap}). The two components can be described by the following equations:

$$B^{ap} = R^y(y, p_y) + R^{q_a}(q_a, p_q) + S(y, d^{ap}) \quad (1)$$

$$C^{ap} = F^{ap} + C(y) + C(q_a) \quad (2)$$

Benefits are described by revenues from tickets and service sold to visitors (R^y and R^{q_a})⁶

and by government subsidies:

$$S(y, d^{ap}) = \alpha C(y) + d^{ap} S^M \quad (1.1)$$

Subsidies depend on the total costs (we assume subsidies are supplied as share of total costs $\alpha C(y)$ with $0 < \alpha < 1$) and on the entry d^{ap} corresponding to the AP-AM link of an Adjacency Matrix⁷ of any potential graph configuration⁸. We assume that a government incentive to the creation of museum networks exists, such that, if an agreement (formal or not formal) between AP and AM is achieved ($d^{ap} = 1$), a lump sum transfer S^M is supplied.

³ Government grants may depend on the specific cultural firm situation, such as on the number of visitors' average, or, as incentive to the creation of a museum system, on the existence of agreements (or informal networks) among local corporations (managing cultural goods) in a territory (non-necessarily contiguous); finally, as incentive to eco-museums constitution (providing strong links among the territory, population and cultural heritage), public funds may be supplied.

⁴ The unit costs will be decreasing in the number of visitors.

⁵ For Dupuit goods (cultural firm case), the optimal prize is zero, for the Hotelling the optimal prize is equal to the marginal variable cost.

⁶ $R^y(y, p_y) = p_y y$; $R^{q_a}(q_a, p_q) = p_q q_a$

⁷ The Adjacency Matrix of a graph on n vertices is an $n \times n$ matrix $A = (a_{ij})$ in which the entry $a_{ij} = 1$ if there is an edge from vertex i to vertex j and is 0 if there is no edge from vertex i to vertex j (see Appendix A.1).

⁸ Since we are dealing with undirected graphs the Adjacency Matrix A will always be symmetric such that $\forall i \forall j \in A, a_{ij} = a_{ji}$.

Total costs in eq. (2) include fixed costs F^{ap} and two variable costs, one associated to visitors (we assume constant marginal costs and we do not consider the congestion problem) and a second one associated to the production of the accessory good/service⁹. In this respect, the cultural firm behaves exactly as a profit firm. We do not make special assumptions on the ownership of the cultural good and we assume that the management of accessory services belongs to the cultural firm.

We assume that both revenues and costs depend on network configuration in this way:

If a link between AP and AM is built, costs (and revenues), associated with the accessory good, are equally shared by the parties. If AP builds a link with the private firm¹⁰, he grants T the right to produce q_a reducing variable costs to zero.

The Utility of AP depends on links structure and is given by the following expression:

$$U^{ap} = R^y(y, p_y) + (1 - d^{ap})(1 - d^T)R^{qa}(q_a, p_q) + d^{ap}(1 - d^T)\frac{(R^{qa}(q_a, p_q))}{2} + S(y, d^{ap}) - \left((1 - d^{ap})(1 - d^T)C(q_a) + d^{ap}(1 - d^T)\frac{C(q_a)}{2} + F^{ap} + C(y) \right) \quad (3)$$

We assume that: AM supplies a dupuit cultural good with high fixed costs (F^{am}) and zero marginal costs. It grants free entry and does not produce any accessory good¹¹. The Utility of AM depends on the balance between fixed costs F^{am} , public subsidies S^M and sponsorship agreements βF^{am} .

$$U^{am} = d^{ap} \left(S^M + \frac{(R^{qa} - C(q_a))}{2} \right) + d^{am} \beta F^{am} - F^{am} \quad (3.1)$$

Firm T Utility function U^T is described by equations (4-4.1)

$$U^T = (1 - d^{ap})\Pi^O + d^{ap}\Pi^M + d^{am}(B^{am} - \beta F^{am} + G^T) \quad (4)$$

$$\Pi = R(p_Q, q_t) - C(q_t) = p q_t - c_2 q_t \quad (4.1)$$

The expressions B^{am} and G^T represent the sponsorship benefits (trademark use and fiscal benefits); βF^{am} represents the sponsorship cost in term of monetary contribution to AM fixed costs (with $0 < \beta < 1$). The terms Π^O and Π^M represent, respectively, profits deriving from the maximization in an oligopoly market structure and from a territorial monopoly.

⁹ The two components of costs $C(y)$, $C(q_a)$ are given by the following equations: $C(y) = c_y y$; $C(q_a) = c_1 q_a$

¹⁰ A link between the private firm T and a cultural firm can take the form of a sponsorship agreement in which the *Sponsee* agrees to grant the sponsor the production of the accessory good (or to use its trademark if no goods are produced) and the *Sponsor* agrees to provide services or to support the cultural firms by monetary transfers. In other words, firm T can give either a “real” contribution to the cultural institution bearing the costs of the accessory good production instead to exploit the positive externalities, or a monetary one.

¹¹ We assume that the average number of visitors of AM does not fulfill the criteria to get access to public grants supporting accessory services.

3.3 The network game

We focus on the formation of cultural agreements amongst heterogeneous (cultural) agents by means of the network formation theory (Bala & Goyal, 2000; Brandes & Erlebach, 2005; Currarini & Morelli 2000; Demange & Wooders, 2005; Dutta & Mutuswami, 1997; Goyal, 2007; Jackson, 2008), analysing the resulting equilibrium networks in which links are formed by agents. The nodes represent players and the edges the relationships between any pair of players. Agents derive utilities from the specific network they belong with. Every player individually can withdraw a connection (undirected link) and every pair of player can build it. The number of links and connected nodes (agents), formed by agents' choices and the resulting networks, can be predicted through equilibrium concepts.

We divide the study of the network formation game in two different analyses. First, we focus on the stability of a type of agreement connecting agents in the general network, as a collection of bilateral and interdependent exchange contracts. Namely, we analyse the pairwise stability¹² of the cultural network formation game. Second, we focus on some specific form of associative contract that binds all the nodes together for the purpose of a common goal, such as an induced (by public governance) cultural district or a cultural system. In other words, we analyse the hyperlink stability¹³ of a museum cultural district and complete cultural district.

Regarding the first type of agent decisions (endogenous agreement formation), we assume that the network formation game (i.e., the market structure in which the interaction among agents takes place) is shaped by agents through link choices and resulting network configurations¹⁴. Two market structures may arise, either a *Mixed Oligopoly* market characterized by the interaction between a private and a public entity (we assume that AM never competes) or a *Territorial Monopoly*¹⁵ for firm T.

We assume that the three agents first make their link decisions and then they maximize their utility getting a specific payoff depending by the resulting network structure. If firm T is the unique producer of the accessory good (e.g., in the complete network structure) since it is providing the service to AM and AP, then it gets the Monopoly Profits¹⁶. When firm T does not gain the right to produce AP accessory good, agents play a *mixed Cournot* game.

We can think of AP as a multiproduct cultural firm welfare maximizing with respect to the number of cultural visits (y) and the accessory good q_a supplied also by firm T (q_t).

The demand function of “cultural visits” depends negatively on the entry price $P(y)$ and positively on the network density Ψ ¹⁷.

$$P(y) = b - y + \Psi \quad (5)$$

$$\Psi = \frac{2|I|}{N(N-1)} \quad (5.1)$$

¹² For a formal definition of Pairwise Stability refer to the Appendix A.2

¹³ For a formal definition of Hyperlink Stability refer to the Appendix A.2

¹⁴ With three heterogeneous nodes we can have eight possible network structures: (g^e) the empty network (no links are formed), (g^c) the complete network (any possible link is formed), (g^M) the museum network (only one link is formed between AP and AM), (g_{AP}^S) the sponsorship network with AP (only one link between T and AP), (g_{AM}^S) the sponsorship network with AM (only one link between T and AM), (g_{AM}^{HS}) the hub-spoke network of AM (both T and AP are linked only with AM), (g_{AP}^{HS}) the hub-spoke network of AP (both T and AM are linked only with AP), (g_T^{HS}) the hub-spoke network of T (both AP and AM are linked only with T).

¹⁵ A territorial monopoly over a product market area arises if an actor is able to control distribution of a good or service.

¹⁶ It represents a situation in which firm T behaves as a Territorial Monopolist (e.g., when T is the only producer of the accessory good in the area) selling the monopoly quantity Q^{Mon} and getting monopoly revenues R^{Mon} . $\Pi^M = \frac{(a-c_2)^2}{4} y^*$

¹⁷ This assumption is based on the observation of consumers' behavior in the Italian regions (over the last twenty years) where, it has been noticed that, the creation of networks and cultural systems has incentivized the concentration and the diffusion of the demand for culture. “*Poli museali di eccellenza nel Mezzogiorno: Analisi della Competitività dell'offerta museale del mezzogiorno e benchmark*”. Mibac, 2009, p. 60.

Where l represents the number of links in the network and N the number of vertices¹⁸.

The demand function of the accessory good depends negatively on price $P(Q)$ and positively on the number of visitors y .

$$P(Q) = a - \frac{Q}{y} \quad (6)$$

Where $Q = (q_a + q_t)$ represents the industry supply given by the sum of the quantities of the accessory good produced by the cultural firm and by the private firm T. If AP and T play Cournot, they simultaneously choose the level of q_a, q_t with T as a profit maximizing firm and AP a welfare maximizing cultural institution (fixing a price equal to marginal costs).

The Welfare function is represented by the following expression:

$$W = U(y) + U(Q) - C(y) - C(Q) \quad (7)$$

The functions $U(y), U(Q)$ represent the gross consumers' surplus in the two markets.

The total costs of the industry $C(Q)$ are represented by eq. (8).

$$C(Q) = C(q_a) + C(q_t) = c_1 q_a + c_2 q_t \quad (8)$$

The First order conditions for AP maximization problem are presented in eq. 9-10.

$$\frac{\partial W}{\partial y}: P(y) = C'(y) \quad (9)$$

$$\frac{\partial W}{\partial q_a}: P(Q) = C'(q_a) \quad (10)$$

From which we obtain¹⁹:

$$y^*(\Psi) = b + \Psi - c_y \quad \text{and} \quad P(y^*) = c_y \quad (11)$$

$$R^{ap}: q_a = [a - c_1]y - q_t \quad (12)$$

Eq. (12) represents the reaction function of the cultural firm AP. Firm T maximizes the profit function yielding the reaction function shown in eq. (14).

$$\frac{\partial \pi}{\partial q_t}: R' = C' \quad (13)$$

$$R^t: q_t = \frac{(a-c_2)}{2}y - \frac{q_a}{2} \quad (14)$$

We assume that the minor attractor AM does not maximize its utility (both in market y and q), however, when a link with AP is built (and an oligopoly market structure is shaped), it shares revenues and the costs as they behave like a single node.

¹⁸ The density is represented by the ratio between the number of links in a network and the maximum number of potential links; given the number of nodes, (it expresses a concept of probability about dyadic connections).

¹⁹ Since $P(Q) = U'(Q)$ and $a - \frac{Q}{y} = c_1$

Simultaneously solving eq. 12 and 14 we obtain the following equilibrium price and quantities:

$$q_a^* = y^*(\Psi)(a - 2c_1 + c_2) \quad (15)$$

$$q_t^* = y^*(\Psi)(c_1 - c_2) \quad (16)$$

$$P(q_a^*, q_t^*) = c_1 \quad (17)$$

If Firm T gains the right to produce AP accessory good he becomes a territorial monopolist and the equilibrium price and quantities are given by:

$$q_M^* = y^*(\Psi) \left(\frac{a - c_2}{2} \right) \quad (18)$$

$$P(q_M^*) = \left(\frac{a + c_2}{2} \right) \quad (19)$$

3.4 Pairwise Stability Results

Considering the results deriving from the strategic interaction between the major attractor and the private firm and its effect on the utility of each agent in the market we can move to the analysis of link decision. In this section, we describe the main propositions on the equilibrium structures resulting from the strategic interaction meanwhile the stability conditions for each network configuration are provided in the Appendix A.3.

Proposition 1. If $B^{am} - \beta F^{am} + G^T \geq 0$ and $S^M \geq \beta F^{am} \geq 0$ either g^C with $c_2 \geq \left(\frac{2c_1\sqrt{Y_2^*} - a\sqrt{Y_3^*}}{2\sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$ or g_{AM}^{HS} with $c_2 < \left(\frac{2c_1\sqrt{Y_2^*} - a\sqrt{Y_3^*}}{2\sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$ are the only PS configurations.

The complete network, in which the three actors are connected, is the only P-Stable structure if public subsidies S^M are greater than the sponsor contribution βF^{am} (so that AP and AM find profitable to build a museum network) and if costs deriving from sponsorship for the firm T do not overcome gains, in terms of direct benefits and fiscal advantages ($B^{am} + G^T \geq \beta F^{am}$). Moreover, firm T also prefers to connect to the two attractors (when AM and AP are already linked) if marginal costs of production are greater than a threshold value, since he can get monopoly profits. On the other hand, if costs are lower enough to prevent firm T to link with AP (exploiting the monopoly power he gets from the agreement), the only PS network is g_{AM}^{HS} .

Proof.

It's easy to show that if g^C P-Stability conditions hold the other networks are not stable. The Museum network g^M and g_{AP}^{HS} are not PS since they both require ($B^{am} - \beta F^{am} + G^T < 0$); g_{AM}^{HS} P-Stability conditions are not compatible with condition 2.3 in Lemma 2 since it requires

$c_2 < \left(\frac{2c_1\sqrt{Y_2^*} - a\sqrt{Y_3^*}}{2\sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$. On the other hand, if the last requirement is fulfilled, then the only PS network is g_{AM}^{HS} since conditions 2.3 in Lemma 2 fail.²⁰ ■

Proposition 2. If $B^{am} - \beta F^{am} + G^T < 0$ either g^M (for $c_2 < \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$ and $(Y_2^* - Y_1^*) < \frac{(\beta F^{am} - (G^T + B^{am}))}{(c_1 - c_2)^2}$) or g_{AP}^{HS} (for $c_2 \geq \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$ and $(Y_3^* - Y_2^*) < \frac{4(\beta F^{am} - (G^T + B^{am}))}{(a - c_2)^2}$) are the only PS networks.

The Museum Network is an equilibrium architecture if Firm T gains from sponsorship are lower than costs and if its marginal costs of production are lower enough to prevent him to exploit the potential gains deriving from a territorial monopoly architecture. On the other hand, if costs are higher of a threshold value, firm T prefers to get the monopoly profits without a sponsorship agreement with AM.

Proof.

If conditions in Lemma 3 are fulfilled, then requirements demanded by conditions in Lemma 2-4 are not satisfied since $B^{am} - \beta F^{am} + G^T < 0$. On the other hand, conditions in Lemma 5 are satisfied only if $c_2 \geq \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$. Moreover, Pairwise stability requirements for the museum network demanded in lemma 3 are not compatible with the last condition in Lemma 5 for g_{AP}^{HS} P-stability since it requires $c_2 < \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$.²¹ ■

3.5- Hypergraph Stability Results

In this section we examine the second type of agreement decisions by means of network stability assuming that agents' strategy space includes also the possibility to join in a more integrated form of connection such as a collective trademark or any sort of association (i.e., in network terms, a Hyperlink). Such associations are, however, neither exclusive nor comprehensive. Some agents may join in several associations, while others may not join any association. Thus, many associations can have overlapping members.

Joining in an association does not only yield (direct and indirect) benefits, but also incurs some costs. For example, a member should pay the membership fee and perform some duty to sustain his association or bear a minimum quality standard cost. A set of associations is defined as a hypergraph (i.e., a generalization of a graph or a network) in graph theory. The main difference consists in the fact that while a link in a network directly connects only a pair of nodes, an association in a hypergraph connects any number of nodes. What more, while a link in a network can be formed by the joint decision of two players, an association in a hypergraph can be formed by the joint decision of any number of players more than one.

²⁰ See the Appendix A.3 for the description of Lemma 2.

²¹ See the Appendix A.3 for the description of Lemma 3 and 5.

We are interested in the stability of two specific hypergraphs, the complete one (induced cultural district) and the one composed by only one hyperlink between AM and AP (induced museum cultural district)²². We can also consider hypergraphs as a sort of formal contracts induced by public governance. In this way, we can compare the complete network stability and the cultural district hypergraph stability as a comparison between induced district and endogenous district formation.

3.6 Hypergraph Stability (The Induced Cultural District)

We assume that the main difference between a museum cultural district (hyperlink between AM and AP) and the museum system (or network) consist in the degree of integration between the two actors. In this model, to simplify the analysis, we assume that if AP and AM join an association (induced by public governance) as a consequence, they also jointly decide whether to build a hyperlink with T. In other words, they act as a coalition. The main implication of this assumption is that if $d^{ap} = 1$ the number of possible deviations from g^{MCD} (the museum cultural district hypergraph) reduces to two (g^e, g^{CD}) since we can consider only the adjacency matrices with $d^T = d^{am} = 1$ or $d^T = d^{am} = 0$. Let's first modify payoffs considering that if agents in the network decide to belong to the complete hyperlink (cultural district), a collective trademark is formed, leading to gains and implying costs (a minimum quality standard) to the agents. In particular, actors' utilities (3-3.1-4) become:

$$U^{ap} = R^y(y, p_y) + (1 - d^{ap})(1 - d^T)R^{qa}(q_a, p_q) + d^{ap}(1 - d^T)\frac{(R^{qa}(q_a, p_q))}{2} + S(y, d^{ap})^{23} - \left[(1 - d^{ap})(1 - d^T)C(q_a) + d^{ap}(1 - d^T)\left(\frac{c'_1 q_a}{2}\right) + d^{ap}d^T d^{am}F^Q + F^{ap} + C(y) \right] \quad (3')$$

$$U^{am} = d^{ap}\left(S^{MCD} + \frac{(R^{qa} - c'_1 q_a)}{2}\right) + d^{am}\beta F^{am} - (F^{am} + d^{ap}d^T d^{am}F^Q) \quad (3.1')$$

$$U^T = (1 - d^{ap})\Pi^O + d^{ap}\Pi^M + d^{am}(B^{am} - \beta F^{am} + G^T) - d^{ap}d^T d^{am}F^Q \quad (4')$$

We assume that $c'_1 < c_1$ since when a Museum Cultural District is formed (MCD) an adequate dimension allows attainment of economies of scale and variety²⁴. The element S^{MCD} represents the public subsidy obtained to belong to an MCD, F^Q is the cost (we assume it is homogeneous across agents) to preserve a minimum quality standard for the product/service Q in the cultural district g^{CD} . The reaction functions and equilibrium values for the accessory good in the museum cultural district are:

²²The most explicit form of cultural district is that constructed around museum networks or within an artistic community. The density of a museum districts in itself creates systemic effects, which attract visitors and tourists. A museum cultural district is the product of a public policy. As in the case of the institutional cultural district, the basic ingredients for a museum cultural district are the presence of a localized culture embedded in the museums' human capital and collections, and an *institutional* start-up represented by a municipal decision to establish a museum district. (Santagata, 2003).

²³ $S(y, d^{ap}) = \alpha C(y) + d^{ap}S^{MCD}$

²⁴ For example, the provision of commercial services may be improved; the collections can be managed with more positive results; activities in strategic areas can be centralized (Santagata, 2003).

$$R^{ap}: q_a = [a - c_1']y - q_t \quad (20)$$

$$R^t: q_t = \frac{(a-c_2)}{2}y - \frac{q_a}{2} \quad (21)$$

$$q_a^* = y^*(\Psi)(a - 2c_1' + c_2) \quad (22)$$

$$q_t^* = y^*(\Psi)(c_1' - c_2) \quad (23)$$

$$P(q_a^*, q_t^*) = c_1' \quad (24)$$

The analysis of hypergraph stability on the “induced” Cultural District (g^{CD}) formation and on the “induced” Museum Cultural District (g^{MCD}) produces the following results:

Proposition 4. The Hypergraphs g^{CD} and g^{MCD} are stable if $S^{MCD} \geq F^Q > \beta F^{am}$ and $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$

Hypergraph stability requirements are: (i) no player has an incentive to exit from an association unilaterally, and (ii) no coalition of more than one player has the incentive to form a new association collectively.

Proof of Proposition 4.

Since from Lemma 9-10 (refer to the Appendix A.4) the induced cultural district g^{CD} is Stable if $S^{MCD} \geq F^Q$ and $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$ and the induced museum cultural district g^{MCD} is Stable if $F^Q > \beta F^{am}$ stability conditions for g^{CD} and g^{MCD} are simultaneously satisfied if $S^{MCD} \geq F^Q > \beta F^{am}$ and $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$. ■

4. Conclusions

This paper analysed the formation and stability of potential cultural networks among heterogeneous cultural and territorial actors. We assumed there are three cultural stakeholders in a delimited territory. Specifically, a Major (or Principal) Attractor as a well-known cultural institution (e.g., the main Museum in a territory); a Minor Attractor as an unexplored cultural heritage in the same territory and a commercial firm T, supplying touristic services or handcrafted goods (exploiting the cultural heritage proximity). We assume that the three agents first make their link decisions and then they maximize their utilities getting a specific payoff depending by the resulting network structure.

We divided the study of the incentives and modalities of interaction between the players involved in two different analyses in order to characterize and distinguish endogenous cultural network formation (bilateral) and induced cultural systems and districts (multilateral). First, we focused on the stability of a type of agreement connecting agents in the general network, as a collection of bilateral and interdependent exchange contracts. Namely, we analysed the pairwise stability of the cultural network formation game. Second, we focused on some specific form of associative contract that binds all the nodes together for the purpose of a common goal, such as an induced (by public governance) cultural

district or a cultural system. In other words, we analyse the hyperlink stability of a museum cultural district and complete cultural district. We assume that the main difference between a museum cultural district (hyperlink between AM and AP) and the museum system (or network) consist in the degree of integration between the two actors. In this model, to simplify the analysis, we assume that if AP and AM join an association (induced by public governance) therefore, they also jointly decide whether to build a hyperlink with T.

The Pairwise stability results first highlight that the empty network and the two sponsorship configurations (i.e., Firm T as sponsor and the two attractors in turn as exclusive sponsee) are not stable equilibria of the game. The two cultural attractors always have incentive to form a link if there exist a positive public subsidy associated to the museum network formation. On the other hand, the touristic firm always prefers to connect to both attractors in order to exploit the territorial monopoly benefits and the positive externality generated by the increase in the number of visitors in the area (as a result of higher network density).

Thus, the complete network, in which the three actors are connected, is the only P-Stable structure if firm T net benefits from sponsorship are greater than zero, public subsidies S^M are greater than the sponsor contribution βF^{am} (such that AM finds profitable to link also to AP while it is already linked to T), and if firm T marginal costs are greater than a specific threshold value. If marginal costs for firm T are lower than the threshold value, only the hub and spoke configuration with AM as central node is P-stable.

If firm T net benefits from sponsorship are lower than zero the Museum Network and the hub-spoke configuration, with AP as central vertex, are the only stable network structures.

If firm T marginal costs are greater than a specific threshold value, AP and T prefers to sign an agreement when they belong to the museum network. On the other hand, they will not sign the agreement if costs are lower.

The results on the hyperlink stability for the cultural district and the museum cultural district, point out the potential for both the architectures to be stable equilibria of the game. The cultural district hypergraph g^{CD} is stable if public subsidies are greater or equal than the minimum quality standard cost (MQS) since, in this case, the two attractors do not have incentives to exit from the association. The firm T, does not have profitable deviations only if the difference between the monopoly profit in g^{CD} and the oligopoly one (in g^e) exceeds the net costs to belong to the cultural district. Since $\Pi^M(y_3^*) > \Pi^O(y_0^*)$ firm T will not exit the association if $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$. In other words, firm T will not exit the association only if net benefits from AM sponsorship are greater than its costs and MQS costs.

The museum cultural district is also a stable equilibrium since both cultural attractors will not exit the museum cultural district if they get a subsidy greater or equal to zero. On the other hand, AM will not join the grand coalition if MQS cost exceeds sponsorship benefits. The Attractor AP, will not join the Cultural District if MQS cost is greater than the difference between subsidies²⁵ in the two network configurations. In other words, greater the positive impact on number of visitors (due to network density), greater the MQS, for the museum district to be stable.

Our results highlight that in order to reach a stable induced cultural district, public subsidies must be set at the MQS cost and the density effect should also be considered as an important element of stability, especially for the P-Stability of the museum cultural district.

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²⁵ The difference between the two subsidies depend on the number of visitors in each configuration. We assume it is increasing in network density.

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APPENDIX

A.1 The Adjacency Matrix of the cultural network configurations

Each entry of the Adjacency Matrix describes the relationship among the three actors and is defined in the following way:

$$d^{ap}(a_{ij} \equiv a_{ji}) = \begin{cases} 1 & \text{if the link APAM} \in g, \\ 0 & \text{otherwise} \end{cases}$$

$$d^{am}(a_{ij} \equiv a_{ji}) = \begin{cases} 1 & \text{if the link AMT} \in g, \\ 0 & \text{otherwise} \end{cases}$$

$$d^T(a_{ij} \equiv a_{ji}) = \begin{cases} 1 & \text{if the link APT} \in g, \\ 0 & \text{otherwise} \end{cases}$$

Fig. 1 The Adjacency Matrix of AP-AM-T Network

	AP	AM	T
AP	0	d^{ap}	d^T
AM	d^{ap}	0	d^{am}
T	d^T	d^{am}	0

A.2 Networks and Network games definition

The first approach on network theory originated from the *Random Graph* literature. According to this literature, network formation is modelled by specifying a stochastic process or by an algorithmic process through which links are formed (Barabasi, 2003; Vega-Redondo, 2007; Watts, 2001). A second approach uses the *Game Theory* and originated from the economic literature (Bala and Goyal, 2000; Brandes and Erlebach, 2005; Currarini and Morelli 2000; Demange and Wooders, 2005; Dutta and Mutuswami, 1997; Goyal, 2007; Jackson, 2008; Myerson, 1977; Wasserman, Faust and Iacobucci, 1994;).

The economic approach to the network formation model focuses on the resulting equilibrium networks in which links are formed by agents controlling nodes or represented by them. The network strategic formation is modelled according to the theory of games. There are two main characteristics of these kinds of models: a) Agents derive utilities from the network. This means that for every possible emerging network there is a different total welfare; b) The links, formed by agents' choices and the resulting networks, can be predicted through equilibrium concepts or through dynamic stochastic processes. A network g is a list of unordered pairs of players linked to each other. A network game is represented by a set of players N , the value function and the allocation rule. The *value function* (v) and the *allocation rule* (Y) represent respectively the value generated by a specific network and the way in which this value is distributed among players. In the network approach these concepts depend on the global structure of the network. The value function can include costs and benefits to links. A simple way to deal with the network equilibrium is to define a stability notion directly over the networks.

A.2.1 The Pairwise stability and Hypergraph stability concepts

In network theory, every player individually can withdraw a connection (undirected link) and every pair of player can build it. Let $N = \{1, 2, \dots, N\}$ denote a finite set of players. The network relations among players are represented by graphs. The nodes represent players and the edges the relationships between any pair of players.

For simple graphs, the stability condition requires that, in case a link which does not belong to the network exists, and a player find it profitable, then the potential partner of the new link would not find it profitable. In other words, a network is *pairwise stable (PS)* if neither players want to cut a link nor any pair of players simultaneously want to add a new link.

We denote by $(g + ij)$ the graph obtained by adding a link ij to the graph g and by $(g - ij)$ the graph obtained by eliminating a link ij to the graph g .

Formally,

A network g is Pairwise stable with respect to v and Y if:

- (i) For every $ij \in g$, $Y_i(g, v) \geq Y_i(g - ij, v)$ and, $Y_j(g, v) \geq Y_j(g - ij, v)$;
- (ii) For every $ij \notin g$ if $Y_i(g, v) < Y_i(g + ij, v)$ then $Y_j(g, v) > Y_j(g + ij, v)$.

Condition (i) requires that for any pair of linked players neither player has an incentive to cut the link. Condition (ii) requires that for any pair of ij (in g) disjointed players at least one of them has no incentive to form a link with the other. This concept takes into account only one link and two players deviations at most, whereas, a profitable deviation involving *more than one link* could also exist. Thus, a pairwise stable network could still be not immune from multilateral links deviations. Moreover, also a *set of players* together can gain from a reorganization of the links.

According to our aims we need to differentiate the case in which agents build autonomously bilateral links from a situation in which a collective trade mark or territorial systems is created translating it in an association in which decisions are taken together. For this reason, we need a different stability concept applied to Hypergraphs. An Hypergraph H is defined by a family of subset of N , $\{A\}$, (representing an Association among nodes) with $|A| \geq 2$ (where $|A|$ represents the size of the association A). In other words, a Hypergraph is a generalized concept of a graph (or a network) because it includes A with $|A| = 2$. Associations are not mutually exclusive, so that a player can participate in more than one association. If $A \subsetneq B$ for some $A, B \in H$, we call A the sub association of B . We follow the hypergraph stability concept defined by (Tackseung-Jeong-Yoo, 2009): a restrictive feature of the stability is that it allows

only a deviation to exit by a single player and a deviation to form a new association with maintaining current associations. It is a weaker stability condition with respect the strong stability notion. Strong stability allows more deviations since it allows any kind of deviation by any coalition.

We denote by $(H + A)$ the hypergraph obtained by adding an association A to H and by $(H - A)$ the hypergraph obtained by eliminating an association A from H . Also, we use the notation of $(H - i(A))$ to mean the hypergraph obtained by player i 's exit from the association A .

A Hypergraph H is stable if

- (i) For any $A \in H$ and for any $i \in A$, $Y_i(H) \geq Y_i(H - i(A))$ and,
- (ii) For any $S \notin H$ and for any $i \in S$, $Y_i(H + S) > Y_i(H)$ implies $Y_j(H + S) < Y_j(H)$ for some $j \in S$.

In words, the stability of a hypergraph requires that no player has an incentive to exit from an association unilaterally, and that no coalition of more than one player has the incentive to form a new association collectively.

A.3 Pairwise Stability Results

Lemma 1. The empty network g^e is not *PS* since the two attractors have always incentive to link together if $S^M > 0$.

Proof.

The empty network is pairwise stable if (i) for any pair of linked players neither player has an incentive to cut the link and (ii) for any pair of disjointed players at least one of them has no incentive to form a link with the other. Condition (i) it is always satisfied since in the empty network linked couple are not present. Conditions (ii) are satisfied if each of the following condition on the three potential linked couples are satisfied:

(ii)

1.1 If $U^{am}(g^e) < U^{am}(g^M)$ (for $S^M > 0$) then

$$U^{AP}(g^e) > U^{AP}(g^M) \text{ (for } S^M < 0 \text{)}^{26};$$

1.2 If $U^{am}(g^e) < U^{am}(g_{AM}^S)$ (for $\beta F^{am} > 0$) then $U^T(g^e) > U^T(g_{AM}^S)$

$$\text{(for } (G^T + B^{am}) < \beta F^{am} \text{ with } c_1 < c_2 \text{);}$$

1.3 If $U^{AP}(g^e) < U^{AP}(g_{AP}^S)$ (always satisfied) then $U^T(g^e) > U^T(g_{AP}^S)$

²⁶ The condition is $S^M < \alpha c_y(Y_0^* - Y_1^*)$ where Y_0^*, Y_1^* represent respectively the number of visitors in the network with zero links and with one link. Since $Y(\Psi)$ is increasing in the number of links $(Y_0^* - Y_1^*) < 0$ and the condition is satisfied also for $S^M < 0$.

$$\left(\text{for } \underline{c}_1^{1.3} < c_1 < \overline{c}_1^{1.3} \right)^{27}.$$

Conditions (ii) fail in 1.1, since if $U^{am}(g^e) < U^{am}(g^M)$ we have also that $U^{AP}(g^e) < U^{AP}(g^M)$ (for $S^M > 0$). In other words, the two attractors have always incentive to link together for any $S^M > 0$. ■

Lemma 2. The Complete network g^C is PS if $c_1 \leq \frac{a\sqrt{Y_3^*}}{2\sqrt{Y_2^*}}$; $B^{am} - \beta F^{am} + G^T \geq 0$ and $S^M \geq \beta F^{am} \geq 0$.

Proof.

Condition (ii) it is always satisfied since each couple is linked. Condition (i) is satisfied if each of the following conditions on the three linked couples are satisfied:

- (i)
- 2.1 $U^{am}(g^C) \geq U^{am}(g_T^{HS})$ (for $S^M \geq \beta F^{am}$) and $U^{AP}(g^C) \geq U^{AP}(g_T^{HS})$ (for $S^M \geq 0$);
- 2.2 $U^{am}(g^C) \geq U^{am}(g_{AP}^{HS})$ (always satisfied) and $U^T(g^C) \geq U^T(g_{AP}^{HS})$ (for $B^{am} - \beta F^{am} + G^T \geq 0$);
- 2.3 $U^{AP}(g^C) \geq U^{AP}(g_{AM}^{HS})$ (always satisfied) and $U^T(g^C) \geq U^T(g_{AM}^{HS})$ (satisfied for $c_2 \geq \left(\frac{2c_1\sqrt{Y_2^*} - a\sqrt{Y_3^*}}{2\sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$)

Conditions 2.1, are simultaneously satisfied for $S^M \geq \beta F^{am} \geq 0$. Conditions 2.2 are satisfied for $B^{am} - \beta F^{am} + G^T \geq 0$. Condition 2.3 is satisfied for $c_2 \geq \left(\frac{2c_1\sqrt{Y_2^*} - a\sqrt{Y_3^*}}{2\sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$, moreover, if $Y_2^* > \frac{Y_3^*}{4}$ and $c_1 \leq \frac{a\sqrt{Y_3^*}}{2\sqrt{Y_2^*}}$ it is always satisfied for $c_2 \geq 0$ ■

Lemma 3. The network g^M is not PS if $B^{am} - \beta F^{am} + G^T \geq 0$

Proof.

Condition (i) is satisfied if the condition on the linked couple is fulfilled:

- (i)
- 3.1 $U^{AP}(g^M) \geq U^{AP}(g^e)$ (always satisfied) and $U^{am}(g^M) \geq U^{am}(g^e)$ (for $S^M \geq 0$);

Condition (ii) demands to fulfil the following conditions on disjoint couples:

- (ii)
- 3.2 If $U^{AP}(g_{AP}^{HS}) > U^{AP}(g^M)$, (always satisfied) then $U^T(g_{AP}^{HS}) < U^T(g^M)$ (for $c_2 < \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$)

²⁷ $\left(\overline{c}_1^{1.3} < c_1 < \underline{c}_1^{1.3} \right) = (c_2 Y_0^*) + \sqrt{(c_2 Y_0^*)^2 + Y_0^* Y_1^* (c_2 - a)^2} < c_1 < (c_2 Y_0^*) - \sqrt{(c_2 Y_0^*)^2 + Y_0^* Y_1^* (c_2 - a)^2}$

3.3 If $U^{am}(g_{AM}^{HS}) > U^{am}(g^M)$ (always satisfied) then $U^T(g_{AM}^{HS}) < U^T(g^M)$ (never satisfied for $B^{am} - \beta F^{am} + G^T \geq 0$)

Conditions (i) are satisfied for $S^M \geq 0$. Condition 3.3 (ii) it is never satisfied since the minor attractor always has an incentive to link to firm T when it belongs to the museum network. On the other hand, firm T accepts to link with AM if the benefits from the sponsorship agreement are greater than the costs to build it. The major attractor AP has always incentives to link to T since the number of visitors increase if the network expands. As a consequence, the museum network is not pairwise stable, if $B^{am} - \beta F^{am} + G^T \geq 0$. ■

Lemma 4. The network g_{AM}^{HS} is PS if $S^M \geq 0$, $(B^{am} - \beta F^{am} + G^T \geq 0)$ and $c_2 < \left(\frac{2 c_1 \sqrt{Y_2^*} - a \sqrt{Y_3^*}}{2 \sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$

Proof.

Condition (i) is satisfied if conditions on the linked couples are fulfilled:

- (i)
- 4.1 $U^{AP}(g_{AM}^{HS}) \geq U^{AP}(g_{AM}^S)$ (always satisfied) and $U^{am}(g_{AM}^{HS}) \geq U^{am}(g_{AM}^S)$ (for $S^M \geq 0$);
- 4.2 $U^{am}(g_{AM}^{HS}) \geq U^{am}(g^M)$ (always satisfied) and $U^T(g_{AM}^{HS}) \geq U^T(g^M)$ (for $B^{am} - \beta F^{am} + G^T \geq 0$)

Condition (ii) demands to fulfil the following conditions on the disjoint couple:

- (ii)
- 4.3 If $U^{AP}(g^C) > U^{AP}(g_{AM}^{HS})$, (always satisfied) then $U^T(g^C) < U^T(g_{AM}^{HS})$ ($c_2 < \left(\frac{2 c_1 \sqrt{Y_2^*} - a \sqrt{Y_3^*}}{2 \sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$).

Conditions 4.1, 4.2 are simultaneously satisfied for $S^M \geq \beta F^{am}$; and $B^{am} - \beta F^{am} + G^T \geq 0$.

Conditions in 4.3 are satisfied if $c_2 < \left(\frac{2 c_1 \sqrt{Y_2^*} - a \sqrt{Y_3^*}}{2 \sqrt{Y_2^*} - \sqrt{Y_3^*}} \right)$ with $Y_2^* > \frac{Y_3^*}{4}$ and $c_1 > \frac{a \sqrt{Y_3^*}}{2 \sqrt{Y_2^*}}$ ■

Lemma 5. The network g_{AP}^{HS} is not PS if $B^{am} - \beta F^{am} + G^T \geq 0$

Proof.

Conditions (i) are fulfilled if the following inequalities on the linked couples utilities are simultaneously satisfied:

- (i)
- 5.1 $U^{AP}(g_{AP}^{HS}) \geq U^{AP}(g_{AP}^S)$ (always satisfied) and $U^{am}(g_{AP}^{HS}) \geq U^{am}(g_{AP}^S)$ (always satisfied)
- 5.2 $U^{AP}(g_{AP}^{HS}) \geq U^{AP}(g^M)$ (always satisfied) and

$$U^T(g_{AP}^{HS}) \geq U^T(g^M) \left(\text{for } c_2 \geq \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right) \right)$$

Condition (ii) demands to fulfil the following conditions on the disjoint couple:

(ii)

5.3 If $U^{am}(g^C) > U^{am}(g_{AP}^{HS})$, (always satisfied) then $U^T(g^C) < U^T(g_{AP}^{HS})$ (never satisfied for $B^{am} - \beta F^{am} + G^T \geq 0$)

Conditions (i) are fulfilled if $c_2 \geq \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$. Conditions (ii) are not satisfied if benefits from the sponsorship agreement are greater than their costs, since firm T and AM will decide to build a link, moving to the complete network configuration. ■

Lemma 6. The network g_{AM}^S is not PS since AP and AM find profitable to form a link deviating to the network g_{AM}^{HS} .

Proof.

Condition (i) is satisfied if conditions on the linked couple is fulfilled:

(i)

6.1 $U^{am}(g_{AM}^S) \geq U^{am}(g^e)$ (for $\beta F^{am} \geq 0$) and
 $U^T(g_{AM}^S) \geq U^T(g^e)$ (for $(G^T + B^{am}) \geq \beta F^{am}$);

Condition (ii) demands to fulfil the following conditions on disjoint couples:

(ii)

6.2 If $U^{am}(g_{AM}^{HS}) > U^{am}(g_{AM}^S)$ (always satisfied) then $U^{AP}(g_{AM}^{HS}) < U^{AP}(g_{AM}^S)$ (never satisfied)

6.3 If $U^{AP}(g_T^{HS}) > U^{AP}(g_{AM}^S)$, (always satisfied) then $U^T(g_T^{HS}) < U^T(g_{AM}^S)$
 $\left(\text{for } c_2 < \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right) \right)$

Conditions 6.2 cannot be satisfied since AM and AP have incentives to build a link when they belong to the network g_{AM}^S . ■

Lemma 7. The network g_{AP}^S is not PS since there exist a profitable deviation for AP and AM, deviating to g_{AP}^{HS} .

Proof.

Condition (i) is satisfied if conditions on the linked couple is fulfilled:

(i)

7.1 $U^{AP}(g_{AP}^S) \geq U^{AP}(g^e)$ (always satisfied) and $U^T(g_{AP}^S) \geq U^T(g^e)$ (for $\overline{c_2}^{7.1} < c_2 < \underline{c_2}^{7.1}$)²⁸;

Condition (ii) requires to fulfil the following conditions on disjoint couples:

(ii)

7.2 If $U^{AP}(g_{AP}^{HS}) > U^{AP}(g_{AP}^S)$ (always satisfied) then $U^{am}(g_{AP}^{HS}) < U^{am}(g_{AP}^S)$ (never satisfied)

7.3 If $U^{am}(g_T^{HS}) > U^{am}(g_{AP}^S)$ (for $\beta F^{am} > 0$), then $U^T(g_T^{HS}) < U^T(g_{AP}^S)$ (never satisfied)

Conditions (ii) fail in condition 7.2 since AP and AM have always incentives to link if AP is already linked to T. ■

Lemma 8. The network g_T^{HS} is not PS since there exist a profitable deviation for AP and AM to form a link deviating to the complete network g^C .

Proof.

Condition (i) is satisfied if the following requirements on linked couples are fulfilled:

(i)

8.1 $U^{AP}(g_T^{HS}) \geq U^{AP}(g_{AM}^S)$ (always satisfied) and $U^T(g_T^{HS}) \geq U^T(g_{AM}^S)$ for $c_2 \geq \left(\frac{2c_1\sqrt{Y_1^*} - a\sqrt{Y_2^*}}{2\sqrt{Y_1^*} - \sqrt{Y_2^*}} \right)$

8.2 $U^{am}(g_T^{HS}) \geq U^{am}(g_{AP}^S)$ (for $\beta F^{am} \geq 0$) and $U^T(g_T^{HS}) \geq U^T(g_{AP}^S)$ (for $B^{am} - \beta F^{am} + G^T \geq 0$)

Condition (ii) demands to fulfil the following requirements on the disjoint couple:

(ii)

8.3 If $U^{AP}(g^C) > U^{AP}(g_T^{HS})$, (always satisfied) then $U^{am}(g^C) < U^{am}(g_T^{HS})$ (never satisfied)

Conditions (ii) fail since AP and AM utilities increase if they build a link when they belong to network g_T^{HS} . ■

A.4 Proof of Proposition 4

Proposition 4. The Hypergraphs g^{CD} and g^{MCD} are stable if $S^{MCD} \geq F^Q > \beta F^{am}$ and $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$

²⁸ $\left(\overline{c_2}^{7.1} < c_2 < \underline{c_2}^{7.1} \right) = \left(\overline{c_2}^{6.3} < c_2 < \underline{c_2}^{6.3} \right) = (a - 2c_1) + \sqrt{2c_1(4c_1 - a)} < c_2 < (a - 2c_1) - \sqrt{2c_1(4c_1 - a)}$

Hypergraph stability requirements are: (i) no player has an incentive to exit from an association unilaterally, and (ii) no coalition of more than one player has the incentive to form a new association collectively. In order to demonstrate Proposition 4, we first introduce Lemma 9 and 10.

Lemma 9. The induced cultural district g^{CD} is Stable if $S^{MCD} \geq F^Q$ and $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$

Proof of lemma 9.

Assume that the induced cultural district is built up only if all the agents participate, otherwise it decays. Players, belonging to the Cultural District hypergraph g^{CD} will not exit the association if:

(i)

$$9.1 \quad U^{AP}(g^{CD}) \geq U^{AP}(g^e) \text{ (for } S^{MCD} \geq F^Q \text{)};$$

$$9.2 \quad U^T(g^{CD}) \geq U^T(g^e) \text{ (for } \Pi^M(y_3^*) - \Pi^O(y_0^*) \geq (F^Q + \beta F^{am}) - (B^{am} + G^T) \text{)}^{29};$$

$$9.3 \quad U^{am}(g^{CD}) \geq U^{am}(g^e) \text{ (for } S^{MCD} + \beta F^{am} \geq F^Q \text{)}.$$

In general, in Hypergraphs, associations are not mutually exclusive, so that a player can participate in more than one association. In the grand-coalition, there are no other association to join since any other possible association among the three players, different from the cultural district, will be a sub-association. Therefore, the hypergraph g^{CD} is stable if conditions 9.1 to 9.3 are simultaneously fulfilled. The two attractors do not have incentives to exit from the association if public subsidy is greater than the minimum quality standard (MQS) cost ($S^{MCD} \geq F^Q$). The firm T, does not have profitable deviations only if the difference between the monopoly profit in g^{CD} and the oligopoly one (in g^e) exceeds the net costs to belong to the cultural district. Moreover, the condition 9.2 is always satisfied if $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$, given that $\Pi^M(y_3^*) > \Pi^O(y_0^*)$. ■

Lemma 10. The induced museum cultural district g^{MCD} is Stable if $F^Q > \beta F^{am}$

Proof of lemma 10.

The main difference between the hyperlink and the bilateral link between AP and AM, in this model, is that in the latter the two agents preserve a decisional autonomy, the former, instead, represents a higher form of integration. In other words, if a hyperlink between AP and AM is formed, they behave as a single node. Players belonging to the Museum Cultural District hypergraph g^{MCD} will not exit the association if:

$$^{29} \Pi^M(y_3^*) = \frac{(a-c_2)^2}{4} Y_3^*; \Pi^O(y_0^*) = (c_1 - c_2)^2 Y_0^*$$

(i)

$$10.1 \quad U^{AP}(g^{MCD}) \geq U^{AP}(g^e) \text{ (always satisfied);}$$

$$10.2 \quad U^{am}(g^{MCD}) \geq U^{am}(g^e) \text{ (always satisfied)}$$

(ii)

10.3 If $U^T(g^{CD}) > U^T(g^{MCD})$ (for $\Pi^M(y_3^*) - \Pi^{O'}(y_1^*) \geq (F^Q + \beta F^{am}) - (B^{am} + G^T)$ ³⁰; then either

$$U^{AP}(g^{CD}) < U^{AP}(g^{MCD}) \text{ (for } S(Y_3^*) - S(Y_1^*) < F^Q \text{)}^{31} \text{ or}$$

$$U^{am}(g^{CD}) < U^{am}(g^{MCD}) \text{ (for } F^Q > \beta F^{am} \text{)}$$

From condition (i) we have that both cultural attractors agents will not exit the museum cultural district if they get a subsidy greater or equal to zero. On the other hand, by condition (ii), AM will not join the grand coalition if it expects to bear MQS cost that exceeds sponsorship benefits. The Attractor AP, will not join the Cultural District if MQS cost is greater than the difference between subsidies in the two network configurations. ■

Proof of Proposition 4.

Consider Lemma 9 and 10, it is straightforward to show that stability conditions for g^{CD} and g^{MCD} (respectively 9.1 to 9.3 and 10.1 to 10.3) are simultaneously satisfied if $S^{MCD} \geq F^Q > \beta F^{am}$ and $(B^{am} + G^T) \geq (F^Q + \beta F^{am})$. ■

³⁰ $\Pi^{O'}(y_1^*) = (c_1' - c_2)^2 Y_1^*$

³¹ $S(Y_3^*) = \alpha c_y Y_3^* + S^{MCD}$; $S(Y_1^*) = \alpha c_y Y_1^* + S^{MCD}$